A Decomposition-based Evolutionary Algorithm with Clustering and Hierarchical Estimation for Multi-objective Fuzzy Flexible Jobshop Scheduling — Supplementary Document

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I. Explanation of energy consumption calculation

We have created the following graph to visually illustrate the calculation of energy consumption. In Fig. 6, the yellow part represents the total energy consumption. W_1^o represents the unit energy consumption of the machine M_1 under load condition, and W_1^I is the unit energy consumption of the machine under no-load conditions. c_1 is the time it takes for machine M_1 to complete all tasks. $p_{1,1,1}$, $p_{2,1,1}$, and $p_{1,3,1}$ represent the processing time of processes $o_{1,1}$, $o_{2,1}$, and $o_{1,3}$ on machine M_1, respectively. The energy consumption of the machine can be calculated by $(p_{1,1,1}+p_{1,2,1}+p_{1,3,1})\times(W_1^o-W_1^I)+c_1\times W_1^I$. Therefore, it is not necessary to calculate the idle time.

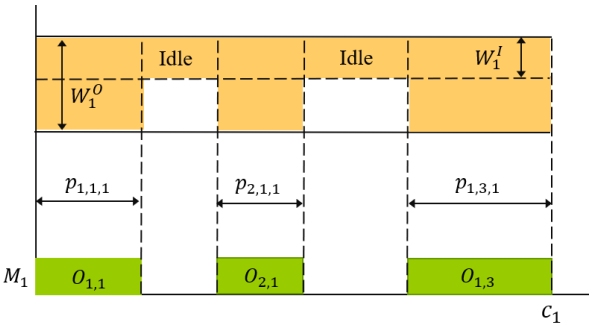

Fig. 6: A simple explanation of energy consumption calculation.

II. Initialization

The execution details of three heuristic initialization rules are described as follows:

Heuristic rule 1:

Let OS and OM be two vectors representing operation code and machine code, respectively. Let J^c be a collection of unfinished jobs. Let NO_j denote the next operation to be processed by job j ($j \in J^c$), $O_{j,k}$ is the k-th operation of job j. h_j is the number of operations for job j. Δ_j is the set of optional machines for job j.

Step 1: Initialization. $J^c = \{1, 2, \ldots, n\}$, $NO_j = 1$ ($j \in J^c$), $iter = 1$.

Step 2: Randomly select j. $j \in J^c$, $m^* = min_{m\in\Delta_j}(\tilde{p}_{j,k,m})$. $\tilde{p}_{j,k,m}$ is the fuzzy processing time of the k-th operation of job j on machines m^*.

Step 3: $OS(iter) = j$, $OM(iter) = OM(m^*)$. $NO_j = NO_j + 1$. $iter = iter + 1$.

Step 4: If $NO_j > h_j$. $J^c = J^c \setminus \{j\}$.

Step 5: If $J^c = \varnothing$, stop. Otherwise, return to Step 2.

Heuristic rule 2:

Let OS and OM be two vectors representing operation code and machine code, respectively. Let J^c be a collection of unfinished jobs. Let NO_j denote the next operation to be processed by job j ($j \in J^c$), $O_{j,k}$ is the k-th operation of job j. h_j is the number of operations for job j. Δ_j is the set of optional machines for job j.

Step 1: Initialization. $J^c = \{1, 2, \ldots, n\}$, $NO_j = 1$ ($j \in J^c$), $iter = 1$.

Step 2: Randomly select j. $j \in J^c$, $m^* \in \Delta_j$ is the earliest machine to complete the job j.

Step 3: $OS(iter) = j$, $OM(iter) = OM(m^*)$. $NO_j = NO_j + 1$. $iter = iter + 1$.

Step 4: If $NO_j > h_j$. $J^c = J^c \setminus \{j\}$.

Step 5: If $J^c = \varnothing$, stop. Otherwise, return to Step 2.

Heuristic rule 3:

Let OS and OM be two vectors representing operation code and machine code, respectively. Let J^c be a collection of unfinished jobs. Let NO_j denote the next operation to be processed by job j ($j \in J^c$), $O_{j,k}$ is the k-th operation of job j. h_j is the number of operations for job j. Δ_j is the set of optional machines for job j.

Step 1: Initialization. $J^c = \{1, 2, \ldots, n\}$, $NO_j = 1$ ($j \in J^c$), $iter = 1$.

Step 2: $\forall j \in J^c$, $m^* \in \Delta_j$ is a random machine.

Step 3: $OS(iter) = j$, $OM(iter) = OM(m^*)$. $NO_j = NO_j + 1$. $iter = iter + 1$.

Step 4: If $NO_j > h_j$. $J^c = J^c \setminus \{j\}$.

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Step 5: If $J^c = \varnothing$, stop. Otherwise, return to Step 2.

III. Offspring generation

In this article, three crossover methods are used. Their execution details are described as follows:

POX:

Let $J = \{1, 2, \ldots, n\}$ be the set of jobs, J_1 is a subset of J, $J_1 = \lfloor n/2 \rfloor$, $J_2 = \{j \mid j \in J, j \notin J_1\}$. S_1 and S_2 are operation codes for two parent solutions. $OS1$ is a vector representing operation code of the offspring.

Step 1: Copy any operations in S_1 that belong to J_1 into $OS1$.

Step 2: Fill the empty positions in $OS1$ with elements belonging to J_2 in S_2.

GPX:

Let $J = \{1, 2, \ldots, n\}$ be the set of job, $0 < l_1 < l_2 <\mid J \mid$, l_1 and l_2 are two random integers. S_1 and S_2 are operation codes for two parent solutions. $OS1$ is a vector representing operation code of the offspring.

Step 1: Randomly select a segment $S_1(l_1 : l_2)$ and copy it into $OS1$ at the position where the first element of the segment occurred in the S_2.

Step 2: Delete the elements belonging to $S_1(l_1 : l_2)$ that first appear in S_2. Fill the remaining elements in S_2 in order to the empty positions in $OS1$.

UX:

Let $M = \{1, 2, \ldots, m\}$ be the set of machines. R is a random vector with elements of 0 or 1. $\mid R \mid = \mid M_1 \mid$, M_1 and M_2 are machine codes for two parent solutions. Exchange the elements in M_1 and M_2 based on R.

Mutation rule 1:

OS and OM are the operation codes and machine codes of the same solution. Let j denote the last element in OS and Δ_j is the set of optional machines for j. Randomly mutate the machine code in OM corresponding to j to another element in Δ_j.

Mutation rule 2:

OS and OM are the operation codes and machine codes of the same solution. m^* is the last machine to complete the task. J^* is a job processed by machine m^* in OS. Randomly mutate the machine code in OM corresponding to J^* to another element in Δ_{J^*}.

Mutation rule 3:

OS and OM are the operation codes and machine codes of the same solution. J^* is a random element in OS, and m^* is the machine with the fastest processing speed for the J^*. Randomly mutate the machine code in OM corresponding to J^* to m^*.

IV. Discussion and analysis of results

We have validated the impact of different initialization methods on algorithm performance through experiments. The experimental details are presented in Table VII. MOEA/DCH1, MOEA/DCH2, and MOEA/DCH3 respectively indicate that the algorithm uses only initialization rules 1, 2, and 3 to generate the initial population. The experimental results indicate that all initialization rules have an undeniable contribution to the performance of the algorithm.

We have verified the contributions of different crossover and mutation methods to the algorithm through experiments, and the results are presented in Table VIII. MOEA/DCH-POX and MOEA/DCH-GPX represent MOEA/DCH independently using POX and GPX as a crossover operator, respectively. MOEA/DCH-MR1, MOEA/DCH-MR2, and MOEA/DCH-MR3 represent the independent use of a mutation operator. From the table, it can be observed that the algorithm's performance is weaker than MOEA/DCH when only one crossover or mutation operator is used. This reveals that different crossover and mutation operators make significant contributions to the algorithm's overall performance.

To validate whether the initialization method proposed by this article can enhance the performance of the comparative algorithms, we conducted a series of experiments. The experimental environment settings are consistent with the revised article, and the comparative results are presented in Tables IX and X. FBEA-I, IAIS-I, and LRVMA-I respectively represent FBEA, IAIS, and LRVMA using the initialization method proposed in our paper. Similarly, MOEA/D-I, NSGA-II-I, and MOEA/D_2N-I respectively represent MOEA/D, NSGA-II, and MOEA/D_2N using the initialization method proposed in our paper. The results of pairwise comparisons shown in the tables reveal that our proposed initialization method does not improve the performance of FBEA, IAIS, and LRVMA. Furthermore, the results presented in the tables indicate that our proposed initialization method can enhance the performance of MOEA/D, NSGA-II, and MOEA/D_2N.

