A trust induced recommendation mechanism for reaching consensus in group decision making

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Abstract

This article addresses the inconsistency problem in group decision making caused by disparate opinions of multiple experts. To do so, a trust induced recommendation mechanism is investigated to generate personalised advices for the inconsistent experts to reach higher consensus level. The concept of trust degree (TD) is defined to identify the trusted opinion from group experts, and then the visual trust relationship is built to help experts 'see' their own trust preferences within the group. Consequently, trust based personalised advices are generated for the inconsistent experts to revisit their opinions. To model the uncertainty of experts, an interval-valued trust decision making space is defined. It includes the novel concepts of interval-valued trust functions, interval-valued trust score (IVTS) and interval-valued knowledge degree (IVKD). The concepts of consensus degree (CD) between an expert and the rest of experts in the group as well as the harmony degree (HD) between the original opinion and the revised opinion are developed for interval-valued trust functions. Combining HD and CD, a more reasonable policy for group consensus is proposed as it should arrive at the threshold value with the maximum value of harmony and consensus degrees simultaneously. Furthermore, because the trust induced recommendation mechanism focuses on changing inconsistent opinions using only opinions from the trusted experts and not from the distrusted ones, the HD based changes cost to reach the threshold value of consensus is lower than previous mechanisms based on the average of the opinion of all experts. Finally, once consensus has been achieved, a ranking order relation for interval-valued trust functions is constructed to select the most appropriate alternative.

Keywords: Group decision making, Recommendation mechanism, Group consensus, Trust degree, Harmony degree

1. Introduction

Group decision making (GDM) problems address decision situations in which a group of experts express preferences, and are aggregated a collective one to to derive a common solution. However, the GDM problems generally involve the situations of inconsistency among group experts. Consequently,

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it is preferable that the set of experts reach agreement before applying aggregation process [2, 28, 31, 44, 47, 58]. Therefore, one key issue to be addressed in GDM is how to deal with inconsistency caused by different and possibly disparate opinions. Inconsistency is usually resolved by consensus process [4, 8, 24, 45, 46, 51] in an effort to achieve a high enough degree of agreement between the set of experts in the group [1, 22, 40, 43, 55]. Most of the developed consensus models incorporate recommendation mechanisms to provide advice to the inconsistent experts with low consensus level in order to increase and ultimately reach a higher acceptable consensus degree by the group of experts [9, 27, 29, 30, 37, 50]. These traditional recommendation mechanisms generate advices using the arithmetic average of opinion derived from individual expert in the group, and then the existence of trust relationship among the experts is not taken into account. Consequently, the inconsistent experts are implicitly forced to implement the given advices without considering whether they trust them or not.

However, trust has been considered an important factor influencing the consensus process in group decision making [12, 17, 23, 32, 33]. Therefore, trust can be used to encourage group experts to reach consensus, and then we should investigate a trust induced recommendation mechanism. To do that, this article defines the concept of trust degree (TD), which is used to build a visual trust relationship to help experts 'see' their own trust preferences within the group. According to this trust relationship, personalised advices are generated for the subset of experts classed as inconsistent. Distrusted opinions are eliminated from these personalised advices, which help their willingly implementation by the inconsistent experts to achieve higher consensus levels. Furthermore, the achievement of the threshold value of consensus via the proposed trust induced recommendation mechanism is less expensive in terms of number of opinion changes than traditional recommendation mechanism based on just the average group opinion. Consequently, the consensus process for GDM developed in this paper is really induced by trust, a diversity of disparate opinions can be unified consistently to lead to high group experts' accessibility.

Furthermore, trust is also an useful technology to deal with uncertainty in GDM. Uncertainty is caused by some qualitative non-functional properties, and most of the existing methods model uncertainty by using fuzzy sets with membership functions of type-1, i.e. membership functions with crisp outputs [5, 6, 25, 48]. However, these fuzzy methods are not enough to capture the uncertainty nature because they are unable to express subjective opinions such as 'trust' or 'distrust' [42]. Therefore, the second objective of this paper is to investigate an interval-valued trust decision making space in which the membership functions of 'trust' and/or 'distrust' are expressed by interval-valued numbers. Specifically, this interval-valued trust decision making space has the ability to express more uncertainty in GDM, such as 'hesitancy' when trust membership and distrust membership sum is less than one, and 'conflict' when trust membership and distrust membership sum exceeds one. In detail, the concepts of interval-valued trust score (IVTS) and interval-valued trust knowledge degree (IVTKD) are defined, and then a strict trust ranking order relation of interval-valued trust function is built. Consequently, the proposed interval-valued trust decision making space is suitable to deal with uncertainty in GDM with the following four tuple information: trust, distrust, hesitancy and conflict.

The rest of paper is set out as follows: Section 2 introduces some definitions associated to the interval-valued trust decision making space: interval-valued trust functions, IVTS and IVTKD. Then, IVTS and IVTKD are combined to build a ranking order relation of interval-valued trust functions. Section 3 proposes the definition of consensus degree on three levels of opinion/preferences and the identification of experts and elements values that contribute less to consensus. The TD concept is defined to construct individual trust relationship, and then a trust induced recommendation mechanism is built to generate personalised advice to the inconsistent experts so that higher consensus is achieved. The concepts of consensus degree (CD) between an expert and the rest of experts in the group as well as the harmony degree (HD) between the original opinion and the revised opinion are developed for interval-valued trust functions. Therefore, the proposed trust induced recommendation mechanism guarantees that it arrives at the threshold value with high consensus and harmony degrees, respectively. Finally, Section 4 provides an analysis of the proposed consensus model highlighting the main differences with respect to traditional consensus models, and then conclusions are drawn.

2. Interval-valued trust decision making space

As aforementioned, fuzzy sets (FSs) are regarded as a useful tool to model uncertainty in the process of decision making [57]. However, the basic component of a FS to model uncertainty is its membership function, although it has been argued its applicability limitation in decision making contexts where it is required to deal with propositions that could be stated as either true, or false, or that it is unknown whether it is true or false. Therefore, it could also be argued its inability to model appropriately vague statements that are assessed using the concepts of 'trust' and 'distrust'. The concept of trust function has been regarded in [34, 35] as a reliable tool to deal with agents' vague by trust degree and distrust degree. Considering that multiple experts might have fuzzier and more uncertainty opinions about alternatives as previously said, this article aims to investigate an interval-valued trust score space in which the trust degree and distrust degree are expressed by interval-valued numbers rather than trip values as FSs allow to. To do so, we first introduce the definition of interval-valued trust functions as follows.

