Strategic weight manipulation in multiple attribute decision making[1]

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Abstract: In some real-world multiple attribute decision making (MADM) problems, a decision maker can strategically set attribute weights to obtain her/his desired ranking of alternatives, which is called the strategic weight manipulation of the MADM. In this paper, we define the concept of the ranking range of an alternative in the MADM, and propose a series of mixed 0-1 linear programming models (MLPMs) to show the process of designing a strategic attribute weight vector. Then, we reveal the conditions to manipulate a strategic attribute weight based on the ranking range and the proposed MLPMs. Finally, a numerical example with real background is used to demonstrate the validity of our models, and simulation experiments are presented to show the better performance of the ordered weighted averaging operator than the weighted averaging operator in defending against the strategic weight manipulation of the MADM problems.

Keywords: multiple attribute decision making, strategic weight manipulation, the ordered weighted averaging operator, ranking

1. Introduction

Multiple attribute decision making (MADM) refers to the problem of ranking alternatives based on the evaluation information of alternatives associated with multiple attributes [9, 10, 16, 25, 31]. The MADM has been widely used in

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engineering, technology, economy, management, and military, and many other fields [12, 15, 18, 22, 40].

The attribute weights play an important role in MADM problems. In the existing literature, there are several approaches to obtain the attribute weights that can be classified into three categories: the subjective approach, the objective approach and the integrated approach.

(1) The subjective approach determines the attribute weights in terms of the decision maker's preference information on attributes [2, 8, 28]. Doyle et al. [8], for example, proposed direct rating and point allocation methods. Meanwhile, several ordinal ranking methods are investigated in [1, 26, 29], and recently, Danielson et al. [5] provided an augmenting ordinal method for obtaining attribute weights.

(2) The objective approach determines the weights of attributes using objective decision matrix information. This approach includes the entropy method [40], the TOPSIS-based method [20, 41] and some mathematical programming based methods (e.g. [3]).

(3) The integrated approach determines the weights of attributes using both decision makers' subjective information and objective decision matrix information. Within these approaches, Cook and Kress [4] proposed the preference-aggregation model based on the use of the Data Envelopment Analysis. Moreover, Fan et al. [11], Horsky and Rao [14] and Pekelman and Sen [23] constructed some optimization-based models to assess the attribute weights based on the use of decision maker's preference information on alternatives.

Generally, in a process of decision making, the decision makers may express their opinions dishonestly to obtain their own interests, which is referred to as strategic manipulation or non-cooperative behavior. The strategic manipulation has been analyzed in-depth with respect to the aggregation function [24, 37, 38], the consensus reaching process [6, 13, 30], and also in large-scale group decision making [7, 21, 36]. It is natural to assume that the process of setting attribute weights in MADM problems is not immune to strategic manipulations, and that a decision maker may strategically set attribute weights in order to obtain her/his desired ranking of the alternative(s). In this study we refer to this kind of strategic manipulations in MADM as the strategic weight manipulation problem.

As mentioned above, there exist different (subjective, objective and integrated) approaches to attribute weights setting. Within these approaches, the decision maker is assumed to be honest, and aims to obtain "best" attribute weights to get a ranking of alternatives. We need to highlight that this paper focuses on the strategic weight manipulation problem in which the decision maker is assumed not to be honest, and she/he aims to strategically set attribute weights to obtain her/his desired ranking of the alternatives.

Although there exist numerous methods to set attribute weights, these approaches do not always consider the general theoretical framework that governs the strategic weight manipulation.

In order to fill this gap, several research challenges are proposed for analysis in this paper:

(1) How to determine the range of the ranking of alternatives when a decision maker strategically set the attribute weights in MADM problems.

(2) When a decision maker wishes to manipulate the ranking of alternatives with a predetermined purpose, how to design a strategic weight vector to achieve this purpose.

(3) How to analyze the performances of two different average operators, the weighted averaging (WA) and the ordered weighted averaging (OWA), in defending against strategic weight manipulation in MADM problems.

In order to do so, the rest of this paper is organized as follows. Section 2 provides the basic knowledge regarding MADM problems and introduces the proposed strategic weight manipulation problem. Then, in Section 3, mixed 0-1 linear programming models are proposed to obtain the ranking range of an alternative under the conditions that the attribute weights being strategically changed, and several desired properties of the ranking range of alternatives are studies. In section 4, mixed 0-1 linear programming models are used to analyze how to design a strategic weight vector to manipulate the ranking of alternative(s) to achieve a desired purpose.

