

Identification of the hydraulic model from operational measurements for supervisory pressure control

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Abstract

The operational pressure control is a cost-effective way to leakage reduction and many pressure control methods and algorithms have been developed. Whilst the pressure control algorithm is model-based, the hydraulic model of the considered distribution network is not always available. Therefore, this paper will focus on the development of an aggregated hydraulic model of the network considered, in particular, identification of a leakage enhanced model using the operational measurements or the available historical data. This will enable a pressure optimisation algorithm to calculate the optimal pressure schedules for the implementation of a pressure control scheme. The identification problem is formulated as a parameter estimation problem in this paper and a least-square based method is derived for estimating the parameters in the model. A case study provided by a UK water company is performed to illustrate the use of the method and the identification results from real operational data are presented.

Introduction

Leakage from water distribution networks is a source of major concern for water companies worldwide and leakage reduction is a key factor of water distribution network management. It is well known that water leakage in a distribution network is directly related to the system service pressure, hence pressure control is an effective way to reduce background leakage and various methods and algorithms have been developed for optimising the operational pressure so as to minimize background leakage and, at the same time, to satisfy pressure requirements at critical points (see e.g. Jowitt and Xu (1990), Ulanicki et al (2000, 2008) and references therein). Before implementing a pressure control scheme, it is necessary to perform planning studies to investigate the economical aspects of such a scheme. Such studies are usually based on the knowledge about the particular network considered and a convenient form of such knowledge

is a hydraulic model of the network. However, it is often the case that such a hydraulic model is not available from the water company and has to be obtained by some identification procedures using measurements or historical operational data.

This paper focuses on the development of an aggregated hydraulic model of the network considered, in particular, identification of a leakage enhanced model using the operational measurements or the available historical data for the implementation of supervisory pressure control. The rest of the paper is organized as follows. Next section gives the problem statement, which is followed by the derivation of a least-square based method for estimating the parameters in both pipe element model and leakage model of the network. A case study provided by a UK water company is then performed to illustrate the use of the method and the identification results from real operational data are presented. The conclusions are given in the last section.

Problem statement

To facilitate monitoring and control activities, it is common practice for water companies to split the water distribution network into district metered areas (DMAs) that are typically small areas and are isolated apart from designated inlets and outlets through which flows are monitored. Pressure reducing valves (PRVs) are used to maintain a specified pressure to the inlets of a DMA against the fluctuations in PRV inlet pressure and the changes of demand from the DMA. This structure also facilitates application of a pressure control scheme.

A prerequisite for pressure control design is an appropriate model of the network to be controlled, but such a model is not always available in practice, therefore some kind of identification procedure is needed so as to estimate an equivalence model from the available measurements or historical data. The equivalence hydraulic model will contain only boundary nodes, target (critical) nodes and fictitious pipes with which these nodes are connected (Ulanicki et al 2000). The data available for model identification are boundary heads and flows, target heads and flows, and the aim of the identification is to obtain a leakage enhanced network model for pressure control. More specifically, the parameters to be identified are the resistance/conductance of the fictitious pipes, the leakage coefficients and leakage exponent.

Hydraulic model identification

The aforementioned equivalence network structure can be conveniently represented by a node-branch incidence matrix $\mathbf{\Lambda}$ (see e.g. Ulanicki et al 2000). The matrix has a row for each node and a column for each branch. A row contains information about which branches are connected to a particular node. The elements of $\mathbf{\Lambda}$ take one of the three values -1 , 0 and $+1$. The value -1 indicates that the flow of the branch is oriented towards the node, the value $+1$ indicates the flow of the branch is leaving the node and value 0 indicates that a branch is not connected to the node. Therefore, each column of $\mathbf{\Lambda}$ contains only two non-zero entries, $+1$ for the branch origin node and -1 for the branch destination node. Let N_{node} denote the total number of the (boundary and target) nodes in the equivalence network, in the most general case, the total number of branches (fictitious pipes) will be $N_{pipe} = \frac{1}{2}(N_{node} - 1)N_{node}$ and $\mathbf{\Lambda}$

will be a $N_{node} \times \frac{1}{2}(N_{node} - 1)N_{node}$ matrix.

Pipe model parameter estimation

The main components in the aforementioned equivalence network are the (fictitious) pipes which can be described by many different formulae. In this paper, the head-flow relationship for the j th pipe element is expressed by Hazen-Williams formula (see e.g. Walski *et al* 2003):

$$\Delta h_j = h_{j,o} - h_{j,d} = R_j q_j |q_j|^{0.852} \quad (1)$$

where $h_{j,o}$ and $h_{j,d}$ are heads in [m] at origin and destination nodes of the j th pipe element respectively; R_j is the resistance of the j th pipe element; q_j is the flow in [l/s] through the element.