***Definition* 1 (Interval-valued trust functions).** A tuple $\lambda=\left(\tilde{t},\tilde{d}\right) = \left(\left[t^-,t^+\right],\left[d^-,d^+\right]\right)$, with first component $\tilde{t}$ representing a trust degree and second component $\tilde{d}$ a distrust degree such that $0 \leq t^- \leq t^+ \leq 1$, $0 \leq d^- \leq d^+ \leq 1$, will be called an interval-valued trust function. The set of interval-valued trust functions will be denoted by

$$\Lambda=\left\{\lambda=\left(\tilde{t},\tilde{d}\right)\middle|\tilde{t},\tilde{d}\subseteq[0,1]\right\}$$

By the above definition of interval-valued trust functions, an interval-valued trust decision making space (IVTDMS) can be established to describe the possible different types of decision making information:

***Definition* 2 (Interval-valued trust decision making space (IVTDMS)).** The interval-valued trust decision making space consists of the following three elements: the set of interval-valued trust functions (Λ), a trust hesitancy space (THS) and a trust conflict space (TCS). It is formally represented as

$$IVTDMS^{\square} = (\Lambda, THS, TCS)$$

with

$$THS = \left\{\lambda \in \Lambda | t^{+} + d^{+} \leq 1\right\}$$

and

$$TCS = \left\{\lambda \in \Lambda | t^{-} + d^{-} > 1\right\}$$

THS involves the following type of information: trust, distrust and hesitancy, while TCS involves a different type of decision information: trust, distrust and conflict. Obviously, $IVTDMS$ comprises THS and TCS simultaneously, and therefore, the possible alternatives in a GDM problem can be evaluated by the above four tuple of information: trust, distrust, hesitancy and conflict.

To compare interval-valued trust functions with interval-valued intuitionistic fuzzy set (IVIFS)[3], we first introduce its definition.

***Definition* 3 (Interval-Valued IFS (IVIFS)).** Let $INT([0,1])$ be the set of all closed subintervals of the unit interval and X be a universe of discourse. An interval-valued IFS (IVIFS) A over X is given as:

$$A = \left\{ \langle x, \widetilde{\mu}_A(x), \widetilde{\nu}_A(x)\rangle \,|x \in X\right\} \tag{1}$$

where $\widetilde{\mu}_A(x), \widetilde{\nu}_A(x) \in INT([0,1])$, represent the membership and the non-membership degrees of the element x to the set A subject to the following constraint

$$0 \leq \sup \widetilde{\mu}_A(x) + \sup \widetilde{\nu}_A(x) \leq 1, \forall x \in X.$$

Denoting by $\widetilde{\mu}_{AL}(x)$, $\widetilde{\mu}_{AU}(x)$, $\widetilde{\nu}_{AL}(x)$ and $\widetilde{\nu}_{AU}(x)$ the lower and upper end points of $\widetilde{\mu}_A(x)$ and $\widetilde{\nu}_A(x)$, respectively, an IVIFS also can be represented as

$$\begin{aligned} A = \Big\{ \langle x, [\widetilde{\mu}_{AL}(x), \widetilde{\mu}_{AU}(x)], [\widetilde{\nu}_{AL}(x), \widetilde{\nu}_{AU}(x)]\rangle \Big| \\ x \in X : 0 \leq \widetilde{\mu}_{AU}(x) + \widetilde{\nu}_{AU}(x) \leq 1, \widetilde{\mu}_{AL}(x) \wedge \widetilde{\nu}_{AL}(x) \geq 0 \Big\} \end{aligned} \tag{2}$$

Then, the hesitancy degree function of an IVIFS is calculated as:

$$\widetilde{\pi}_A(x) = [1 - \widetilde{\mu}_{AU}(x) - \widetilde{\nu}_{AU}(x), 1 - \widetilde{\mu}_{AL}(x) - \widetilde{\nu}_{AL}(x)], \quad \widetilde{\pi}_A(x) \subseteq [0,1]. \tag{3}$$

Obviously, IVIFS can be used to represent three tuples of decision making information: membership, non-membership and hesitation, which is similar to THS. However, IVIFS is different to TCS because this last one can deal with conflict decision information. Therefore, the proposed interval-valued trust functions can be regarded as a generalization of IVIFS.

To determine the most optimal alternative, we need propose a new ranking method for interval-valued trust functions. First, the concept of trust score and knowledge degree associated to interval-valued trust functions are defined as follows.

***Definition* 4 (Interval-valued Trust Score (IVTS) Function).** The mapping on the set of interval-valued trust functions, Λ:

$$IVTS(\lambda) = \frac{t^- + t^+ - d^- - d^+}{2} \tag{4}$$

is called the interval-valued trust score function. $IVTS(\lambda) \in [-1, 1]$ represents the normalised dominance that the trust value has over the corresponding distrust value of an interval-valued trust function value of an expert, i.e. the strict trust value contained in an interval-valued trust function.

If two experts have the same IVTS, the uncertainty degree associated to their respective interval-valued trust functions as represented in the following definition can be used to further differentiate them.

***Definition* 5 (Interval-valued Knowledge Degree (IVKD)).** The interval-valued knowledge degree is a mapping on the ser of interval-valued trust functions, Λ as follows:

$$IVKD(\lambda) = \left| 1 - \frac{t^- + t^+ + d^- + d^+}{2} \right| \tag{5}$$

where $IVKD(\lambda) \in [0, 1]$. If $IVKD(\lambda) = 0$, then it has perfect knowledge or complete trust state, otherwise there exists trust knowledge uncertainty. Thus, IVKD is a supplement to IVTS in ranking interval-valued trust functions.

Notice that when information is an interval-value trust function as for example $([0.6, 0.8], [0.5, 0.7])$, we calculate $\widetilde{\pi}_A(x) = [-0.5, -0.1] \nsubseteq [0, 1]$, which clearly does not represent the hesitancy degree of an interval-valued intuitionistic fuzzy set. Therefore, this case is not appropriate for the three tuples of information: trust, distrust and hesitancy, but for another type of decision information: trust, distrust and conflict.

By combining IVTS and IVKD, an interval-valued trust order space is defined as a model that allows to compare and preserve information about the provenance of interval-valued trust functions as follow:

***Definition* 6 (Interval-valued Order Space (IVTOS)).** An interval-valued trust order space

$$IVTOS^{\square} = (\Lambda, \leq_{IVTS}, \leq_{IVKD}, \neg)$$

Consists of the set of interval-valued trust functions, a trust ordering $\leq_{IVTS}$, a knowledge ordering $\leq_{IVKD}$, and a negation operator $\neg$ that verify the following properties

$$\begin{aligned} &\lambda_1 \leq_{IVTS} \lambda_2 \quad \text{iff} IVTS_1 \leq IVTS_2 \\ &\lambda_1 \leq_{IVKD} \lambda_2 \quad \text{iff} IVKD_1 \geq IVKD_2 \\ &\neg(\tilde{t}, \tilde{d}) = (\tilde{d}, \tilde{t}) \end{aligned}$$

In the above interval-valued order space, IVTSs are used to evaluate the degree of strict trust an expert may have on alternatives under one criterion when providing his interval-valued trust functions, while IVKDs are designed to determine the uncertainty contained in the associated interval-valued trust functions. Their role for ranking interval-valued trust functions is similar to the mean and the variance in statistics. Therefore, the interval-valued order space has the following order relation on the set of interval-valued trust functions, Λ, to be defined:

***Definition* 7 (Order relation of Interval-valued Trust Functions).** Given two interval-valued trust functions, λ_1 and λ_2, λ_1 precedes λ_2,

$$\lambda_1 \prec \lambda_2$$

if and only if one of the following conditions is true:

1. $IVTS(\lambda_1) < IVTS(\lambda_2)$;
2. $IVTS(\lambda_1) = IVTS(\lambda_2) \wedge IVKD(\lambda_1) > IVKD(\lambda_2)$.

When comparing two trust scores, the one with the higher trust score function is ordered higher, and in case of equal trust score functions, the lower knowledge degree prevails. Therefore, this order relation makes a contribution to the decision making problem with IVTS information. It can determine the final optimized alternative.