In summary, to ensure fairness, our proposed hybrid initialization strategy is employed by MOEA/D, MOEA/D_ 2N, and NSGA-II. On the other hand, FBEA, IAIS, and LRVMA implement initialization strategies independently designed by their respective original papers.

The calculation of HV heavily depends on the selection of reference points, and unreasonable choices for these points may not accurately reflect the algorithm's performance. The issue you pointed out is a significant concern that must be addressed. In our work, to ensure fairness in the experimental environment, all solutions are normalized using the same standard. Specifically, f_{max} and f_{min} are determined from the set of non-dominated solutions obtained by merging all algorithms. After normalization, the reference point is established as (1.1, 1.1, 1.1). Under this premise, zooming in on the reference point will only equally amplify the HV value without changing the original.

To reinforce the credibility of the preceding conclusion, we conducted a series of experiments where reference points were set at (1.05, 1.05, 1.05), (1.2, 1.2, 1.2), and (1.5, 1.5, 1.5), with the corresponding HV values presented in Tables XI, XII, and XIII, respectively. The experimental results indicate that as the reference point increases, the HV also increases. Importantly, altering the reference point do not modify the original relationship between advantages and disadvantages. vantages.

TABLE VII: Comparison results of HV values obtained by MOEA/DCH under different initialization rules.

	MOEA/DCH	MOEA/DCH1	MOEA/DCH2	MOEA/DCH3
J20M6	7.1373E-01(1.41E-01)	4.7032E-01(8.16E-02)+	4.7780E-01(8.16E-02)+	**7.1421E-01(8.80E-02)=**
J20M7	7.4548E-01(3.19E-02)	2.3657E-01(1.21E-01)+	1.6173E-01(5.48E-02)+	**7.5956E-01(6.12E-02)-**
J20M8	**1.1715E-01(2.17E-02)**	5.1344E-03(4.88E-03)+	1.6963E-03(4.44E-03)+	1.1231E-01(2.30E-02)+
J20M9	**7.7057E-01(8.62E-02)**	5.9332E-02(3.50E-02)+	1.4188E-02(1.89E-02)+	7.4673E-01(1.57E-01)+
J20M10	**5.2989E-01(3.67E-02)**	5.0782E-02(1.78E-02)+	4.4842E-03(6.98E-03)+	4.9630E-01(4.98E-02)+
J30M6	**7.5207E-01(1.13E-01)**	2.5118E-01(9.62E-02)+	2.4652E-01(7.08E-02)+	7.0275E-01(8.49E-02)+
J30M7	**7.0470E-01(5.58E-02)**	1.8423E-01(5.37E-02)+	9.1387E-02(5.31E-02)+	6.9852E-01(3.13E-02)+
J30M8	**4.8146E-01(4.07E-02)**	4.2157E-02(1.50E-02)+	5.4743E-03(7.12E-03)+	4.8174E-01(3.94E-02)+
J30M9	**8.6900E-01(4.34E-02)**	8.5835E-02(4.40E-02)+	2.7027E-03(7.41E-03)+	8.4742E-01(4.41E-02)+
J30M10	**7.2590E-01(4.41E-02)**	9.3394E-02(3.04E-02)+	3.3713E-03(4.40E-03)+	7.2129E-01(2.97E-02)+
J40M6	1.2427E-01(1.12E-02)	8.8678E-03(5.03E-03)+	4.7160E-03(6.22E-03)+	1.2191E-01(7.94E-03)+
J40M7	8.2474E-01(7.81E-02)	5.5508E-02(2.76E-02)+	8.4491E-03(1.46E-02)+	**8.2968E-01(6.29E-02)=**
J40M8	**6.7607E-01(3.19E-02)**	9.3149E-02(3.74E-02)+	1.1744E-02(1.61E-02)+	6.7667E-01(3.19E-02)+
J40M9	**8.7708E-01(2.51E-02)**	5.4917E-02(2.63E-02)+	2.5101E-03(4.95E-03)+	8.5638E-01(6.44E-02)+
J40M10	**8.3427E-01(3.91E-02)**	1.3966E-01(6.04E-02)+	6.9054E-03(1.24E-02)+	8.2610E-01(3.93E-02)+
J50M6	**2.8052E-01(2.77E-02)**	3.0169E-02(1.73E-02)+	8.4848E-03(5.53E-03)+	2.6730E-01(3.14E-02)+
J50M7	8.5107E-01(2.87E-02)	5.5362E-02(2.76E-02)+	1.0100E-02(1.47E-02)+	**8.6164E-01(6.11E-02)-**
J50M8	**5.1173E-01(3.16E-02)**	3.3845E-02(1.25E-02)+	1.5264E-03(3.57E-03)+	5.0597E-01(2.43E-02)=
J50M9	6.1908E-01(4.79E-02)	7.7619E-02(2.36E-02)+	8.4509E-03(6.48E-03)+	**6.4506E-01(5.24E-02)-**
J50M10	6.5885E-01(1.98E-02)	8.8875E-02(2.01E-02)+	7.9728E-03(3.95E-03)+	6.4355E-01(2.15E-02)+
J80M6	**8.7443E-02(4.18E-03)**	4.0520E-03(1.16E-03)+	6.6845E-04(8.87E-04)+	8.6549E-02(5.70E-03)+
J80M7	**9.0396E-01(4.26E-02)**	9.8006E-02(3.40E-02)+	3.2894E-02(2.55E-02)+	8.8746E-01(5.62E-02)+
J80M8	**6.6110E-01(3.22E-02)**	4.4162E-02(1.11E-02)+	4.2941E-03(3.89E-03)+	6.4407E-01(2.69E-02)+
J80M9	8.8971E-01(3.05E-02)	1.3617E-01(4.19E-02)+	1.7102E-02(1.83E-02)+	**9.0446E-01(4.18E-02)-**
J80M10	**7.0541E-01(2.49E-02)**	3.0891E-02(1.01E-02)+	4.0930E-03(3.30E-03)+	6.9095E-01(3.18E-02)+
J100M6	**2.6237E-01(1.48E-02)**	6.8823E-03(4.48E-03)+	1.1512E-03(1.28E-03)+	2.6156E-01(1.28E-02)+
J100M7	**9.1583E-01(5.51E-02)**	1.0013E-01(2.80E-02)+	2.4272E-02(2.39E-02)+	9.0824E-01(3.57E-02)+
J100M8	**4.3565E-01(2.82E-02)**	1.0674E-02(3.65E-03)+	7.0383E-04(1.36E-03)+	4.2676E-01(3.44E-02)+
J100M9	**9.2433E-01(3.94E-02)**	4.5016E-02(2.23E-02)+	8.9490E-03(1.25E-02)+	9.1929E-01(4.91E-02)+
J100M10	6.3643E-01(1.74E-02)	1.1811E-03(1.21E-03)+	1.4018E-03(1.75E-03)+	6.1632E-01(3.02E-02)+
	+/=/-	30/0/0	30/0/0	24/2/4

TABLE VIII: Comparison results of HV values obtained by MOEA/DCH under different crossover and mutation operators.