Let $\mathbf{\Lambda}_i$ denote the i th row of $\mathbf{\Lambda}$; $\mathbf{\Lambda}_j^T$ denote the j th column of $\mathbf{\Lambda}$; the heads at each node and the flows in each branch are collected in the nodal head vector \mathbf{h} and the branch flow vector \mathbf{q} respectively, i.e. $\mathbf{h} = [h_1, h_2, \dots, h_{N_{node}}]^T$, $\mathbf{q} = [q_1, q_2, \dots, q_{N_{pipe}}]^T$. The head-flow relationship for the pipe elements in the network can then be written as follows:

$$\mathbf{\Lambda}_j^T \mathbf{h} = R_j q_j |q_j|^{0.852} \quad j = 1, 2, \dots, N_{pipe} \quad (2)$$

The mass balance equations at nodes can be written as:

$$\mathbf{\Lambda}_i \mathbf{q} = -q_{t,i} + q_{b,i} \quad i = 1, 2, \dots, N_{node} \quad (3)$$

where $q_{t,i}$ is the total target flow (i.e. demand and leakage) for node i , $q_{b,i}$ is the boundary flow ($q_{b,i} = 0$ for non-boundary node) for node i . The above mass balance equations at nodes can be written in a compact form:

$$\mathbf{\Lambda} \mathbf{q} = \underbrace{\begin{bmatrix} -q_{t,1} + q_{b,1} \\ -q_{t,2} + q_{b,2} \\ \vdots \\ -q_{t,N_{node}} + q_{b,N_{node}} \end{bmatrix}}_{\mathbf{q}_{t,b}} \quad (4)$$

To estimate the pipe resistance R_j , the pipe model (1) is rewritten in a reverse form:

$$q_j = G_j \Delta h_j |\Delta h_j|^{-0.46} \quad j = 1, 2, \dots, N_{pipe} \quad (5)$$

where $G_j = 1/R_j^{0.54}$ is the conductance of the j th pipe in the network. The pipe equations (5) can be assembled in a vector form as follows:

$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_{N_{pipe}} \end{bmatrix} = \underbrace{\begin{bmatrix} \Delta h_1 |\Delta h_1|^{-0.46} & 0 & \dots & 0 \\ 0 & \Delta h_2 |\Delta h_2|^{-0.46} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Delta h_{N_{pipe}} |\Delta h_{N_{pipe}}|^{-0.46} \end{bmatrix}}_{\mathbf{\Delta H}} \underbrace{\begin{bmatrix} G_1 \\ G_2 \\ \vdots \\ G_{N_{pipe}} \end{bmatrix}}_{\mathbf{G}} \quad (6)$$

Substituting (6) into (4), the following compact network model can be derived:

$$\mathbf{\Lambda} \mathbf{\Delta H} \mathbf{G} = \mathbf{q}_{t,b} \quad (7)$$

Since all heads, target flows and boundary flows are measured or available from historical data, the model equation (7) is linear with respect to the parameter vector \mathbf{G} (from which the resistance $R_j, j = 1, 2, \dots, N_{pipe}$ can be derived). Assume that a time history measurement data set of length K (i.e. $\{\Delta h_j(k), q_{t,i}(k), q_{b,i}(k)\}_{k=1}^K$ for $j = 1, 2, \dots, N_{pipe}$ and $i = 1, 2, \dots, N_{node}$) is selected, the parameter estimation can then be formulated as a least-square problem. For the model defined by (7), it is straightforward to write the explicit linear regression form in terms of the parameter vector \mathbf{G} to be estimated:

$$\underbrace{\begin{bmatrix} \mathbf{q}_{t,b}(1) \\ \mathbf{q}_{t,b}(2) \\ \vdots \\ \mathbf{q}_{t,b}(K) \end{bmatrix}}_{\mathbf{Q}_{t,b}} = \underbrace{\begin{bmatrix} \Lambda \Delta \mathbf{H}(1) \\ \Lambda \Delta \mathbf{H}(2) \\ \vdots \\ \Lambda \Delta \mathbf{H}(K) \end{bmatrix}}_{\Phi} \mathbf{G} \tag{8}$$