3. Trust based recommendation mechanism for group consensus

As aforementioned, in the group decision making process there exists inconsistency caused by different experts' opinions, and consequently, it is preferable that the set of experts reach consensus before aggregating individual opinions into a collective one. To do that, this article defines the consensus degree (CD) associated to interval-valued trust functions at three levels: (1) decision matrix; (2) alternatives; and (3) element values. When the consensus degree reaches a threshold value, agreed by the group of experts, the resolution process of the GDM is carried out; otherwise the inconsistent experts (with a consensus degree below the threshold value) are identified and a recommendation mechanism is activated to provide advices to improve their consensus degree. Most of the existing recommendation advices are produced based on the arithmetic average of all individual opinions [9, 29, 30, 37, 50]. It could be argued that inconsistent experts are forced to implement recommendation advices and make changes in their opinion as these recommendation mechanisms neglect trust relationship among the group of experts. However, in practice, the individual expert might have different trust degree with other experts, and therefore a more reasonable and suitable policy should rest on this premise and, consequently, it should allow the experts to revisit his/her evaluations according to the advice from the experts the trust.

To achieve this aim, a trust based consensus model is here developed to help experts 'see' their relative trust position within the group, and then build the trust relationship for individual expert. By doing this, the inconsistent experts can implement the advice from their trusted others, while they can select or not consider at all the advice provided by other experts they do not trust enough, i.e. experts with a trust degree below a fixed threshold value. Hence, the personalised advices are produced by this trust induced recommendation mechanism to help the inconsistent increase consensus. Finally, a visual graphical simulation of future consensus status if the trust based personalised advices were to be implemented is provided. In the light of this visual extra information, the inconsistent experts can revisit their evaluations and make changes if considered appropriate to increase consensus.

The trust induced group consensus decision making model interval-valued trust functions is depicted in Fig.1. Specifically, it consists of the following four steps: (1) Constructing the interval-valued trust decision making space; (2) Determining the consensus degree at three levels; (3) Visual consensus identification, trust induced recommendation and rationality analysis; and (4) Selection Process. The first step has already been covered in Sections 2. The remaining steps will be presented in more detail in the following subsections. A step-by-step example is also provided to illustrate the computation processes involved in each step. For the sake of simplicity, a low number of experts and alternatives are assumed.

3.1. The definition of consensus degree (CD) with interval-valued trust functions

To evaluate agreement within a group of experts could be facilitated with the provision of a measurement of the consensus level [11, 26]. According to the decision information expressed by interval-valued trust functions, we will introduce the definition of consensus degree (CD) on three levels: Consensus degree on elements of alternatives, Consensus degree on alternatives and Consensus degree on the decision matrix.

Level 1. *Consensus degree on elements of alternatives.* The trust degree between experts U^h and U^k on the elements of alternatives x_i under attribute c_j is:

$$CE_{ij}\left(\tilde{R}^h, \tilde{R}^k\right) = 1 - \frac{\left|t_{ij}^{h-} - t_{ij}^{k-}\right| + \left|t_{ij}^{h+} - t_{ij}^{k+}\right| + \left|d_{ij}^{h-} - d_{ij}^{k-}\right| + \left|d_{ij}^{h+} - d_{ij}^{k+}\right|}{4} \tag{6}$$

Then, the average consensus degree of the expert U^h to the group on the alternatives x_i under attribute c_j is defined as:

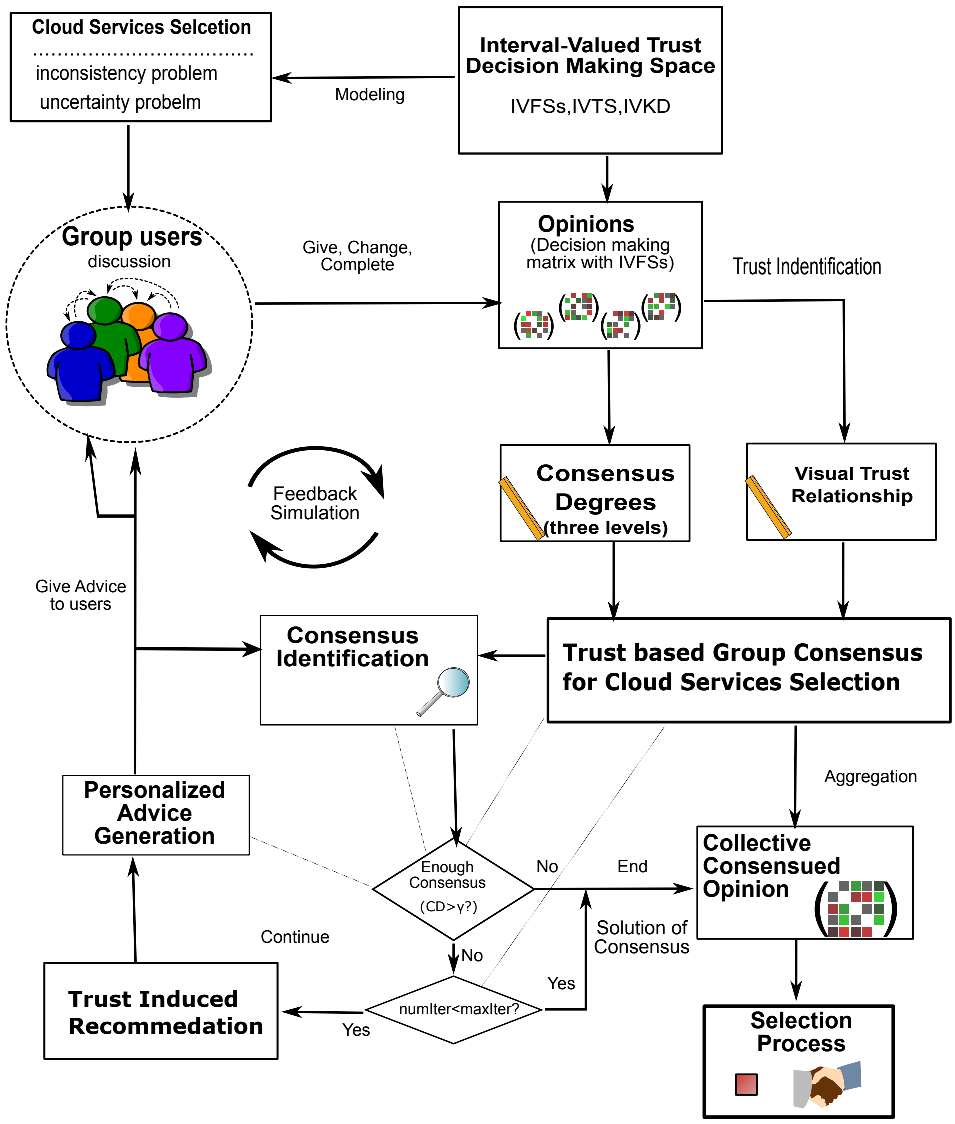


Figure 1: Trust induced recommendation mechanism for group consensus

$$ACE_{ij}^{h} = \frac{1}{l-1} \sum_{h \neq k, k=1}^{l} CE_{ij} \left(\tilde{R}^{h}, \tilde{R}^{k} \right) \tag{7}$$