	MOEA/DCH	MOEA/DCH-POX	MOEA/DCH2-GPX	MOEA/DCH-MR1	MOEA/DCH-MR2	MOEA/DCH-MR3
J20M6	**6.6165E-01(1.17E-01)**	6.3205E-01(1.43E-01)+	5.4309E-01(8.48E-02)+	3.0886E-01(5.38E-02)+	4.6328E-01(3.66E-02)+	4.8163E-01(7.79E-02)+
J20M7	**6.0240E-01(8.28E-02)**	5.4468E-01(8.37E-02)+	5.1471E-01(1.24E-01)+	3.5062E-01(9.62E-02)+	4.2227E-01(1.24E-01)+	4.2967E-01(9.69E-02)+
J20M8	**5.0682E-01(2.71E-01)**	4.6424E-01(2.09E-01)+	2.4174E-01(1.74E-01)+	1.1690E-01(7.82E-02)+	2.3164E-01(2.21E-01)+	3.6873E-01(2.49E-01)+
J20M9	**5.9257E-01(6.90E-02)**	5.5249E-01(1.31E-01)+	2.8678E-01(1.66E-01)+	1.8802E-01(7.13E-02)+	4.3244E-01(1.43E-01)+	3.1839E-01(1.50E-01)+
J20M10	3.9791E-01(6.53E-02)	**4.0181E-01(2.67E-01)-**	8.8292E-02(1.16E-01)+	1.0962E-01(2.02E-01)+	7.4003E-02(7.16E-02)+	2.4900E-01(2.50E-01)+
J30M6	6.2073E-01(2.24E-01)	**6.5228E-01(1.32E-01)-**	4.7137E-01(7.54E-02)+	2.5329E-01(8.46E-02)+	4.0792E-01(1.35E-01)+	4.0070E-01(1.30E-01)+
J30M7	**7.1087E-01(4.55E-02)**	6.3899E-01(1.39E-01)+	3.9451E-01(1.00E-01)+	2.6387E-01(1.60E-01)+	5.0006E-01(1.28E-01)+	3.2478E-01(1.47E-01)+
J30M8	**6.9243E-01(1.76E-01)**	4.6467E-01(1.85E-01)+	3.6816E-01(9.99E-02)+	3.9390E-01(5.36E-02)+	3.8540E-01(2.12E-01)+	4.9810E-01(9.51E-02)+
J30M9	**8.0540E-01(2.85E-01)**	5.6420E-01(4.19E-02)+	3.7408E-01(2.55E-01)+	1.8997E-01(7.88E-02)+	3.5824E-01(1.75E-01)+	6.5431E-01(2.53E-01)+
J30M10	3.5680E-01(1.63E-01)	**4.4766E-01(2.64E-01)-**	9.4695E-02(8.62E-02)+	2.2073E-01(1.18E-01)+	3.2851E-01(1.88E-01)+	2.1021E-01(1.26E-01)+
J40M6	5.8908E-01(5.50E-02)	**5.9926E-01(1.58E-01)=**	5.3239E-01(7.69E-02)+	2.7771E-01(5.12E-02)+	4.0962E-01(2.18E-01)+	2.2643E-01(3.87E-02)+
J40M7	**6.0543E-01(1.64E-01)**	4.4275E-01(2.20E-01)+	3.5090E-01(1.48E-01)+	2.2389E-01(1.45E-01)+	5.0294E-01(7.30E-02)+	1.4610E-01(6.22E-02)+
J40M8	**6.5723E-01(1.42E-01)**	4.2453E-01(8.41E-02)+	3.1277E-01(1.99E-01)+	1.6437E-01(7.30E-02)+	4.4654E-01(1.21E-01)+	3.7080E-01(1.02E-01)+
J40M9	**6.0819E-01(1.34E-01)**	4.8916E-01(6.23E-02)+	2.5731E-01(8.05E-02)+	2.1448E-01(7.90E-02)+	4.1546E-01(1.38E-01)+	3.8059E-01(6.87E-02)+
J40M10	**8.2983E-01(2.69E-01)**	5.2240E-01(1.90E-01)+	1.5269E-01(1.11E-01)+	1.0551E-01(6.08E-02)+	3.2287E-01(1.71E-01)+	1.2062E-01(6.72E-02)+
J50M6	**5.9740E-01(1.34E-01)**	5.8159E-01(1.02E-01)+	6.0191E-01(1.18E-01)+	3.2309E-01(5.16E-02)+	3.8568E-01(8.51E-02)+	3.6473E-01(9.11E-02)+
J50M7	**4.2235E-01(4.64E-02)**	2.9977E-01(6.32E-02)+	2.9095E-01(8.73E-02)+	3.5602E-01(1.76E-01)+	1.4300E-01(7.55E-02)+	1.9361E-01(6.11E-02)+
J50M8	**7.2001E-01(1.98E-01)**	4.2890E-01(1.99E-01)+	2.8590E-01(2.87E-01)+	1.7202E-01(9.96E-02)+	7.1117E-02(1.12E-01)+	2.0088E-01(1.63E-01)+
J50M9	**8.2583E-01(1.25E-01)**	3.1266E-01(2.00E-01)+	3.6101E-01(1.48E-01)+	1.9579E-01(5.77E-02)+	3.1941E-01(5.70E-02)+	3.4689E-01(9.00E-02)+
J50M10	**9.7799E-01(4.16E-01)**	4.7817E-01(1.86E-01)+	1.9652E-01(1.27E-01)+	1.4487E-01(7.09E-02)+	1.1765E-01(9.15E-02)+	1.2668E-01(7.02E-02)+
J80M6	**6.9772E-01(1.78E-01)**	3.6722E-01(1.20E-01)+	4.9698E-01(1.43E-01)+	4.0331E-01(1.75E-01)+	2.3141E-01(1.10E-01)+	4.1210E-01(1.17E-01)+
J80M7	**6.9134E-01(8.35E-02)**	3.8106E-01(2.12E-01)+	3.2985E-01(1.10E-01)+	1.6415E-01(6.96E-02)+	2.1962E-01(9.85E-02)+	1.9172E-01(1.79E-01)+
J80M8	**6.7145E-01(2.21E-01)**	2.9587E-01(1.49E-01)+	4.5975E-01(1.22E-01)+	4.2003E-02(4.70E-02)+	2.9993E-01(1.26E-01)+	1.1829E-01(1.42E-01)+
J80M9	**8.9013E-01(1.69E-01)**	3.3803E-01(1.53E-01)+	2.7560E-01(1.74E-01)+	2.7635E-01(1.95E-01)+	1.3787E-01(1.28E-01)+	2.8636E-01(2.46E-01)+
J80M10	**7.5595E-01(3.08E-01)**	4.0911E-01(1.74E-01)+	2.3754E-01(1.31E-01)+	1.0909E-01(8.97E-02)+	1.9229E-01(1.27E-01)+	2.5777E-01(8.07E-02)+
J100M6	**5.1431E-01(1.72E-01)**	3.2351E-01(2.25E-01)+	3.0297E-01(1.71E-01)+	1.1105E-01(6.34E-02)+	1.0129E-01(1.39E-01)+	2.2271E-01(1.75E-01)+
J100M7	**6.5369E-01(2.85E-01)**	2.6078E-01(1.50E-01)+	5.1228E-01(1.52E-01)+	2.8594E-01(6.74E-02)+	2.6547E-01(1.41E-01)+	2.1971E-01(1.10E-01)+
J100M8	**9.5797E-01(1.88E-01)**	4.9119E-01(2.36E-01)+	4.7392E-01(9.61E-02)+	6.0427E-02(8.34E-02)+	3.1191E-01(1.59E-02)+	1.3014E-01(1.09E-01)+
J100M9	**8.5797E-01(8.23E-02)**	4.0873E-01(3.04E-01)+	3.5800E-01(1.33E-01)+	2.1625E-01(5.96E-02)+	2.2888E-01(1.94E-01)+	2.7058E-01(6.48E-02)+
J100M10	**9.1797E-01(5.40E-01)**	2.9849E-01(1.75E-01)+	2.0063E-01(1.34E-01)+	1.1200E-01(1.10E-01)+	1.3745E-01(3.01E-02)+	1.0858E-01(1.02E-01)+
	+/=/-	2026/2/3	30/0/0	30/0/0	30/0/0	30/0/0

TABLE IX: Comparison results of HV values obtained by FBEA, IAIS, and LRVMA under different initialization rules.