where, for $k = 1, 2, \dots, K$

$$\mathbf{q}_{t,b}(k) = \begin{bmatrix} -q_{t,1}(k) + q_{b,1}(k) \\ -q_{t,2}(k) + q_{b,2}(k) \\ \vdots \\ -q_{t,N_{node}}(k) + q_{b,N_{node}}(k) \end{bmatrix} \tag{9}$$

and

$$\Delta \mathbf{H}(k) = \begin{bmatrix} \Delta h_1(k) |\Delta h_1(k)|^{-0.46} & 0 & \dots & 0 \\ 0 & \Delta h_2(k) |\Delta h_2(k)|^{-0.46} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Delta h_{N_{pipe}}(k) |\Delta h_{N_{pipe}}(k)|^{-0.46} \end{bmatrix} \tag{10}$$

\mathbf{G} can then be estimated by solving the above linear regression problem with the LS method as follows:

$$\hat{\mathbf{G}} = \Phi^+ \mathbf{Q}_{t,b} \tag{11}$$

where Φ^+ is the pseudo-inverse of Φ . From equations (8), (9) and (10), it is clear that the matrices $\mathbf{Q}_{t,b}$ and Φ in equation (11) are formed entirely using the available measurement data $\{\Delta h_j(k), q_{t,i}(k), q_{b,i}(k)\}_{k=1}^K$ for $j = 1, 2, \dots, N_{pipe}$ and $i = 1, 2, \dots, N_{node}$.

Equations (8)~(11) provide a method for identifying an equivalence network model in the general case. If, however, the mass balance equation (4) can be solved, hence branch flows $q_j, j = 1, 2, \dots, N_{pipe}$ are available; the identification algorithm can be greatly simplified. Since q_j is available, it is straightforward to write the explicit linear regression form in terms of the R_j from equation (1) for each pipe:

$$\underbrace{\begin{bmatrix} \Delta h_j(1) \\ \Delta h_j(2) \\ \vdots \\ \Delta h_j(K) \end{bmatrix}}_{\Delta \mathbf{H}_j} = \underbrace{\begin{bmatrix} q_j(1) |q_j(1)|^{0.852} \\ q_j(2) |q_j(2)|^{0.852} \\ \vdots \\ q_j(K) |q_j(K)|^{0.852} \end{bmatrix}}_{\mathbf{Q}_j} R_j \quad j = 1, 2, \dots, N_{pipe} \tag{12}$$

Similar to (11), the resistance R_j can then be estimated as follows:

$$\hat{R}_j = \mathbf{Q}_j^+ \Delta \mathbf{H}_j \quad j = 1, 2, \dots, N_{pipe} \quad (13)$$

Assume that the measurement errors are independent, identically distributed (i.i.d.), the estimation error, i.e. the standard deviation of the estimate \hat{R}_j can also be evaluated, this is given by:

$$\sigma(\hat{R}_j) = \sqrt{\frac{S}{K-1} (\mathbf{Q}_j^T \mathbf{Q}_j)^{-1}} \quad (14)$$

where S is the sum of squared residuals defined as follows:

$$S = \sum_{k=1}^K (\Delta h_j(k) - q_j(k) |q_j(k)|^{0.852} \hat{R}_j)^2 \quad (15)$$

Leakage model parameter estimation

The pressure management for leakage reduction requires an enhanced hydraulic model, which incorporate pressure dependent leakage terms. Several mathematical models relating the leakage and the operating pressure based on experimental results have been proposed. The leakage-pressure relationship as shown in equation (16) (Ulanicki *et al* 2008) is assumed in this paper:

$$l_i = k_i p_i^{\alpha_i} = k_i (h_i - H_i)^{\alpha_i} \quad (16)$$

where k_i is the leakage coefficient; α_i is the leakage exponent, h_i is the head of node i and H_i is the elevation of node i . The leakage exponent α_i ranges from 0.5 to 2.5 depending on many factors described in the literature. In this paper, the leakage exponent will be estimated along with the leakage coefficient k_i using the leakage measurement data $\{h_i(k), H_i, l_i(k)\}_{k=1}^K$. In practice, the leakage $l_i, i = 1, 2, \dots, N_{node}$ may not be available, some kinds of estimation or approximation have to be used. The minimum night flow (MNF) will be used in the following case study to approximate the leakage.