Level 2. *Consensus degree on alternatives.* The consensus degree between experts U^h and U^k on the alternative x_i is:

$$CA_i \left(\tilde{R}^{h}, \tilde{R}^{k} \right) = \frac{1}{n} \sum_{j=1}^{n} \left(1 - \frac{\left| t_{ij}^{h-} - t_{ij}^{k-} \right| + \left| t_{ij}^{h+} - t_{ij}^{k+} \right| + \left| d_{ij}^{h-} - d_{ij}^{k-} \right| + \left| d_{ij}^{h+} - d_{ij}^{k+} \right|}{4} \right) \tag{8}$$

Then, the average consensus degree of the expert U^h to the group on the alternatives x_i is defined as:

$$ACA_{i}^{h} = \frac{1}{l-1} \sum_{h \neq k, k=1}^{l} CA_{i} \left(\tilde{R}^{h}, \tilde{R}^{k} \right) \tag{9}$$

Level 3. *Consensus degree on decision matrix.* The consensus degree between experts U^h and U^k on the decision matrix is:

$$CD \left(\tilde{R}^{h}, \tilde{R}^{k} \right) = \frac{1}{m \cdot n} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(1 - \frac{\left| t_{ij}^{h-} - t_{ij}^{k-} \right| + \left| t_{ij}^{h+} - t_{ij}^{k+} \right| + \left| d_{ij}^{h-} - d_{ij}^{k-} \right| + \left| d_{ij}^{h+} - d_{ij}^{k+} \right|}{4} \right) \tag{10}$$

Thus, the average consensus degree of the expert U^h to the group is defined as:

$$ACD^{h} = \frac{1}{l-1} \sum_{h \neq k, k=1}^{l} CD \left(\tilde{R}^{h}, \tilde{R}^{k} \right) \tag{11}$$

The greater the value of ACD^h ($0 \leq ACD^h \leq 1$), the greater the agreement between the individual expert U^h and the group. When ACD^h ($h = 1, \ldots, l$) satisfies a minimum satisfaction threshold value $\gamma \in [0.5, 1)$; then the consensus reaching process ends, and the selection process is applied to achieve the solution of consensus. Otherwise, a trust induced recommendation mechanism could be activated to provide personalized advice to the inconsistent experts.

Example 1. An electronic enterpriser is to select the most appropriate cloud services from three alternatives (A_1, A_2, A_3): three criteria (c_1, c_2, c_3) are considered as: Performance; Security and privacy; Usability, with associated weighting vector $\omega = (0.3, 0.5, 0.2)^T$. This company has a group of experts $\left(U^1, U^2, U^3, U^4, U^5\right)$ from five different departments. The five decision matrix with interval-valued trust functions given by the five experts being:

$$\tilde{R}^{(1)} = \left(\begin{array}{cccc} & c_1 & c_2 & c_3 \\ A_1 & ([0.4, 0.6], [0.2, 0.6]) & ([0.2, 0.5], [0.6, 0.8]) & ([0.5, 0.6], [0.4, 0.7]) \\ A_2 & ([0.5, 0.3], [0.4, 0.5]) & ([0.3, 0.7], [0.4, 0.6]) & ([0.5, 0.9], [0.3, 0.4]) \\ A_3 & ([0.6, 0.8], [0.5, 0.7]) & ([0.3, 0.5], [0.1, 0.3]) & ([0.4, 0.7], [0.6, 0.8]) \end{array} \right)$$

$$\tilde{R}^{(2)} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \left(\begin{array}{ccc} c_1 & c_2 & c_3 \\ ([0.2,0.5],[0.3,0.8]) & ([0.5,0.9],[0.4,0.5]) & ([0.3,0.4],[0.3,0.5]) \\ ([0.4,0.6],[0.3,0.7]) & ([0.3,0.6],[0.5,0.7]) & ([0.5,0.6],[0.3,0.4]) \\ ([0.3,0.6],[0.4,0.8]) & ([0.4,0.7],[0.2,0.4]) & ([0.4,0.5],[0.3,0.4]) \end{array} \right)$$

$$\tilde{R}^{(3)} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \left(\begin{array}{ccc} c_1 & c_2 & c_3 \\ ([0.4,0.5],[0.5,0.7]) & ([0.3,0.7],[0.4,0.6]) & ([0.4,0.5],[0.2,0.6]) \\ ([0.3,0.6],[0.2,0.5]) & ([0.5,0.6],[0.4,0.7]) & ([0.4,0.9],[0.5,0.6]) \\ ([0.3,0.5],[0.2,0.4]) & ([0.4,0.6],[0.2,0.5]) & ([0.4,0.9],[0.5,0.7]) \end{array} \right)$$

$$\tilde{R}^{(4)} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \left(\begin{array}{ccc} c_1 & c_2 & c_3 \\ ([0.3,0.6],[0.2,0.5]) & ([0.6,0.9],[0.3,0.5]) & ([0.4,0.6],[0.2,0.5]) \\ ([0.5,0.7],[0.4,0.8]) & ([0.2,0.6],[0.4,0.7]) & ([0.5,0.7],[0.3,0.6]) \\ ([0.4,0.6],[0.4,0.8]) & ([0.5,0.8],[0.3,0.5]) & ([0.2,0.3],[0.1,0.2]) \end{array} \right)$$

$$\tilde{R}^{(5)} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \left(\begin{array}{ccc} c_1 & c_2 & c_3 \\ ([0.2,0.4],[0.3,0.5]) & ([0.2,0.3],[0.2,0.7]) & ([0.6,0.8],[0.8,0.9]) \\ ([0.5,0.7],[0.2,0.4]) & ([0.6,0.8],[0.3,0.5]) & ([0.5,0.9],[0.1,0.3]) \\ ([0.8,0.9],[0.3,0.4]) & ([0.1,0.3],[0.3,0.7]) & ([0.4,0.9],[0.6,0.7]) \end{array} \right)$$

According to expression (7), the average consensus degree on elements of alternatives are:

$$ACE_{ij}^{1} = \begin{pmatrix} 0.881 & 0.750 & 0.838 \\ 0.825 & 0.900 & 0.906 \\ 0.794 & 0.819 & 0.794 \end{pmatrix} \quad ACE_{ij}^{2} = \begin{pmatrix} 0.869 & 0.800 & 0.806 \\ 0.869 & 0.894 & 0.875 \\ 0.825 & 0.863 & 0.775 \end{pmatrix}$$

$$ACE_{ij}^{3} = \begin{pmatrix} 0.850 & 0.831 & 0.844 \\ 0.857 & 0.900 & 0.838 \\ 0.769 & 0.875 & 0.806 \end{pmatrix} \quad ACE_{ij}^{4} = \begin{pmatrix} 0.881 & 0.775 & 0.844 \\ 0.844 & 0.894 & 0.881 \\ 0.831 & 0.825 & 0.625 \end{pmatrix}$$

$$ACE_{ij}^{5} = \begin{pmatrix} 0.869 & 0.744 & 0.669 \\ 0.856 & 0.813 & 0.850 \\ 0.731 & 0.744 & 0.800 \end{pmatrix}$$

Then, by expression (9), the average consensus degrees on alternatives are:

$$\begin{aligned} &ACA_i^1 = (0.823, 0.877, 0.802); \quad ACA_i^2 = (0.825, 0.879, 0.821); \\ &ACA_i^3 = (0.842, 0.865, 0.817); \quad ACA_i^4 = (0.833, 0.873, 0.760); \\ &ACA_i^5 = (0.760, 0.840, 0.758). \end{aligned}$$

Finally, the experts' average consensus degree are:

$$ACD^1 = 0.834,\ ACD^2 = 0.842,\ ACD^3 = 0.841,\ ACD^4 = 0.822,\ ACD^5 = 0.786.$$

If the threshold value is set at $\gamma = 0.8$, then the recommendation mechanism is activated to generate advice to assist expert U^5 to increase his/her consensus degree.