	FBEA	FBEA-I	IAIS	IAIS-I	LRVMA	LRVMA-I
J20M6	**4.3491E-01(5.19E-02)**	4.2629E-01(5.11E-02)	**8.2106E-01(7.52E-02)**	8.0481E-01(6.40E-02)	**6.4300E-01(6.10E-02)**	6.3027E-01(5.46E-02)
J20M7	2.2634E-01(3.72E-02)	**2.3285E-01(4.10E-02)**	**6.6794E-01(7.01E-02)**	6.5471E-01(7.09E-02)	**4.5786E-01(9.36E-02)**	4.4880E-01(9.45E-02)
J20M8	**5.6727E-02(1.03E-02)**	5.5604E-02(1.08E-02)	**1.2051E-01(2.03E-02)**	1.1813E-01(2.06E-02)	**5.8508E-02(1.55E-02)**	5.7350E-02(1.71E-02)
J20M9	**2.3708E-01(4.00E-02)**	2.3238E-01(3.36E-02)	**4.5738E-01(1.00E-01)**	4.4833E-01(1.06E-01)	**2.1290E-01(9.90E-02)**	2.0868E-01(8.70E-02)
J20M10	**6.4672E-02(2.03E-02)**	6.3391E-02(2.00E-02)	**1.7677E-01(3.08E-02)**	1.7327E-01(3.03E-02)	**6.9722E-02(1.59E-02)**	6.8342E-02(1.73E-02)
J30M6	1.4579E-01(2.20E-02)	**1.4702E-01(2.31E-02)**	**4.1594E-01(4.34E-02)**	4.0770E-01(4.51E-02)	**3.0515E-01(3.62E-02)**	2.9911E-01(3.46E-02)
J30M7	**2.1741E-01(3.30E-02)**	2.1311E-01(3.25E-02)	**3.5218E-01(7.93E-02)**	3.4521E-01(7.46E-02)	1.8170E-01(7.35E-02)	**2.0750E-01(7.48E-02)**
J30M8	3.6069E-02(1.64E-02)	**3.6355E-02(1.54E-02)**	**1.6040E-01(4.27E-02)**	1.5723E-01(3.73E-02)	**4.8077E-02(2.77E-02)**	4.7125E-02(2.81E-02)
J30M9	**2.3888E-01(7.02E-02)**	2.3415E-01(6.19E-02)	**4.7681E-01(1.70E-01)**	4.6736E-01(1.52E-01)	**1.5028E-01(8.83E-02)**	1.4730E-01(8.93E-02)
J30M10	**1.0236E-01(1.15E-01)**	1.0033E-01(1.06E-01)	**1.1869E-01(4.02E-02)**	1.1634E-01(4.12E-02)	**3.5556E-02(2.62E-02)**	3.4852E-02(2.64E-02)
J40M6	3.5826E-02(6.35E-03)	**3.6016E-02(7.40E-03)**	**6.9653E-02(1.07E-02)**	6.8273E-02(1.08E-02)	**3.4914E-02(9.26E-03)**	3.4223E-02(8.65E-03)
J40M7	3.40771E-01(3.24E-02)	**3.4782E-01(2.99E-02)**	**8.4457E-01(4.94E-02)**	8.2784E-01(4.25E-02)	**6.0565E-01(6.00E-02)**	5.9366E-01(6.60E-02)
J40M8	2.1394E-01(1.84E-01)	**2.1763E-01(1.72E-01)**	**7.5774E-01(7.42E-02)**	7.4273E-01(7.38E-02)	**4.4075E-01(6.61E-02)**	4.3203E-01(6.31E-02)
J40M9	**4.5125E-01(5.85E-02)**	4.4231E-01(6.19E-02)	**9.3599E-01(3.20E-02)**	9.1745E-01(2.86E-02)	**6.5384E-01(4.45E-02)**	6.4090E-01(4.66E-02)
J40M10	**8.4944E-02(6.51E-02)**	8.3262E-02(5.84E-02)	**8.0907E-01(6.51E-02)**	7.9305E-01(7.54E-02)	**3.3196E-01(9.16E-02)**	3.2539E-01(9.14E-02)
J50M6	1.7264E-01(2.19E-02)	**1.7517E-01(2.17E-02)**	**3.9798E-01(3.09E-02)**	3.9010E-01(3.39E-02)	**2.3922E-01(3.06E-02)**	2.3448E-01(3.01E-02)
J50M7	**2.1617E-01(2.91E-02)**	2.1189E-01(2.65E-02)	**3.7366E-01(1.24E-01)**	3.6626E-01(1.20E-01)	**1.0613E-01(9.22E-02)**	1.0403E-01(9.23E-02)
J50M8	**4.8622E-02(1.15E-02)**	4.7659E-02(1.22E-02)	**1.2662E-01(2.04E-02)**	1.2411E-01(2.11E-02)	**4.3010E-02(2.13E-02)**	4.2158E-02(2.20E-02)
J50M9	9.0960E-02(1.28E-02)	**9.1159E-02(1.36E-02)**	**1.8883E-01(2.76E-02)**	1.8509E-01(3.16E-02)	1.0501E-01(2.51E-02)	**1.1293E-01(2.56E-02)**
J50M10	1.0138E-01(6.39E-02)	**1.0335E-01(5.87E-02)**	**1.3448E-01(8.29E-02)**	1.3182E-01(9.26E-02)	**5.0790E-02(1.59E-02)**	4.9784E-02(1.43E-02)
J80M6	**2.0965E-02(7.20E-02)**	2.0550E-02(7.34E-02)	**1.8201E-02(1.81E-03)**	1.7841E-02(1.87E-03)	**7.3722E-03(3.03E-03)**	7.2262E-03(3.53E-03)
J80M7	2.1396E-01(4.18E-02)	**2.1764E-01(4.19E-02)**	**8.2161E-01(5.37E-02)**	8.0534E-01(5.78E-02)	**4.1304E-01(7.62E-02)**	4.0486E-01(7.77E-02)
J80M8	6.7272E-02(1.36E-01)	**6.8514E-02(1.40E-01)**	**1.3225E-01(1.69E-01)**	1.2963E-01(1.88E-01)	**3.1188E-02(1.41E-02)**	3.0571E-02(1.43E-02)
J80M9	**4.5627E-01(2.78E-02)**	4.4723E-01(2.72E-02)	**9.2793E-01(3.90E-02)**	9.0956E-01(3.65E-02)	**6.6549E-01(6.20E-02)**	6.5231E-01(5.76E-02)
J80M10	**7.7628E-02(8.75E-02)**	7.6091E-02(9.14E-02)	**1.2029E-01(2.52E-02)**	1.1791E-01(2.68E-02)	**6.3491E-02(2.59E-02)**	6.2234E-02(2.78E-02)
J100M6	**5.7102E-02(3.23E-02)**	5.5971E-02(3.41E-02)	**1.2116E-01(1.20E-02)**	1.1877E-01(1.28E-02)	**7.0415E-02(1.10E-02)**	6.9021E-02(1.03E-02)
J100M7	**2.2322E-01(3.67E-02)**	2.1880E-01(3.60E-02)	**3.4382E-01(9.45E-02)**	3.3701E-01(8.70E-02)	**1.0693E-01(8.68E-02)**	1.0481E-01(9.94E-02)
J100M8	**4.9745E-02(1.55E-01)**	4.8760E-02(1.64E-01)	**6.1936E-02(1.16E-02)**	6.0710E-02(1.06E-02)	**2.8138E-02(1.02E-02)**	2.7581E-02(1.01E-02)
J100M9	**1.2149E-01(8.10E-03)**	1.1908E-01(8.11E-03)	**1.0555E-01(2.28E-02)**	1.0346E-01(2.09E-02)	**1.0035E-01(3.42E-02)**	9.8360E-02(3.58E-02)
J100M10	**4.2106E-02(1.15E-02)**	4.1272E-02(1.07E-02)	**9.7721E-02(1.48E-02)**	9.5786E-02(1.46E-02)	**5.7467E-02(1.47E-02)**	5.6329E-02(1.52E-02)

TABLE X: Comparison results of HV values obtained by MOEA/D, MOEA/D_2N, and NSGA-II under different initialization rules.