The linear least-square method can not directly be used for estimating the parameters k_i and α_i in leakage model (16) as the model is not linear in terms of α_i . However, the problem can be resolved by taking logarithms of equation (16):

$$\ln l_i = \ln k_i + \alpha_i \ln p_i \quad (17)$$

For the given data set $\{h_i(k), H_i, l_i(k)\}_{k=1}^K$ of size K , the following standard linear regression form in terms of the transformed variables $\boldsymbol{\theta} = [\ln k_i \quad \alpha_i]^T$ can readily be derived from equation (17):

$$\underbrace{\begin{bmatrix} \ln l_i(1) \\ \ln l_i(2) \\ \vdots \\ \ln l_i(K) \end{bmatrix}}_{\mathbf{L}} = \underbrace{\begin{bmatrix} 1 & \ln p_i(1) \\ 1 & \ln p_i(2) \\ \vdots & \vdots \\ 1 & \ln p_i(K) \end{bmatrix}}_{\boldsymbol{\Phi}} \boldsymbol{\theta} \quad (18)$$

The LS estimates of k_i and α_i is then given by:

$$\hat{\theta} = \begin{bmatrix} \ln \hat{k}_i \\ \hat{\alpha}_i \end{bmatrix} = \Phi^+ \mathbf{L} \quad \text{and} \quad \hat{k}_i = e^{\ln \hat{k}_i} \quad (19)$$

The variance-covariance matrix of the estimate $\hat{\theta}$ is given by:

$$\sigma(\hat{\theta}) = \frac{S_T}{K-2} (\Phi^T \Phi)^{-1} \quad (20)$$

where S_T is the sum of squared transformed residuals defined as follows:

$$S_T = \sum_{k=1}^K (\ln l_i(k) - \ln \hat{k}_i - \hat{\alpha}_i \ln(p_i(k)))^2 \quad (21)$$

The standard deviations of the model parameter estimates is then given by (see e.g. Bevington and Robinson 2003):

$$\sigma(\hat{k}_i) = \sigma(\hat{\theta})_{11} / \frac{d(\ln \hat{k}_i)}{d\hat{k}_i} = \hat{k}_i \sigma(\hat{\theta})_{11} \quad \text{and} \quad \sigma(\hat{\alpha}_i) = \sigma(\hat{\theta})_{22} \quad (22)$$

where $\sigma(\hat{\theta})_{ii} (i = 1, 2)$ is the main diagonal element of the matrix $\sigma(\hat{\theta})$ defined by (20).

Case study

Pressure management in UK water industry has been an ongoing process for a period of time and an event-driven pressure management trial was recently carried out at a UK water company. As a part of the UK Engineering & Physical Science Research Council (EPSRC)-funded project NEPTUNE, a supervisory pressure control scheme covering wide areas and the associated implementation through the hardware used in the aforementioned trial are proposed for minimizing leakage and customer interruptions. However, the water company does not have a hydraulic model for the area considered. Without a model it is also difficult to estimate how much savings would result from the implementation of the proposed pressure control scheme.

In this section, an aggregated hydraulic model for the area considered will be developed using the available measurements for the implementation of the proposed supervisory pressure control scheme. The network extracted from the schematics of the considered area is shown in Figure 1. It has one boundary node (PRV in the figure) which is connected to six target (critical) nodes (DMAs in the figure). As there is no connections between target (DMA) nodes, the mass balance equation (4) can be solved and the branch flows $q_j, j = 1, 2, \dots, 6$ will be available, which are equal to the measured target flows in this case. The PRV node is the common origin for all the pipe elements in the network, and the destination nodes are the associated DMAs. The total leakage for each target (DMA) node will be approximated by its MNF for estimation of parameters in leakage model (16).

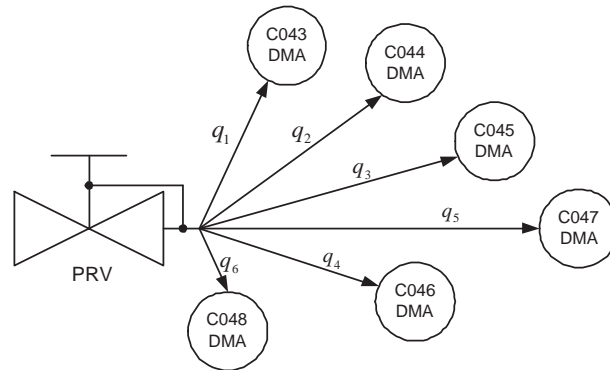


Figure 1: Schematic structure of the considered network

Historical data for parameter estimation

The available historical data files associated with the considered area are provided by a UK water company and they are summarized in Table 1. As can be seen, from this table, the data is not complete. Figure 2 shows the typical plots of the daily PRV outlet head and a DMA node flow over a week.