3.2. Trust induced recommendation mechanism

The trust induced recommendation mechanism includes three steps: (1) Identification of the elements of alternatives values that should be changed; (2) Construction of the visual trust relationship; and (3) Generation of trust based personalised advice.

3.2.1. Identification of the elements of alternatives values

To identify the elements of alternatives values that are contributing less to the consensus, the following three steps of are carried out:

Step 1. The experts with a consensus index lower than the threshold value γ are identified:

$$EXPCH = \{h \mid ACD^h < \gamma\}$$

Step 2. For the identified experts, their alternatives with ACA_i^h lower than the satisfaction threshold γ are identified:

$$ALT = \{(h, i) \mid e_h \in EXPCH \ \wedge \ ACA_i^h < \gamma\}$$

Step 3. Finally, the elements of alternatives to be changed are:

$$APS = \{(h, i, j) \mid (h, i) \in ALT \ \wedge \ ACE_{ij}^h < \gamma\}.$$

Example 2. (Example 1 continuation) The following APS set is obtained:

$$APS = \{(5, 1, 2), (5, 1, 3), (5, 3, 1), (5, 3, 2)\}$$

Thus, based trust personalised advice for elements of alternatives r_{12}^5, r_{13}^5, r_{31}^5 and r_{32}^5 are generated for expert U^5.

3.2.2. Construction of visual trust relationship

The proposed recommendation mechanism uses trust relationship to generate personalised advices. In most practical cases, a decision maker may trust opinions coming from trusted experts close to him/her. Therefore, this article defines the concept of trust degree (TD) based on a distance between two elements expressed by interval-valued trust functions.

***Definition* 8 (Trust Degree (TD)).** Let U^h and U^k be any two experts, then their consensus degree on the decision matrix is a measure of the trust degree between them, i.e.:

$$TD^{hk} = CD\left(\tilde{R}^h, \tilde{R}^k\right) \tag{12}$$

Obviously, U^h completely trusts U^k when TD^{hk}=1, and U^h completely distrusts U^k when $TD^{hk} = 0$. The following trust matrix can be constructed:

$$TD = \begin{bmatrix} 1 & TD_{12} & \cdots & TD_{1l} \\ TD_{21} & 1 & \cdots & TD_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ TD_{l1} & TD_{l2} & \cdots & 1 \end{bmatrix}$$

As aforementioned, an expert will trust opinions which are close to him/her, and therefore the following simple rule can be implemented to ascertain whether there exists a trust relationship between experts in a group:

"If $TD^{hk} > \gamma$, then there exists a trust relationship between U^h and U^k, otherwise, there is no trust."

Therefore, the application of such a rule allow to visualise the trust relationship among a group of experts and to see which experts are trusted by U^h (UT^h), as the following example illustrates.

Example 3. (Example 2 continuation) We can calculate the following trust matrix as:

$$TD = \begin{pmatrix} 1.000 & 0.836 & 0.850 & 0.814 & 0.836 \\ 0.836 & 1.000 & 0.869 & 0.906 & 0.756 \\ 0.850 & 0.869 & 1.000 & 0.831 & 0.814 \\ 0.814 & 0.906 & 0.831 & 1.000 & 0.739 \\ 0.836 & 0.756 & 0.814 & 0.739 & 1.000 \end{pmatrix}$$

Because $\gamma = 0.8$, the trust relationship is constructed as:

$$TR = \begin{pmatrix} - & \odot & \odot & \odot & \odot \\ \odot & - & \odot & \odot & \times \\ \odot & \odot & - & \odot & \odot \\ \odot & \odot & \odot & - & \times \\ \odot & \times & \odot & \times & - \end{pmatrix}$$

where $\odot$ means trust relationship, which is graphically illustrated in Fig.2.

This example clearly identifies the inconsistent expert U^5 trusted ones: $UT^5 = \{U^1, U^3\}$. Therefore, combining the opinions from U^1 and U^3, the recommendation mechanism can generate trust based personalised advices for U^5, as it is described in detail in the following section.

3.2.3. Generation of trust based personalised advice

If $(i, j) \in EV^h$ the trust based personalised advice generated for U^h is:

"you are advised to change your evaluation value for the alternatives A_i under attribute c_j, $r_{ij}^h = \left(\tilde{t}_{ij}^h, \tilde{d}_{ij}^h\right)$, to the value $rr_{ij}^h = \left(r\tilde{t}_{ij}^h, r\tilde{d}_{ij}^h\right)$."

$$\left(r\tilde{t}_{ij}^h, r\tilde{d}_{ij}^h\right) = \left((1-\delta)\cdot \tilde{t}_{ij}^h + \delta \cdot \bar{\tilde{t}}_{ij}^{\odot} \quad , \quad (1-\delta)\cdot \tilde{d}_{ij}^h + \delta \cdot \bar{\tilde{d}}_{ij}^{\odot}\right) \tag{13}$$

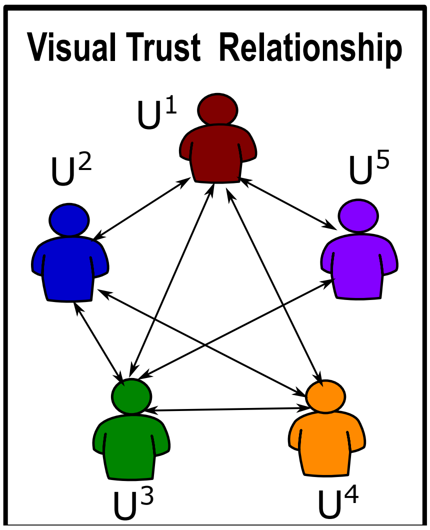


Figure 2: Visual trust relationship

where

$$\bar{\tilde{t}}_{ij}^{\odot} = \frac{1}{k}\sum_{s=1}^{k} \tilde{t}_{ij}^{h_s}, \quad \bar{\tilde{d}}_{ij}^{\odot} = \frac{1}{k}\sum_{s=1}^{k} \tilde{d}_{ij}^{h_s} \tag{14}$$

with $UT^h = \{U^{h_1}, \ldots, U^{h_k}\}$, and $\delta \in [0,1]$ is a parameter to control the degree of recommendation.

Example 4. (Example 3 continuation) Taking a value of $\delta = 0.5$, the trust based recommendation advices for expert U^5 are:

- You are advised to change your evaluation value of alternative A_1 under attribute c_2 to a value closer to $([0.23, 0.45], [0.35, 0.70])$.
- You are advised to change your evaluation value of alternative A_1 under attribute c_3 to a value closer to $([0.53, 0.68], [0.55, 0.78])$.
- You are advised to change your evaluation value of alternative A_3 under attribute c_1 to a value closer to $([0.63, 0.78], [0.33, 0.48])$.
- You are advised to change your evaluation value of alternative A_3 under attribute c_2 to a value closer to $([0.23, 0.43], [0.23, 0.55])$.