	NSGA-II	NSGAII-I	MOEA/D	MOEA/D-I	MOEA/D_2N	MOEA/D_2N-I
J20M6	3.2287E-01(4.82E-02)	**3.4514E-01(4.92E-02)**	4.6793E-01(4.95E-02)	**4.9651E-01(5.28E-02)**	**4.6102E-01(5.30E-02)**	4.5480E-01(5.01E-02)
J20M7	1.4980E-01(3.09E-02)	**1.5495E-01(3.29E-02)**	3.2009E-01(4.24E-02)	**3.2947E-01(4.45E-02)**	2.4217E-01(6.73E-02)	**2.6312E-01(6.53E-02)**
J20M8	8.3239E-02(1.88E-01)	**9.1469E-02(1.96E-01)**	**1.1272E-01(2.00E-01)**	1.0639E-01(2.01E-01)	1.2135E-01(2.13E-01)	**1.2624E-01(1.97E-01)**
J20M9	**8.9431E-02(2.93E-02)**	8.8928E-02(3.10E-02)	2.1308E-01(4.54E-02)	**2.2004E-01(4.47E-02)**	1.9304E-01(3.95E-02)	**2.0166E-01(4.16E-02)**
J20M10	4.1775E-02(5.87E-02)	**4.2423E-02(5.52E-02)**	1.2673E-01(1.99E-01)	**1.3157E-01(2.10E-01)**	8.4243E-02(1.27E-01)	**8.5805E-02(1.22E-01)**
J30M6	1.3491E-01(1.11E-01)	**1.4060E-01(1.10E-01)**	2.3576E-01(1.68E-01)	**2.5628E-01(1.86E-01)**	**1.9939E-01(2.89E-02)**	1.8057E-01(2.61E-02)
J30M7	1.6236E-01(2.55E-02)	**1.6913E-01(2.71E-02)**	2.4549E-01(4.33E-02)	**2.5832E-01(4.27E-02)**	2.3159E-01(3.18E-02)	**2.3175E-01(3.34E-02)**
J30M8	2.0516E-02(5.89E-02)	**2.2620E-02(5.50E-02)**	6.0479E-02(7.89E-02)	**6.4345E-02(7.51E-02)**	1.3116E-01(1.80E-01)	**1.3670E-01(1.67E-01)**
J30M9	7.0058E-02(2.10E-02)	**7.6058E-02(2.07E-02)**	2.1951E-01(5.34E-02)	**2.2702E-01(5.70E-02)**	1.8171E-01(4.81E-02)	**1.9150E-01(4.41E-02)**
J30M10	1.1380E-01(8.08E-02)	**1.0658E-01(7.39E-02)**	1.4181E-01(1.21E-01)	**1.4963E-01(1.23E-01)**	**2.4719E-01(8.15E-02)**	2.3896E-01(9.38E-02)
J40M6	5.2433E-02(1.29E-01)	**5.6580E-02(1.27E-01)**	7.1451E-02(1.26E-01)	**7.3966E-02(1.22E-01)**	1.1506E-01(2.52E-01)	**1.2318E-01(2.35E-01)**
J40M7	2.4445E-01(4.34E-02)	**2.7096E-01(4.09E-02)**	4.3368E-01(4.15E-02)	**4.6269E-01(4.55E-02)**	3.9455E-01(3.52E-02)	**4.0236E-01(3.51E-02)**
J40M8	2.1608E-01(2.01E-01)	**2.1946E-01(2.02E-01)**	3.2283E-01(1.63E-01)	**3.2971E-01(1.70E-01)**	2.9334E-01(1.75E-01)	**3.2073E-01(1.72E-01)**
J40M9	3.8865E-01(4.55E-02)	**3.9909E-01(4.79E-02)**	**5.8206E-01(3.45E-02)**	5.7142E-01(3.38E-02)	5.3410E-01(5.15E-02)	**5.4225E-01(4.34E-02)**
J40M10	**5.1903E-02(1.22E-01)**	5.3803E-02(1.19E-01)	2.7321E-01(1.98E-01)	**2.9929E-01(2.04E-01)**	**2.2624E-01(1.52E-01)**	2.2043E-01(1.49E-01)
J50M6	1.5151E-01(3.51E-02)	**1.6651E-01(3.78E-02)**	2.2626E-01(2.27E-02)	**2.4975E-01(2.40E-02)**	2.3238E-01(9.12E-02)	**2.5488E-01(9.77E-02)**
J50M7	5.9421E-02(2.14E-02)	**5.9805E-02(1.98E-02)**	2.0453E-01(5.59E-02)	**2.0611E-01(5.30E-02)**	1.7261E-01(4.27E-02)	**1.7615E-01(3.67E-02)**
J50M8	5.6666E-02(1.20E-01)	**5.6940E-02(1.15E-01)**	**1.8967E-01(2.40E-01)**	1.7586E-01(2.42E-01)	**1.9592E-01(2.05E-01)**	1.9003E-01(2.08E-01)
J50M9	1.2124E-01(1.23E-02)	**1.2587E-01(1.23E-02)**	1.2116E-01(1.06E-02)	**1.2471E-01(1.14E-02)**	2.1903E-01(9.98E-03)	**2.2729E-01(1.06E-02)**
J50M10	6.2269E-02(4.75E-02)	**6.3522E-02(4.76E-02)**	1.7786E-01(4.10E-02)	**1.7821E-01(4.19E-02)**	2.3201E-01(1.46E-02)	**2.3384E-01(1.37E-02)**
J80M6	1.0575E-01(2.51E-01)	**1.0629E-01(2.48E-01)**	6.2477E-02(1.53E-01)	**6.3551E-02(1.56E-01)**	1.7479E-01(3.19E-01)	**1.8214E-01(3.29E-01)**
J80M7	7.2077E-02(3.62E-02)	**7.9604E-02(3.70E-02)**	3.2976E-01(4.77E-02)	**3.6155E-01(5.13E-02)**	2.9381E-01(5.70E-02)	**3.2392E-01(6.32E-02)**
J80M8	1.6475E-02(3.91E-02)	**1.7696E-02(3.55E-02)**	7.4979E-02(1.12E-01)	**7.7554E-02(1.08E-01)**	7.2175E-02(8.52E-02)	**7.6817E-02(9.49E-02)**
J80M9	3.8545E-01(4.29E-02)	**3.8695E-01(4.70E-02)**	5.5249E-01(4.13E-02)	**6.0461E-01(4.46E-02)**	**5.9326E-01(2.60E-02)**	5.6942E-01(2.66E-02)
J80M10	6.6806E-02(4.80E-02)	**7.3283E-02(5.46E-02)**	1.0943E-01(1.21E-01)	**1.2036E-01(1.16E-01)**	2.2947E-01(4.70E-02)	**2.5589E-01(4.12E-02)**
J100M6	9.0405E-02(1.91E-01)	**9.9677E-02(1.76E-01)**	**8.6479E-02(2.58E-02)**	8.4080E-02(2.36E-02)	1.4396E-01(2.35E-01)	**1.4740E-01(2.52E-01)**
J100M7	7.5175E-02(3.29E-02)	**8.2182E-02(3.73E-02)**	3.3458E-01(3.80E-02)	**3.3681E-01(4.32E-02)**	**2.9696E-01(3.29E-02)**	2.9219E-01(3.43E-02)
J100M8	5.1532E-02(1.51E-01)	**5.1914E-02(1.56E-01)**	2.9089E-02(4.04E-03)	**2.9559E-02(3.81E-03)**	9.3289E-02(1.91E-01)	**1.0400E-01(1.93E-01)**
J100M9	**1.3475E-01(9.79E-03)**	1.3390E-01(1.03E-02)	**1.5883E-01(7.42E-03)**	1.4902E-01(7.77E-03)	3.2340E-01(8.70E-03)	**3.2342E-01(8.19E-03)**
J100M10	**1.3734E-01(7.92E-02)**	1.2941E-01(8.06E-02)	2.5019E-01(6.34E-02)	**2.7737E-01(6.10E-02)**	3.6281E-01(4.62E-02)	**4.0435E-01(4.53E-02)**

TABLE XI: HV values obtained by all algorithms, the reference point is set to (1.05, 1.05, 1.05).