Table 1: Available data for identification

Nodes	DG2 pressure data	DG2 elevation	Flow data
C043	04/05/07~04/08/08	130[m]	12/06/08 (Thur)
C044	01/04/07~04/08/08	140[m]	01/04/07~13/06/08
C045	01/04/07~04/08/08	148.6[m]	01/04/07~07/07/08
C046	01/04/04~19/06/08	148.55[m]	01/04/07~07/07/08
C047	01/04/07~04/08/08	141.38[m]	01/04/07~31/07/08
C048	09/07/08~18/07/08	141.9[m]	09/07/08~18/07/08
PRV	25/09/07~12/02/08	149[m]	01/01/06~10/10/07

It can be seen that the flow and head plots on weekdays approximately follow the same pattern which are obvious different from the plots on weekends. The minimum night flow occurs between 2:00am and 3:00am and there are two peaks in a daily flow plot. The first peak occurs around 6:45am in weekdays and two hours later on the weekends, the second peak occurs around 6:00pm.

The data collected between 11/01/2008 and 06/02/2008 are selected for modelling because this is the latest period of time over which the measurements for the PRV and most of the DMAs are available as shown in Table 1.

Identification results

Since there is no overlap between the PRV outlet head data and the C043, C048 head data as shown in Table 1, it would not be possible to estimate the resistances

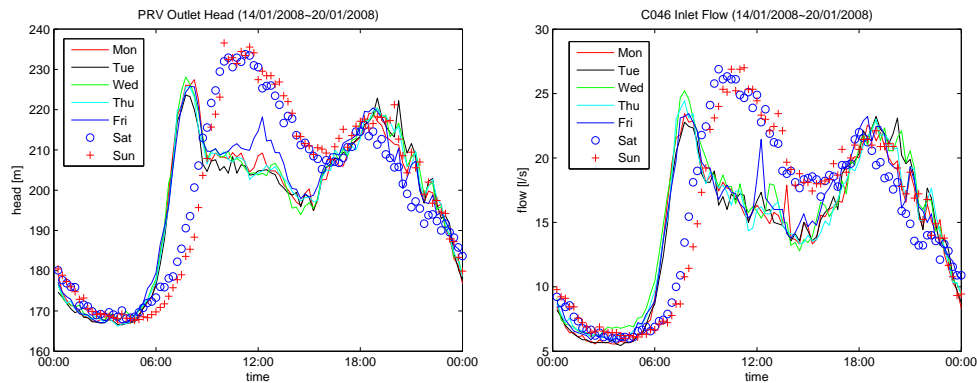


Figure 2: Daily PRV outlet head and DMA flow over a week

between PRV and C043, C048 DMAs. However, as shown in Figure 2, the PRV outlet heads on the weekdays approximately follow the same pattern, the PRV outlet heads taken in the different dates will be used as the replacement data for estimating the resistances between PRV and C043, C048 DMAs. Having removed all the outliers detected in the original data sets, model identification is performed and the results of parameter estimation are summarized in Table 2.

Table 2: Results of parameter estimation with the outliers being removed

DMA	R_j	$\sigma(R_j)$	k_j	$\sigma(k_j)$	α_j	$\sigma(\alpha_j)$
C043	0.7739	4.0945×10^{-2}	3.4763×10^{-2}		1.1	
C044	2.7862	1.8876×10^{-2}	9.6198×10^{-5}	1.0191×10^{-3}	2.5111	3.0285
C045	1.3865	1.4331×10^{-2}	2.2914×10^{-2}		1.1	
C046	0.1495	5.4148×10^{-4}	1.9783×10^{-1}	7.1931×10^{-1}	1.0091	1.0915
C047	0.4480	3.2228×10^{-3}	5.3411×10^{-2}	7.2589×10^{-1}	0.9741	3.8450
C048	0.3971	1.5554×10^{-2}	4.7983×10^{-2}		1.1	

The R_j s in the table are estimated with equation (13) with the associated standard deviations $\sigma(R_j)$ being estimated with equation (14). Figure 3 shows the results of data fitting for C046 and C047 using their R_j estimates. The k_j s and α_j s of the leakage model in Table 2 are estimated by equation (19) with the associated standard deviations being evaluated with equations (20)~(22), where the leakage measurements are approximated by their MNFs.