Example 5. (Example 4 continuation) (Second Consensus Round). Once expert U^5 implements the trust based changes in his/her evaluation values, a new consensus process round starts.

Assuming U^5 implements the above trust based personalised values, the new decision matrix would be:

$$\overline{R}^{(5)} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{pmatrix} c_1 & c_2 & c_3 \\ ([0.20, 0.40], [0.30, 0.50]) & ([0.23, 0.45], [0.35, 0.70]) & ([0.53, 0.68], [0.55, 0.78]) \\ ([0.50, 0.70], [0.20, 0.40]) & ([0.60, 0.80], [0.30, 0.50]) & ([0.50, 0.90], [0.10, 0.30]) \\ ([0.63, 0.78], [0.33, 0.48]) & ([0.23, 0.43], [0.23, 0.55]) & ([0.40, 0.90], [0.60, 0.70]) \end{pmatrix}$$

and applying the computation process in Example (1), the new consensus degrees would become: $\overline{ACD}^1 = 0.846$, $\overline{ACD}^2 = 0.854$, $\overline{ACD}^3 = 0.852$, $\overline{ACD}^4 = 0.833$, $\overline{ACD}^5 = 0.831$, and all experts will have a consensus degree above the threshold value $\gamma = 0.8$.

From the above computation result, we find that the trust based recommendation mechanism can improve the consensus level of the inconsistent expert U^5 with the feedback parameter $\delta = 0.5$. To show a more general effect of the proposed trust based recommendation mechanism, we draw the following proposition.

***Proposition* 1.** *Let $\widetilde{R}^1, \widetilde{R}^2, \widetilde{R}^3, \widetilde{R}^4, \widetilde{R}^5$ be the original trust decision making matrices in the above example and, after expert U^5 implements the above trust recommendation advices, the new trust decision making matrices be $\widetilde{R}^1, \widetilde{R}^2, \widetilde{R}^3, \widetilde{R}^4, \overline{R}^5$. Then we have*

$$ACD^5 < \overline{ACD}^5 \tag{15}$$

Proof. From expression (11), we obtain that

$$ACD^5 = \frac{1}{4}\left(4 - \left(\left|\tilde{R}^5 - \tilde{R}^1\right| + \left|\tilde{R}^5 - \tilde{R}^2\right| + \left|\tilde{R}^5 - \tilde{R}^3\right| + \left|\tilde{R}^5 - \tilde{R}^4\right|\right)\right) < \gamma$$

Let γ=$1 - \alpha$, then the above equation can be rewritten as:

$$\left|\tilde{R}^5 - \tilde{R}^1\right| + \left|\tilde{R}^5 - \tilde{R}^2\right| + \left|\tilde{R}^5 - \tilde{R}^3\right| + \left|\tilde{R}^5 - \tilde{R}^4\right| > 4\alpha$$

then

$$\delta(\left|\tilde{R}^5 - \tilde{R}^1\right| + \left|\tilde{R}^5 - \tilde{R}^2\right| + \left|\tilde{R}^5 - \tilde{R}^3\right| + \left|\tilde{R}^5 - \tilde{R}^4\right|) > 4\delta\alpha, \quad \delta, \alpha \in [0, 1]$$

Since U^5 just trusts experts U^1 and U^3 by Fig 2, then according to expression (13), the new decision matrix $\overline{R}^5$ is:

$$\overline{R}^5 = (1-\delta)\tilde{R}^5 + \delta\frac{\tilde{R}^1 + \tilde{R}^3}{2}$$

then we have

$$\left|\tilde{R}^5 - \tilde{R}^1\right| = \left|(1-\delta)\tilde{R}^5 + \delta\frac{\tilde{R}^1 + \tilde{R}^3}{2} - \tilde{R}^1\right| \leq (1-\delta)\left|\tilde{R}^5 - \tilde{R}^1\right| + \frac{\delta}{2}\left|\tilde{R}^3 - \tilde{R}^1\right|,$$

$$\left|\bar{R}^5 - \tilde{R}^3\right| \leq (1-\delta)\left|\tilde{R}^5 - \tilde{R}^3\right| + \frac{\delta}{2}\left|\tilde{R}^3 - \tilde{R}^1\right|,$$

$$\left|\bar{R}^5 - \tilde{R}^2\right| \leq (1-\delta)\left|\tilde{R}^5 - \tilde{R}^2\right| + \frac{\delta}{2}\left(\left|\tilde{R}^3 - \tilde{R}^2\right| + \left|\tilde{R}^1 - \tilde{R}^2\right|\right)$$

and

$$\left|\bar{R}^5 - \tilde{R}^4\right| \leq (1-\delta)\left|\tilde{R}^5 - \tilde{R}^4\right| + \frac{\delta}{2}\left(\left|\tilde{R}^3 - \tilde{R}^4\right| + \left|\tilde{R}^1 - \tilde{R}^4\right|\right)$$

Because there is full trust relationship between group experts U^1, U^2, U^3, U^4 in Fig 2, and by expression (12) in Definition 8, we obtain that

$$TD^{hk} = CD(\tilde{R}^h, \tilde{R}^k) = 1 - |R^h - R^k| > \gamma, \quad h,k = 1,2,3,4, h \neq k$$

then

$$\left|\tilde{R}^h - \tilde{R}^k\right| < \alpha, \quad h,k = 1,2,3,4, h \neq k$$

thus

$$\begin{aligned}\left|\bar{R}^5 - \tilde{R}^1\right| + \left|\bar{R}^5 - \tilde{R}^3\right| + \left|\bar{R}^5 - \tilde{R}^2\right| + \left|\bar{R}^5 - \tilde{R}^4\right| \leq \\ (1-\delta)\left(\left|\tilde{R}^5 - \tilde{R}^1\right| + \left|\tilde{R}^5 - \tilde{R}^3\right| + \left|\tilde{R}^5 - \tilde{R}^2\right| + \left|\tilde{R}^5 - \tilde{R}^4\right|\right) + 3\delta\alpha\end{aligned}$$

Since $\delta(\left|\tilde{R}^5 - \tilde{R}^1\right| + \left|\tilde{R}^5 - \tilde{R}^2\right| + \left|\tilde{R}^5 - \tilde{R}^3\right| + \left|\tilde{R}^5 - \tilde{R}^4\right|) > 4\delta\alpha$, then

$$\begin{aligned}\left|\tilde{R}^5 - \tilde{R}^1\right| + \left|\tilde{R}^5 - \tilde{R}^3\right| + \left|\tilde{R}^5 - \tilde{R}^2\right| + \left|\tilde{R}^5 - \tilde{R}^4\right| > \\ (1-\delta)\left(\left|\tilde{R}^5 - \tilde{R}^1\right| + \left|\tilde{R}^5 - \tilde{R}^3\right| + \left|\tilde{R}^5 - \tilde{R}^2\right| + \left|\tilde{R}^5 - \tilde{R}^4\right|\right) + 4\delta\alpha\end{aligned}$$

then

$$\begin{aligned}\left|\bar{R}^5 - \tilde{R}^1\right| + \left|\bar{R}^5 - \tilde{R}^3\right| + \left|\bar{R}^5 - \tilde{R}^2\right| + \left|\bar{R}^5 - \tilde{R}^4\right| < \\ \left|\tilde{R}^5 - \tilde{R}^1\right| + \left|\tilde{R}^5 - \tilde{R}^3\right| + \left|\tilde{R}^5 - \tilde{R}^2\right| + \left|\tilde{R}^5 - \tilde{R}^4\right|\end{aligned}$$

and according to expression (11), we obtain that

$$ACD^5 < \overline{ACD}^5$$

which finishes the proof of Proposition 1. □

Therefore, it can be concluded that the trust based recommendation mechanism can guarantee the inconsistent expert U^5 will increase his/her consensus level irrespective of the value of the parameter δ.