	MOEA/DCH	FBEA	IAIS	NSGAII	MOEA/D	LRVMA	MOEA/D_2N
J20M6	**7.9257E-01(5.74E-02)**	4.0860E-01(5.07E-02)+	8.0451E-01(7.26E-02)+	3.1815E-01(4.63E-02)+	4.7109E-01(5.60E-02)+	6.2119E-01(6.06E-02)+	4.2864E-01(5.46E-02)+
J20M7	**6.6418E-01(3.67E-02)**	1.9807E-01(4.14E-02)+	6.4662E-01(6.96E-02)+	1.2855E-01(3.18E-02)+	3.0185E-01(4.54E-02)+	4.3208E-01(8.89E-02)+	2.3565E-01(6.45E-02)+
J20M8	**1.1481E-01(1.01E-02)**	4.2283E-02(8.61E-03)+	9.2332E-02(1.81E-02)+	7.8833E-02(1.85E-01)+	9.0212E-02(2.01E-01)+	3.7991E-02(1.54E-02)+	1.0991E-01(2.09E-01)+
J20M9	**7.0129E-01(6.88E-02)**	2.1167E-01(3.67E-02)+	4.3119E-01(1.13E-01)+	7.0915E-02(2.91E-02)+	1.9552E-01(4.72E-02)+	1.8920E-01(8.74E-02)+	1.7787E-01(3.96E-02)+
J20M10	**1.9780E-01(2.27E-02)**	5.0423E-02(1.79E-02)+	1.4495E-01(3.03E-02)+	3.0091E-02(5.76E-02)+	1.1558E-01(2.03E-01)+	4.8711E-02(1.63E-02)+	7.1188E-02(1.33E-01)+
J30M6	**3.7064E-01(3.00E-02)**	1.1395E-01(2.16E-02)+	3.9003E-01(4.80E-02)+	1.1645E-01(1.04E-01)+	2.3109E-01(1.79E-01)+	2.7793E-01(3.49E-02)+	1.5465E-01(2.74E-02)+
J30M7	**4.9360E-01(2.93E-02)**	1.9376E-01(3.09E-02)+	3.2301E-01(8.04E-02)+	1.4404E-01(2.34E-02)+	2.2928E-01(4.10E-02)+	1.8294E-01(7.02E-02)+	2.0719E-01(3.29E-02)+
J30M8	**1.6062E-01(2.13E-02)**	2.5813E-02(1.41E-02)+	1.2791E-01(3.74E-02)+	1.6844E-02(5.48E-02)+	4.9281E-02(7.73E-02)+	3.0421E-02(2.47E-02)+	1.1835E-01(1.67E-01)+
J30M9	**7.0491E-01(6.45E-02)**	2.1438E-01(6.56E-02)+	4.5083E-01(1.65E-01)+	5.9210E-02(2.11E-02)+	2.0177E-01(5.85E-02)+	1.2687E-01(8.48E-02)+	1.6875E-01(4.41E-02)+
J30M10	**1.7926E-01(1.79E-02)**	8.4964E-02(1.11E-01)+	8.0843E-02(3.92E-02)+	8.1748E-02(7.71E-02)+	1.2261E-01(1.25E-01)+	1.9095E-02(2.02E-02)+	1.0848E-01(8.82E-02)+
J40M6	7.7490E-02(7.21E-03)	2.5730E-02(5.92E-03)+	5.4189E-02(1.08E-02)+	4.6752E-02(1.27E-01)+	6.1013E-02(1.26E-01)+	2.3616E-02(8.14E-03)+	**1.1093E-01(2.43E-01)-**
J40M7	**7.4340E-01(4.71E-02)**	3.2071E-01(3.01E-02)+	6.2875E-01(4.75E-02)+	2.4454E-01(4.50E-02)+	4.3733E-01(4.44E-02)+	5.8311E-01(6.49E-02)+	3.7608E-01(3.64E-02)+
J40M8	**5.0153E-01(5.81E-02)**	1.9462E-01(1.85E-01)+	4.3923E-01(7.08E-02)+	1.9700E-01(2.07E-01)+	3.0390E-01(1.60E-01)+	4.1540E-01(7.01E-02)+	2.9518E-01(1.70E-01)+
J40M9	**7.6864E-01(2.88E-02)**	4.2435E-01(5.84E-02)+	7.2330E-01(3.32E-02)+	3.7171E-01(4.73E-02)+	5.4758E-01(3.38E-02)+	6.3208E-01(4.93E-02)+	5.1751E-01(4.85E-02)+
J40M10	**5.0969E-01(6.24E-02)**	5.9050E-02(5.46E-02)+	4.1219E-01(7.26E-02)+	4.4775E-02(1.26E-01)+	2.7113E-01(2.07E-01)+	3.0398E-01(8.65E-02)+	1.9188E-01(1.47E-01)+
J50M6	**3.8640E-01(3.10E-02)**	1.5106E-01(2.28E-02)+	3.6365E-01(3.20E-02)+	1.4263E-01(3.60E-02)+	2.2424E-01(2.19E-02)+	2.1458E-01(3.26E-02)+	2.2872E-01(8.78E-02)+
J50M7	**5.5729E-01(3.31E-02)**	1.9043E-01(2.77E-02)+	3.2550E-01(1.30E-01)+	4.4410E-02(1.87E-02)+	1.8154E-01(5.34E-02)+	8.5633E-02(8.64E-02)+	1.5280E-01(3.91E-02)+
J50M8	1.3072E-01(1.71E-02)	3.5851E-02(9.49E-03)+	9.3962E-02(2.14E-02)+	4.1575E-02(1.16E-01)+	1.5825E-01(2.38E-01)=	2.3618E-02(1.76E-02)+	**1.5824E-01(2.24E-01)-**
J50M9	**1.8542E-01(2.40E-02)**	7.3791E-02(1.14E-02)+	1.5198E-01(2.88E-02)+	9.8650E-02(1.02E-02)+	1.0169E-01(9.69E-03)+	7.5407E-02(2.37E-02)+	1.9390E-01(1.04E-02)
J50M10	9.6718E-02(1.12E-02)	8.5628E-02(5.95E-02)=	1.0291E-01(9.12E-02)=	4.3771E-02(4.14E-02)+	1.5337E-01(3.91E-02)-	2.9932E-02(1.25E-02)+	**2.0159E-01(1.46E-02)-**
J80M6	1.1077E-02(1.35E-03)	1.2475E-02(6.46E-02)=	**1.0023E-02(1.59E-03)-**	1.0147E-01(2.46E-01)+	5.6639E-02(1.58E-01)-	2.9780E-03(2.21E-03)+	1.7536E-01(3.50E-01)=
J80M7	**5.9169E-01(6.80E-02)**	1.9087E-01(4.21E-02)+	8.0494E-01(6.10E-02)+	6.0323E-02(3.32E-02)+	3.3469E-01(5.28E-02)+	3.8706E-01(7.51E-02)+	2.9669E-01(6.11E-02)+
J80M8	7.1912E-02(1.23E-02)	5.8274E-02(1.28E-01)+	**1.0841E-01(1.91E-01)-**	1.2320E-02(3.47E-02)+	6.3809E-02(1.09E-01)+	1.7227E-02(1.08E-02)+	5.5270E-02(9.22E-02)+
J80M9	**6.8067E-01(4.41E-02)**	4.3032E-01(2.90E-02)+	9.1497E-01(3.83E-02)+	3.6052E-01(4.59E-02)+	5.8174E-01(4.21E-02)+	6.4435E-01(6.39E-02)+	5.4544E-01(2.84E-02)+
J80M10	1.0598E-01(1.38E-02)	6.3599E-02(9.23E-02)+	8.1474E-02(2.43E-02)+	5.1804E-02(5.04E-02)+	9.7835E-02(1.16E-01)+	3.8744E-02(2.36E-02)+	**2.2329E-01(4.39E-02)-**
J100M6	9.3422E-02(1.18E-02)	4.2945E-02(2.85E-02)+	1.0296E-01(1.19E-02)=	8.7156E-02(1.71E-01)+	6.7722E-02(1.84E-02)+	5.6013E-02(1.02E-02)+	**1.3253E-01(2.35E-02)=**
J100M7	**5.8807E-01(7.82E-02)**	1.9713E-01(3.62E-02)+	3.1310E-01(9.03E-02)+	6.4988E-02(3.34E-02)+	3.0970E-01(3.98E-02)+	8.6489E-02(8.21E-02)+	2.6626E-01(3.31E-02)-
J100M8	3.7670E-02(1.07E-02)	4.3443E-02(1.60E-01)+	4.0342E-02(1.03E-02)-	4.7144E-02(1.45E-01)+	1.9356E-02(3.20E-03)+	1.5524E-02(8.57E-03)+	**8.6311E-02(2.08E-01)-**
J100M9	1.6994E-01(1.55E-02)	9.8793E-02(7.02E-03)+	6.4891E-02(2.15E-02)-	1.0190E-01(9.18E-03)+	**1.2105E-01(7.25E-03)-**	7.1271E-02(2.87E-02)+	1.0459E-01(8.28E-03)+
J100M10	8.0994E-02(8.81E-03)	2.9162E-02(9.11E-03)+	5.9739E-02(1.41E-02)+	1.0503E-01(7.34E-02)-	**2.5002E-01(6.91E-02)-**	3.3551E-02(1.26E-02)+	2.1362E-01(4.69E-02)-
	+/=/-	28/2/0	24/2/4	29/0/1	25/1/4	30/0/0	23/1/6

TABLE XII: HV values obtained by all algorithms, the reference point is set to (1.2, 1.2, 1.2).