Section 5 presents a numerical example to illustrate the proposed models, and simulation experiments are presented to compare the performances of the WA and OWA [32, 39] operators in defending against strategic weight manipulation in MADM problems. Concluding remarks and future research agenda are provided in Section 6.

2. Background

This section introduces the MADM problem and the concept of ranking range of an alternative, which will provide a basis to study the strategic weight manipulation problem in MADM.

2.1 MADM problem

Let $X = \{x_1,x_2,\dots,x_n\}$ be the set of alternatives, $A = \{a_1,a_2,\dots,a_m\}$ the set of predefined attributes, and $w = (w_1,w_2,\dots,w_m)$ the associated weight vector of the attributes, such that $w_j \geq 0$ and $\sum_{j=1}^{m} w_j = 1$. Let $V = [v_{ij}]_{n \times m}$ be the decision matrix given by the decision maker, where v_{ij} denotes the preference value for the alternative $x_i \in X$ with respect to the attribute $a_j \in A$, representing how well alternative x_i verifies attribute a_j.

Generally, the resolution process of MADM problems includes three steps:

(1) Normalization of the decision matrix

In MADM problems, attributes are classified into two categories: benefit attributes and cost attributes. The decision maker's decision matrix $V = [v_{ij}]_{n \times m}$ needs to be normalized into a corresponding standardized individual's decision matrix $\overline{V} = [\overline{v}_{ij}]_{n \times m}$, where

$$\overline{v}_{ij} = \frac{v_{ij} - \min_i(v_{ij})}{\max_i(v_{ij}) - \min_i(v_{ij})} \tag{1}$$

if $a_j \in A$ is a benefit attribute, and

$$\overline{v}_{ij} = \frac{\max_i(v_{ij}) - v_{ij}}{\max_i(v_{ij}) - \min_i(v_{ij})} \tag{2}$$

if $a_j \in A$ is a cost attribute.

(2) Aggregation of the standardized decision matrix

Let $D(x_i)$ be the decision evaluation value of the alternative x_i, which is obtained by aggregating its associated attribute preference values using Eq. (3) and an appropriate aggregation operator F:

$$D(x_i) = F(\overline{v}_{i1}, \overline{v}_{i2}, \ldots, \overline{v}_{im}) \tag{3}$$

In MADM problems, the aggregation operators frequently used are the WA operator and the OWA operator [32, 39].

When F is a WA operator with an associated weight vector $w = (w_1, w_2, \ldots, w_m)$, Eq. (3) can be rewritten as follows:

$$D(x_i) = WA_w(\overline{v}_{i1}, \overline{v}_{i2}, \ldots, \overline{v}_{im}) = \sum_{j=1}^{m} w_j \overline{v}_{ij} \tag{4}$$

While, when F is a OWA operator with an associated weight vector $w = (w_1, w_2, \ldots, w_m)$, Eq. (3) can be rewritten as follows:

$$D(x_i) = OWA_w(\overline{v}_{i(1)}, \overline{v}_{i(2)}, \ldots, \overline{v}_{i(m)}) = \sum_{j=1}^{m} w_j \overline{v}_{i(j)} \tag{5}$$

where $\overline{v}_{i(j)}$ is the jth largest value in $\{\overline{v}_{i1}, \overline{v}_{i2}, \ldots, \overline{v}_{im}\}$.

(3) Ranking of alternatives

Let $Q_k = \{x_i | D(x_i) > D(x_k),\ i = 1,2,\ldots,n\}$ be the set of the alternatives whose decision evaluation value is greater than that of the alternative x_k, and $|Q_k|$ be its cardinality. Clearly, for Q_k, because $x_k \notin Q_k$, then alternative x_k such that $D(x_k) = \max\{D(x_1), \ldots, D(x_n)\}$ might verify as well that $|Q_k| = 0$, while alternative $x_j \notin Q_j$, such that $D(x_j) = \min\{D(x_1), \ldots, D(x_n)\}$ might as well have $|Q_j| = n - 1$, and therefore this alternative will be ranked in 1-st and n-th positions, i.e., it is justified the following definition of the ranking position of an alternative in terms of $|Q_k|$: $r_k = |Q_k| + 1$, i.e.,

$$r(x_k) = \left|\left\{x_i \,|\, D(x_i) > D(x_k),\ \ i = 1,2,\ldots,n\right\}\right| + 1 \tag{6}$$

Based on the ranking of alternatives, we can easily obtain the following results.

(1) Let $x_i \succ x_j \Leftrightarrow r(x_i) < r(x_j)$, then we have $x_i \succ x_j \Leftrightarrow D(x_i) > D(x_j)$.