3.3. Rationality analysis of recommendation mechanisms

This section first introduces the traditional recommendation mechanism based on the average method already mentioned. Secondly, the definition of harmony degree (HD) is proposed to determine the deviation degree before and after making a change of the inconsistent opinion. The HD is used to compare our trust induced recommendation mechanism with respect to the traditional average based recommendation mechanism.

3.3.1. Traditional recommendation mechanism without trust

Most traditional recommendation mechanisms generate advices using the average value of the opinion of all experts in the group opinions with group trust relationship being neglected [20, 21, 36, 49, 52], as follows:

If $(i, j) \in EV^h$ the personalized advice generated for e^h is:
"you are advised to change your evaluation value for the alternatives A_i under attribute c_j, $r^h_{ij} = \left(\tilde{t}^h_{ij}, \tilde{d}^h_{ij}\right)$, to a value $ar^h_{ij} = \left(a\tilde{t}^h_{ij}, a\tilde{d}^h_{ij}\right)$."

$$\left(a\tilde{t}^h_{ij}, a\tilde{d}^h_{ij}\right) = \left((1-\delta)\cdot \tilde{t}^h_{ij} + \delta \cdot \tilde{t}_{ij}, (1-\delta)\cdot \tilde{d}^h_{ij} + \delta \cdot \tilde{d}_{ij}\right) \tag{16}$$

with $(\tilde{t}_{ij}, \tilde{d}_{ij})$ being represents the average of all the opinions

$$\tilde{t}_{ij} = \frac{1}{l}\sum_{h=1}^{l} \tilde{t}^h_{ij}, \quad \tilde{d}^h_{ij} = \frac{1}{l}\sum_{h=1}^{l} \tilde{d}^h_{ij} \tag{17}$$

and $\delta \in [0, 1]$ is a parameter to control the degree of recommendation.

Taking a value of $\delta = 0.5$, the advice for expert U^5 by the traditional recommendation mechanism would be:

- You should change your evaluation value of alternative A_1 under attribute c_2 to a value closer to $([0.30, 0.53], [0.31, 0.65])$.
- You should change your evaluation value of alternative A_1 under attribute c_3 to a value closer to $([0.50, 0.66], [0.54, 0.74])$.
- You should change your evaluation value of alternative A_3 under attribute c_1 to a value closer to $([0.60, 0.76], [0.34, 0.54])$.
- You should change your evaluation value of alternative A_3 under attribute c_2 to a value closer to $([0.25, 0.48], [0.25, 0.56])$.

After U^5 implements the above values, the new consensus degrees result in: $ACD'_1 = 0.846$, $ACD'_2 = 0.857$, $ACD'_3 = 0.854$, $ACD'_4 = 0.837$ and $ACD'_5 = 0.841$. As before, all consensus degrees reach the threshold value $\gamma = 0.8$. However, the inconsistent expert U^5 affords more changes cost than the result of Example 5, which is verified in the following section.

3.3.2. Comparison of different recommendation mechanisms

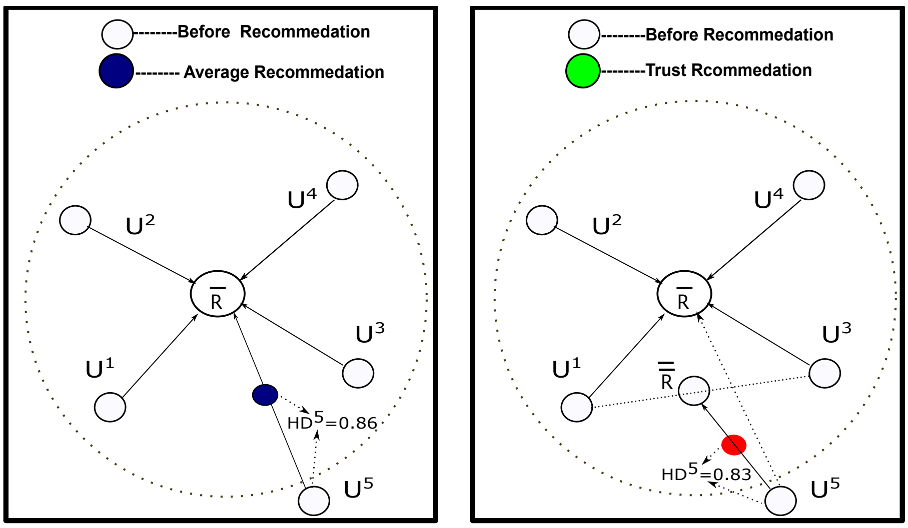
Apart from consensus, decision makers intend to keep individuals original opinions so that they can keep their independence [13]. In other words, decision makers are willing to reach a fixed threshold of consensus degree with lower changes cost for implementing recommended advices [14–16, 18, 19].

To determine changes cost, this article defines the concept of harmony degree (HD) for interval-valued trust functions, which is based on the deviation degree between the original opinion R^h and the recommended opinion $rR^h = \left(r\tilde{t}_{ij}^h, r\tilde{d}_{ij}^h\right)$ that replaces it [41].

***Definition* 9 (Harmony Degree (HD)).** The harmony degree (HD) of inconsistent expert U^h if the recommended advices were implemented is:

$$HD^h = 1 - \frac{1}{\#ASP} \sum_{(h,i,j)\in ASP} \left(\frac{\left|t_{ij}^{h-} - rt_{ij}^{h-}\right| + \left|t_{ij}^{h+} - rt_{ij}^{h+}\right| + \left|d_{ij}^{h-} - rd_{ij}^{h-}\right| + \left|d_{ij}^{h+} - rd_{ij}^{h+}\right|}{4} \right) \quad (18)$$

Notice that it is $0 \leq HD^h \leq 1$, and that the bigger the value of HD^h, the smaller the deviation from the original opinions provided by U^h and the new opinions derived from the implementation of the recommended advices. According to expression (18), the harmony degrees of U^5 using the traditional recommendation mechanism and using our proposed trust induced recommendation mechanism are $HD^5 = 0.86$ and $HD^5 = 0.89$, respectively. The feedback simulation for these two different recommendation mechanism is shown in Fig. 3(a) and Fig. 3(b), respectively.