However, the estimate of leakage exponent for C045 takes negative value ($\hat{\alpha}_j = -3.4095 \times 10^{-1}$). To investigate the problem, the results of leakage data fitting for C045 DMA with the outlier being removed is plotted in Figure 4 and it shows, in overall, a slightly negative correlation between the MNF and pressure. This may indicate that the MNF is not a good estimate of the leakage for C045 DMA. As a remedy, the minimum MNF over the dates between 11/01/2008 and 06/02/2008 (excluding the outlier) is used as the estimate of leakage for C045 DMA and α_j takes the value of 1.1. k_j is then

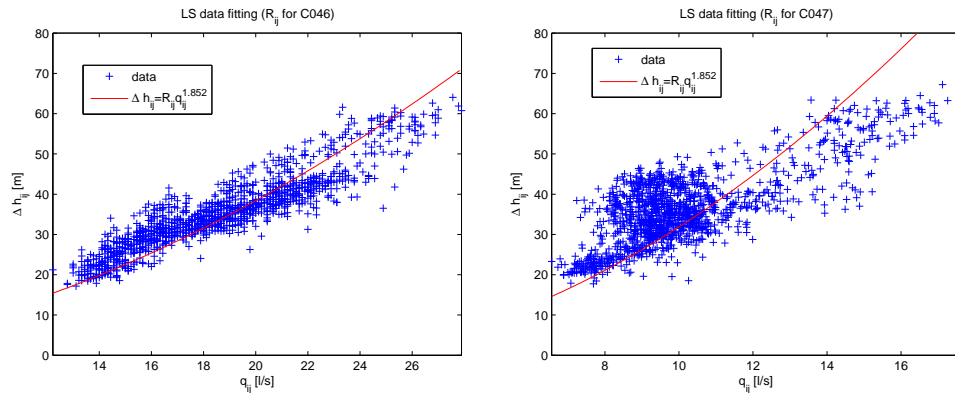


Figure 3: Results of R_j data fitting for C046 and C047 with outliers being removed

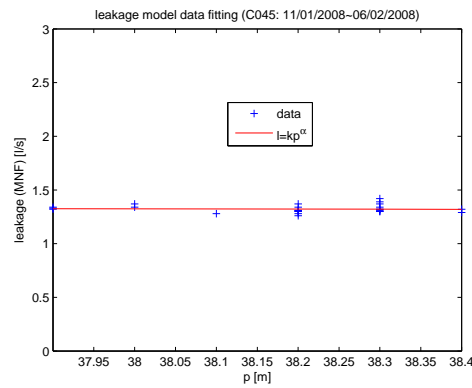


Figure 4: C045 MNF-pressure plot with outlier being removed

obtained as:

$$k_j = \frac{\text{minimum MNF}}{p_{\min MNF}^{1.1}} \tag{23}$$

and this result is shown in Table 2.

The α_j s for C043 and C048 are also assumed to be 1.1 as shown in Table 2 and the corresponding k_j s are estimated with equation (23). This is because: (i) only one-day flow data (i.e. only one MNF) is available for C043 DMA; (ii) there are many missing data in flow and DG2 files for C048, therefore the averages of MNFs and the associated pressures over 09/07/2008-18/07/2008 are used for estimating k_j for C048 DMA with equation (23).

It needs to be pointed out that the estimation errors (standard deviations) of leakage model parameters k_j and α_j evaluated with equations (20)~(22) are quite large in comparison with the values of the corresponding parameter estimates as shown in Table 2. This is because the matrix $\Phi^T \Phi$ in (20) is ill-conditioned. This occurs when the measurements have only a marginal effect on the estimated parameters. In our case study, this is because we do not have the leakage measurements over a sufficient range

of pressure points. The MNFs have to be used in this study to approximate the leakage measurements for parameter estimation and the corresponding pressure range is very small (the pressure variations are not more than 1m at the most). This results in a nearly rank-deficient Φ matrix defined in (18) which implies that the measurements are not “informative” enough and the identified leakage model will not be reliable when the pressure is outside the range of the measured pressure data used for identification.

Conclusions

A method for identifying a steady-state leakage enhanced hydraulic model of a water distribution network from measurements or historical operational data is presented in this paper. Only the head and flow data of the considered nodes are required by the method for identification. The method has been applied for a case study where a model of a real water distribution network provided by a UK water company has been identified from the available historical operational data. The case study also shows that the MNF alone is not “informative” enough for identifying a reliable leakage model and, sometimes, it is even not a good approximate to the leakage.

Further work is being carried out to validate and refine the identified model and to use the identified model for supervisory pressure control. The research is also being performed to develop the appropriate method for pre-processing the raw data, for example, to detect outliers and to deal with missing data etc, so as to facilitate the practical application of the method.

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