	MOEA/DCH	FBEA	IAIS	NSGAII	MOEA/D	LRVMA	MOEA/D_2N
J20M6	**8.1760E-01(5.09E-02)**	4.7023E-01(4.78E-02)+	8.2778E-01(6.43E-02)+	3.8466E-01(4.41E-02)+	5.2831E-01(5.19E-02)+	6.6408E-01(5.50E-02)+	4.8938E-01(5.07E-02)+
J20M7	**7.0407E-01(3.29E-02)**	2.7203E-01(4.03E-02)+	6.8738E-01(6.20E-02)+	2.0056E-01(3.46E-02)+	3.7082E-01(4.30E-02)+	4.9156E-01(8.21E-02)+	3.0636E-01(6.23E-02)+
J20M8	**1.8410E-01(1.29E-02)**	8.5605E-02(1.24E-02)+	1.6954E-01(2.03E-02)+	1.1607E-01(1.84E-01)+	1.3649E-01(1.93E-01)+	9.7787E-02(1.82E-02)+	1.5591E-01(2.03E-01)+
J20M9	**7.3671E-01(6.17E-02)**	2.7785E-01(3.75E-02)+	4.9169E-01(1.06E-01)+	1.2309E-01(3.42E-02)+	2.6025E-01(4.84E-02)+	2.5343E-01(9.25E-02)+	2.4116E-01(4.00E-02)+
J20M10	**2.7322E-01(2.57E-02)**	9.2707E-02(2.43E-02)+	2.2954E-01(3.20E-02)+	7.0496E-02(6.44E-02)+	1.6037E-01(2.00E-01)+	1.0913E-01(1.98E-02)+	1.1387E-01(1.33E-01)+
J30M6	**4.3697E-01(2.79E-02)**	1.8482E-01(2.24E-02)+	4.5130E-01(4.56E-02)+	1.8301E-01(1.01E-01)+	2.9619E-01(1.65E-01)+	3.4645E-01(3.39E-02)+	2.2438E-01(2.89E-02)+
J30M7	**5.4752E-01(2.75E-02)**	2.5600E-01(3.37E-02)+	3.9476E-01(7.60E-02)+	2.1319E-01(2.75E-02)+	3.0480E-01(4.26E-02)+	2.5954E-01(6.70E-02)+	2.7144E-01(3.48E-02)+
J30M8	**2.3176E-01(2.53E-02)**	5.8226E-02(2.08E-02)+	2.1477E-01(4.43E-02)+	3.7701E-02(6.85E-02)+	9.4528E-02(8.73E-02)+	8.4253E-02(3.59E-02)+	1.6931E-01(1.76E-01)+
J30M9	**7.3966E-01(5.80E-02)**	2.7807E-01(6.58E-02)+	5.1009E-01(1.50E-01)+	1.0876E-01(2.52E-02)+	2.6830E-01(5.68E-02)+	1.9348E-01(9.01E-02)+	2.2953E-01(4.58E-02)+
J30M10	**2.5685E-01(2.07E-02)**	1.3446E-01(1.11E-01)+	1.8295E-01(3.90E-02)+	1.5283E-01(7.59E-02)+	2.0412E-01(1.19E-01)+	7.8099E-02(3.76E-02)+	2.8830E-01(8.07E-02)+
J40M6	1.2493E-01(8.65E-03)	5.7562E-02(8.34E-03)+	9.9588E-02(1.30E-02)+	7.7107E-02(1.35E-01)+	9.9359E-02(1.36E-01)+	5.9273E-02(1.12E-02)+	**1.4584E-01(2.42E-01)-**
J40M7	**7.7432E-01(4.19E-02)**	3.8718E-01(2.88E-02)+	6.4933E-01(4.21E-02)+	3.1225E-01(4.44E-02)+	4.9556E-01(4.15E-02)+	6.2917E-01(5.92E-02)+	4.3893E-01(3.40E-02)+
J40M8	**5.5541E-01(5.35E-02)**	2.5629E-01(1.76E-01)+	4.6968E-01(6.34E-02)+	2.5643E-01(2.00E-01)+	3.7083E-01(1.45E-01)+	4.7437E-01(6.63E-02)+	3.5904E-01(1.58E-01)+
J40M9	**7.9694E-01(2.54E-02)**	4.8688E-01(5.34E-02)+	7.3273E-01(2.92E-02)+	4.3731E-01(4.38E-02)+	5.9815E-01(3.08E-02)+	6.7446E-01(4.44E-02)+	5.7152E-01(4.40E-02)+
J40M10	**5.6520E-01(5.66E-02)**	1.3759E-01(5.91E-02)+	4.1672E-01(6.44E-02)+	8.9484E-02(1.34E-01)+	3.4243E-01(1.92E-01)+	3.7344E-01(8.21E-02)+	2.6688E-01(1.38E-01)+
J50M6	**4.4778E-01(2.91E-02)**	2.1745E-01(2.41E-02)+	4.0139E-01(3.09E-02)+	2.0758E-01(4.11E-02)+	2.9040E-01(2.17E-02)+	2.7962E-01(3.29E-02)+	2.9621E-01(9.59E-02)+
J50M7	**6.0738E-01(3.06E-02)**	2.5814E-01(2.90E-02)+	3.7392E-01(1.20E-01)+	9.0800E-02(2.50E-02)+	2.4699E-01(5.36E-02)+	1.4587E-01(1.03E-01)+	2.1598E-01(4.01E-02)+
J50M8	2.0211E-01(2.14E-02)	7.4834E-02(1.44E-02)+	1.8243E-01(2.21E-02)+	9.6319E-02(1.16E-01)+	2.0588E-01(2.37E-01)=	8.3903E-02(2.60E-02)+	**2.4406E-01(2.03E-01)-**
J50M9	**2.5729E-01(2.73E-02)**	1.2311E-01(1.42E-02)+	2.4897E-01(2.89E-02)+	1.7551E-01(1.37E-02)+	1.6543E-01(1.16E-02)+	1.5721E-01(2.44E-02)+	1.8216E-01(1.18E-02)+
J50M10	1.6533E-01(1.52E-02)	1.3345E-01(6.79E-02)+	1.8833E-01(8.72E-02)=	1.0422E-01(4.74E-02)+	2.2045E-01(3.81E-02)-	9.0804E-02(1.88E-02)+	**2.8628E-01(1.41E-02)-**
J80M6	1.8660E-02(2.69E-03)	3.2346E-02(8.06E-02)=	**2.1198E-02(2.74E-03)-**	1.1495E-01(2.60E-01)+	7.8270E-02(1.69E-01)-	2.0035E-02(4.96E-03)+	1.9272E-01(3.52E-01)=
J80M7	**6.3946E-01(6.24E-02)**	2.6213E-01(4.26E-02)+	4.2849E-01(5.40E-02)+	1.1706E-01(4.06E-02)+	4.0031E-01(4.93E-02)+	4.4862E-01(7.00E-02)+	3.6466E-01(5.95E-02)+
J80M8	1.2409E-01(1.66E-02)	8.9498E-02(1.38E-01)+	**1.7394E-01(1.78E-01)-**	3.1401E-02(4.58E-02)+	1.0430E-01(1.15E-01)+	6.1522E-02(1.81E-02)+	1.1899E-01(9.07E-02)+
J80M9	**7.1785E-01(3.95E-02)**	4.9027E-01(2.70E-02)+	5.2536E-01(3.38E-02)+	4.2419E-01(4.32E-02)+	6.2870E-01(3.83E-02)+	6.8473E-01(5.75E-02)+	5.9646E-01(2.62E-02)+
J80M10	1.7483E-01(1.82E-02)	1.0500E-01(9.87E-02)+	1.2587E-01(2.51E-02)+	1.1652E-01(5.33E-02)+	1.6080E-01(1.21E-01)+	1.1135E-01(3.26E-02)+	**3.0823E-01(4.19E-02)-**
J100M6	1.4312E-01(1.36E-02)	8.5102E-02(4.48E-02)+	1.5392E-01(1.36E-02)=	1.2322E-01(1.87E-01)+	1.1494E-01(3.17E-02)+	9.8731E-02(1.26E-02)+	**1.7364E-01(2.31E-01)=**
J100M7	**6.3551E-01(7.15E-02)**	2.6555E-01(3.68E-02)+	3.8909E-01(8.06E-02)+	1.1548E-01(4.03E-02)+	3.7696E-01(3.88E-02)+	1.4862E-01(1.04E-01)+	3.3190E-01(3.30E-02)-
J100M8	7.8506E-02(1.66E-02)	6.4137E-02(1.63E-01)+	1.0234E-01(1.27E-02)-	6.3057E-02(1.57E-01)+	5.1900E-02(5.88E-03)+	5.5459E-02(1.37E-02)+	**1.3762E-01(1.97E-01)-**
J100M9	2.4874E-01(1.84E-02)	1.6173E-01(9.09E-03)+	1.7449E-01(2.10E-02)+	9.1169E-01(1.31E-02)+	**1.9674E-01(7.59E-03)-**	1.5584E-01(3.54E-02)+	1.8345E-01(1.03E-02)+
J100M10	1.4969E-01(1.29E-02)	6.8900E-02(1.48E-02)+	1.1280E-01(1.65E-02)+	1.7392E-01(7.53E-02)-	**3.2004E-01(6.51E-02)-**	1.0247E-01(1.73E-02)+	2.4818E-01(4.41E-02)+
	+/=/-	29/1/0	25/2/3	29/0/1	25/1/4	30/0/0	23/1/6

TABLE XIII: HV values obtained by all algorithms, the reference point is set to (1.5, 1.5, 1.5).