(a) HD of traditional recommendation mechanism

(b) HD of trust induced recommendation mechanism

Figure 3: Visual simulation of harmony degree before and after implementing recommendation mechanism

Table 1: ACD index of two recommendation mechanisms with different parameters δ_i

δ_i	0.1	0.3	0.5
$ACD_5'(Trust)$	0.795	0.81	0.83
$ACD_5'(Traditional)$	0.798	0.82	0.84

We also can choose different feedback parameters to compare the effect of the trusted induced recommendation mechanism with respect to the traditional one, which are shown in Table 1 and in

Table 2: HD index of two recommendation mechanisms with different parameters δ_i

δ_i	0.1	0.3	0.5
$HD_5(Trust)$	0.96	0.93	0.89
$HD_5(Traditional)$	0.95	0.92	0.86

Table 2, when $\delta = 0.1$, the HD index of the trusted induced recommendation mechanism is still bigger than the obtained with the traditional recommendation mechanism although their consensus degrees are lower than the accepted threshold value and need to further interaction. When $\delta = 0.3$, we have the same conclusion.

Therefore, the trust induced recommendation mechanism has lower changes costs than the traditional recommendation mechanism for reaching a consensus threshold value. In other words, the first one can keep more of the initial opinions from the inconsistent experts in reaching the consensus threshold. The main reason for this is that the trust induced recommendation just adopts the closer trusted opinions and ignores the further distrusted opinions. On the contrary, the traditional recommendation mechanism forces the inconsistent expert to adopt all the opinions, and then it neglects a reality that may happen when the recommended advice makes experts deviate too much from their original opinions, making these unacceptable. Hence, the proposed trust induced recommendation mechanism has a more reasonable policy that it arrives at the consensus threshold value but at the same time with higher harmony degrees than with respect to the traditional one, making the implementation of their recommended advices more plausible.

4. Selection process

Once each expert's consensus degree $ACD^h\ (h = 1, ..., l)$ satisfies threshold value γ, then the individual decision making matrixes with interval-valued trust functions, $\tilde{R}^{(h)}$, are aggregated to a collective decision matrix, $\bar{R}$. To do that, we use consensus degree as a reliable source to assign weight or importance values to each expert so that the higher the consensus degree associated to an expert, the higher the importance associated to him/her. This methodology can be implemented via the induced ordered weighted average (OWA) operator [56], which in turn can be guided by a linguistic quantifier [54] to model the concept of majority in the decision making resolution [53]. In detail, the linguistic quantifier is represented mathematically by a basic unit-monotonic (BUM) function $Q : [0, 1] \to [0, 1]$ such that $Q\,(0) = 0,\ \ Q\,(1) = 1$ and $Q\,(x) \geq Q\,(y)$ if $x \geq y$. Thus, the weight of the OWA operator can be obtained as follows:

$$w_h = Q\left(\frac{S(h)}{S(l)}\right) - Q\left(\frac{S(h-1)}{S(l)}\right),\ h = 1, 2, ..., l. \tag{19}$$

with $S(h) = \sum_{k=1}^{h} ACD^{\sigma(k)}$ and σ is the permutation used to induce the ordering of the values to aggregate, which is obtained by ordering from highest to lowest the consensus degrees of the experts in the group [10].

Yager [53] considered the parameterised family of regular increasing monotone (RIM) quantifiers $Q(r) = r^a \, (a \geq 0)$ for such representation. This family of functions guarantees that: (1) all the experts contribute to the final aggregated value (strict monotonicity property), and (2) associates, when $a \in [0, 1]$, higher weight values to the aggregated values with associated higher importance values(concavity property) [56]. In particular, the value $a = 1/2$ is used to represent the fuzzy linguistic quantifier ‘most of’.

Example 6. (Example 5 continuation) According to the final consensus degree, we have that

$$ACD^2 > ACD^3 > ACD^1 > ACD^4 > ACD^5$$

Expression (19) using $Q(r) = r^{1/2}$ derives the following weighting vector:

$$w = (0.14, 0.45, 0.19, 0.12, 0.11)^T$$

The corresponding collective decision matrix $\bar{R} = (\bar{r}_{ij})$ is:

$$\bar{R} = \left(\begin{array}{cccc} & c_1 & c_2 & c_3 \\ A_1 & ([0.28, 0.52], [0.32, 0.69]) & ([0.41, 0.77], [0.42, 0.59]) & ([0.39, 0.51], [0.31, 0.58]) \\ A_2 & ([0.42, 0.59], [0.30, 0.62]) & ([0.36, 0.64], [0.44, 0.67]) & ([0.49, 0.75], [0.32, 0.46]) \\ A_3 & ([0.39, 0.64], [0.37, 0.68]) & ([0.38, 0.64], [0.20, 0.44]) & ([0.38, 0.63], [0.39, 0.53]) \end{array} \right)$$

Using the attributes weighting vector $\omega = (0.3, 0.5, 0.2)^T$, the weighted collective overall opinions values associated to the alternatives $A_i \, (i = 1, 2, 3)$ are:

$$\widetilde{r}_1 = ([0.365, 0.640], [0.365, 0.618]);$$

$$\widetilde{r}_2 = ([0.405, 0.647], [0.373, 0.613]);$$

$$\widetilde{r}_3 = ([0.385, 0.637], [0.285, 0.529]).$$

According to expression (4), the associated interval-valued trust scores are:

$$TS_{r_1} = 0.011; \; TS_{r_2} = 0.033; \; TS_{r_3} = 0.101.$$

Consequently, it is concluded that

$$A_3 \succ A_2 \succ A_1$$

Thus, the best alternative is A_3.

5. Conclusion

This article aims to resolve the inconsistency problem in GDM due to various experts' opinions. To do that, it proposes a trust induced recommendation mechanism to help group experts to reach consensus. It has the following main advantages with respect to previous models proposed in the literatures.

1. It investigates interval-valued trust decision making space to model uncertainty in GDM including the novel concepts of interval-valued trust functions, interval-valued trust score, interval-valued knowledge degree, and the ranking order relation for interval-valued trust functions. Then, it can express subjective opinions such as: trust, distrust, hesitancy and conflict, and therefore it is suitable to deal with uncertainty in GDM. It defines the consensus degrees with interval-valued trust functions decision making information within a group at three levels: on elements of alternatives, on alternatives and on decision matrix. Then, a consensus model is designed following a top to bottom methodology by these three levels of consensus degrees.

2. It develops a trust induced recommendation mechanism to generate personalised advice for the inconsistent experts to reach higher consensus degree. The definition of trust degree (TD) between every two experts is introduced, and it is used to identify which opinion is trusted or not. Then, the visual trust relationship among group experts is built. Thus, the trust induced recommendation mechanism allows the inconsistent expert to implement the advice up to his/her personalised trust. Moreover, it can guarantee the inconsistent expert move to a higher consensus level after he/she implements this recommended advice. Therefore, it can overcome the drawback of forcing the inconsistent expert to implement the advice associated to the traditional recommendation mechanism.

3. It defines the concept of harmony degree (HD) to determine the deviation degree between the original opinion and the changed opinion. The proposed trust induced recommendation mechanism is proved to have bigger HD than the traditional one, which means less changes cost. Combining consensus degree and harmony degree, this article proposes a novel policy for group consensus that it arrives at the threshold value with the high value of harmony degree simultaneously.

The trust induced recommendation mechanism for consensus in GDM is based on the posterior computation of trust information by the distance between any two experts' opinions. Other priori factors influencing trust relationship, such as historical interaction and reputation of experts are not considered [38, 39]. A potential avenue to explore in future to address this issue is to construct trust relationship by combining a posteriori and a priori trust information to enhance the reliability of trust relationship in the recommendation mechanism in GDM.

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