	MOEA/DCH	FBEA	IAIS	NSGAII	MOEA/D	LRVMA	MOEA/D_2N
J20M6	**8.5307E-01(4.15E-02)**	5.6248E-01(4.19E-02)+	8.6092E-01(5.24E-02)+	4.8675E-01(3.94E-02)+	6.1270E-01(4.49E-02)+	7.2634E-01(4.62E-02)+	5.8009E-01(4.38E-02)+
J20M7	**7.6092E-01(2.73E-02)**	3.8838E-01(3.70E-02)+	7.4607E-01(5.10E-02)+	3.1959E-01(3.61E-02)+	4.7641E-01(3.81E-02)+	5.8040E-01(7.05E-02)+	4.1767E-01(5.65E-02)+
J20M8	**2.9950E-01(1.54E-02)**	1.7596E-01(1.66E-02)+	2.9386E-01(2.19E-02)+	1.9746E-01(1.76E-01)+	2.2809E-01(1.74E-01)+	2.0820E-01(2.03E-02)+	2.4624E-01(1.87E-01)+
J20M9	**7.8730E-01(5.10E-02)**	3.8538E-01(3.65E-02)+	5.8129E-01(9.25E-02)+	2.2441E-01(3.78E-02)+	3.6803E-01(4.65E-02)+	3.6809E-01(8.44E-02)+	3.4780E-01(3.84E-02)+
J20M10	**3.9024E-01(2.74E-02)**	1.8051E-01(3.14E-02)+	3.5817E-01(3.27E-02)+	1.7115E-01(6.65E-02)+	2.4850E-01(1.87E-01)+	2.1889E-01(2.43E-02)+	2.0175E-01(1.27E-01)+
J30M6	**5.3573E-01(2.42E-02)**	3.0317E-01(2.25E-02)+	5.4422E-01(4.06E-02)+	2.9719E-01(9.28E-02)+	4.0218E-01(1.41E-01)+	4.5280E-01(3.09E-02)+	3.3968E-01(2.88E-02)+
J30M7	**6.2743E-01(2.41E-02)**	3.6012E-01(3.61E-02)+	5.0117E-01(6.74E-02)+	3.3265E-01(3.41E-02)+	4.2343E-01(4.53E-02)+	3.8194E-01(5.88E-02)+	3.7719E-01(3.58E-02)+
J30M8	**3.4650E-01(2.83E-02)**	1.3501E-01(2.88E-02)+	3.4646E-01(4.95E-02)+	1.0087E-01(8.34E-02)+	1.8873E-01(9.81E-02)+	1.9016E-01(4.48E-02)+	2.6439E-01(1.82E-01)+
J30M9	**7.8939E-01(4.82E-02)**	3.8260E-01(6.26E-02)+	5.9739E-01(1.27E-01)+	2.0784E-01(2.84E-02)+	3.7798E-01(5.14E-02)+	3.1727E-01(8.41E-02)+	3.3340E-01(4.58E-02)+
J30M10	**3.7684E-01(2.25E-02)**	2.2928E-01(1.06E-01)+	3.3198E-01(3.79E-02)+	2.7862E-01(6.89E-02)+	3.4756E-01(9.89E-02)+	2.0561E-01(4.14E-02)+	4.1149E-01(6.87E-02)+
J40M6	2.1756E-01(1.00E-02)	1.3317E-01(1.13E-02)+	1.9094E-01(1.53E-02)+	1.4972E-01(1.38E-01)+	1.8155E-01(1.41E-01)+	1.4298E-01(1.37E-02)+	**2.2216E-01(2.29E-01)-**
J40M7	**8.1816E-01(3.43E-02)**	4.8912E-01(2.59E-02)+	6.7854E-01(3.43E-02)+	4.1985E-01(4.12E-02)+	5.8291E-01(3.62E-02)+	6.9660E-01(5.01E-02)+	5.3463E-01(2.95E-02)+
J40M8	**6.3482E-01(4.58E-02)**	3.6099E-01(1.56E-01)+	5.1333E-01(5.24E-02)+	3.5893E-01(1.81E-01)+	4.8629E-01(1.19E-01)+	5.6366E-01(5.86E-02)+	4.6089E-01(1.37E-01)+
J40M9	**8.3685E-01(2.05E-02)**	5.7893E-01(4.54E-02)+	7.4600E-01(2.35E-02)+	5.3534E-01(3.78E-02)+	6.7193E-01(2.60E-02)+	7.3554E-01(3.69E-02)+	6.5035E-01(3.69E-02)+
J40M10	**6.4565E-01(4.75E-02)**	2.7084E-01(5.44E-02)+	5.5174E-01(5.26E-02)+	2.1488E-01(1.22E-01)+	4.5245E-01(1.65E-01)+	4.7895E-01(7.29E-02)+	3.8481E-01(1.21E-01)+
J50M6	**5.4104E-01(2.56E-02)**	3.2931E-01(2.40E-02)+	4.2137E-01(2.81E-02)+	3.1871E-01(4.49E-02)+	3.9781E-01(2.03E-02)+	3.8930E-01(3.15E-02)+	4.0408E-01(9.92E-02)+
J50M7	**6.8013E-01(2.64E-02)**	3.6809E-01(2.89E-02)+	4.1590E-01(1.02E-01)+	1.8691E-01(3.17E-02)+	3.5735E-01(4.85E-02)+	2.6614E-01(1.17E-01)+	3.2394E-01(3.91E-02)+
J50M8	3.1895E-01(2.51E-02)	1.5993E-01(2.03E-02)+	2.1765E-01(2.26E-02)+	2.1801E-01(1.07E-01)+	2.9539E-01(2.25E-01)+	1.9853E-01(3.09E-02)+	**3.8073E-01(1.68E-01)-**
J50M9	**3.7125E-01(2.94E-02)**	2.1840E-01(1.69E-02)+	2.8900E-01(2.75E-02)+	3.0873E-01(1.76E-02)+	2.7567E-01(1.34E-02)+	2.8877E-01(2.38E-02)+	4.1748E-01(1.35E-02)+
J50M10	2.8076E-01(1.88E-02)	2.2651E-01(7.46E-02)+	1.9012E-01(7.85E-02)=	2.2686E-01(4.93E-02)+	3.3334E-01(3.48E-02)-	2.0245E-01(2.50E-02)+	**4.1591E-01(1.37E-02)-**
J80M6	1.8933E-02(4.57E-03)	8.4121E-02(9.67E-02)+	**2.0766E-01(4.30E-03)-**	1.5847E-01(2.67E-01)+	1.3776E-01(1.76E-01)-	7.7853E-02(8.03E-03)+	1.8227E-01(3.41E-01)=
J80M7	**7.0791E-01(5.32E-02)**	3.7624E-01(4.08E-02)+	5.6185E-01(4.38E-02)+	2.2597E-01(4.57E-02)+	5.0060E-01(4.31E-02)+	5.4206E-01(6.08E-02)+	4.6983E-01(5.42E-02)+
J80M8	2.2297E-01(2.11E-02)	1.6275E-01(1.44E-01)+	**2.8542E-01(1.55E-01)-**	1.0385E-01(5.53E-02)+	2.2439E-01(1.06E-01)+	1.5760E-01(2.41E-02)+	2.4054E-01(8.25E-02)+
J80M9	**7.7121E-01(3.27E-02)**	5.7958E-01(2.35E-02)+	6.4003E-01(2.72E-02)+	5.2133E-01(3.80E-02)+	6.9716E-01(3.24E-02)+	7.4322E-01(4.77E-02)+	6.7089E-01(2.25E-02)+
J80M10	2.8993E-01(2.22E-02)	1.9173E-01(1.02E-01)+	2.3694E-01(2.47E-02)+	2.4360E-01(5.30E-02)+	2.7195E-01(1.25E-01)+	2.3790E-01(3.53E-02)+	**4.3750E-01(3.76E-02)-**
J100M6	2.3754E-01(1.51E-02)	1.7255E-01(6.15E-02)+	1.7934E-01(1.51E-02)=	2.0053E-01(1.98E-01)+	2.0743E-01(4.57E-02)+	1.8767E-01(1.47E-02)+	**2.5620E-01(2.16E-01)=**
J100M7	**7.0397E-01(6.07E-02)**	3.7602E-01(3.55E-02)+	5.0012E-01(6.64E-02)+	2.1556E-01(4.49E-02)+	4.8081E-01(3.58E-02)+	2.8174E-01(1.09E-01)+	4.3598E-01(3.14E-02)-
J100M8	1.6524E-01(2.33E-02)	1.2397E-01(1.60E-01)+	1.1320E-01(1.51E-02)-	1.3279E-01(1.61E-01)+	1.2947E-01(9.30E-03)+	1.4460E-01(1.85E-02)+	**2.3877E-01(1.75E-01)-**
J100M9	3.7043E-01(2.05E-02)	2.7104E-01(1.10E-02)+	2.3140E-01(1.95E-02)+	3.4128E-01(1.74E-02)+	**3.1807E-01(7.96E-03)-**	2.9515E-01(3.26E-02)+	5.2405E-01(1.21E-02)+
J100M10	2.6592E-01(1.68E-02)	1.5546E-01(2.12E-02)+	1.1389E-01(1.82E-02)+	2.9666E-01(7.29E-02)-	**4.3026E-01(5.71E-02)-**	2.2222E-01(2.13E-02)+	5.5713E-01(3.88E-02)-
	+/=/-	30/0/0	25/2/3	29/0/1	26/0/4	30/0/0	23/1/6