(2) Let $x_i \precsim x_j \Leftrightarrow r(x_i) \geq r(x_j)$, then we have $x_i \precsim x_j \Leftrightarrow D(x_i) \leq D(x_j)$.

2.2 The proposed research problem: Strategic weight manipulation

Let $r_w(x_k)$ be the ranking of alternative x_k when setting the associated weight vector of the attributes $w = (w_1, w_2, ..., w_m)$. Clearly, $r_w(x_k)$ can change when the weight vector $w = (w_1, w_2, ..., w_m)$ is changed, in other words, the manipulation of the weight vector can lead to a change in the ranking order of the alternatives. The following example clearly illustrates this issue.

Example 1: Assume three alternatives $\{x_1, x_2, x_3\}$ and four attributes $\{a_1, a_2, a_3, a_4\}$ with the following standardized decision matrix $\bar{V} = [\bar{v}_{ij}]_{3\times 4}$ is:

$$\bar{V} = \begin{bmatrix} 0.59 & 1 & 0.8 & 0.63 \\ 0.6 & 0.8 & 1 & 0.46 \\ 1 & 0.5 & 0.4 & 1 \end{bmatrix}$$

Different $w = (w_1, w_2, ..., w_m)$ lead to different rankings $r_w(x) = \{r_w(x_1), r_w(x_2), r_w(x_3)\}$ of the alternatives $X = \{x_1, x_2, x_3\}$. Indeed,

1) If we set $w = (0.3, 0.2, 0.1, 0.4)$, then we have $r_w(x) = \{3, 2, 1\}$;

2) If we set $w = (0.1, 0.45, 0.24, 0.21)$, then it is $r_w(x) = \{1, 2, 3\}$;

3) While if we set $w = (0.3, 0.1, 0.5, 0.1)$, then $r_w(x) = \{2, 1, 3\}$ is obtained.

Because different attribute weights yield different ranking of alternatives, in this paper, we give the definition of ranking range of an alternative as follows:

Definition 1: In MADM problems, $R(x_k) = [\underline{r}(x_k), \bar{r}(x_k)]$ is known as the ranking range of the alternative x_k, with $\underline{r}(x_k) = \min_{w\in W} r_w(x_k)$ and $\bar{r}(x_k) = \max_{w\in W} r_w(x_k)$ being the best and worst rankings of alternative x_k, respectively, and

$$W = \{w = (w_1, w_2, ..., w_m) | \sum_{j=1}^{m} w_j = 1, \quad 0 \le w_j \le 1\}.$$

In addition, in this paper, we introduce the concept of attribute ranking and attribute ranking range to analyze the properties of the ranking range of an alternative.

Let $O_j(x_k) = \{x_i | \bar{v}_{ij} > \bar{v}_{kj}\}$ (i=1,2,…,n; j=1,2,…,m) be the set of alternatives whose decision evaluation value is greater than that of the alternative x_k associated with the attribute a_j, and $|O_j(x_k)|$ be its cardinality. Let $\bar{O}_j(x_k) = \{x_i | \bar{v}_{ij} \le \bar{v}_{kj}\}$ (i=1,2,…,n; j=1,2,…,m) be the set of alternatives whose decision evaluation value is not greater than that of the alternative x_k associated with the attribute a_j, and $|\bar{O}_j(x_k)|$ be its cardinality.

Based on the sets $O_j(x_k)$ and $\overline{O}_j(x_k)$, the concept of attribute ranking and attribute ranking range can be formally presented as follows:

Definition 2: In MADM problems, $c_j(x_k) = |O_j(x_k)| + 1$, i.e.,

$$c_j(x_k) = \left|\{x_i \left|\overline{v}_{ij} > \overline{v}_{kj}\right.\}\right| + 1, \quad (j = 1, 2, \ldots, m) \tag{7}$$

is the attribute ranking of the alternative x_k associated with the attribute a_j. Then, let $\underline{c}(x_k) = \min_j c_j(x_k)$ and $\bar{c}(x_k) = \max_j c_j(x_k)$, $C(x_k) = [\underline{c}(x_k), \bar{c}(x_k)]$ is the attribute ranking range of the alternative x_k.

As mentioned above, in MADM problems, a decision maker could strategically set an attribute weight vector to obtain her/his desired ranking of alternative(s), which in this paper is referred to as the strategic weight manipulation in MADM.

In the following, based on the concept of ranking range, we investigate some issues on the strategic weight manipulation of the MADM to deal with the challenges presented in the introduction section.

In order to improve readability, the main notation used in this paper is listed as follows.

X: The set of alternatives;

A: The set of attributes;

$V = [v_{ij}]_{n \times m}$: Decision matrix;

$\overline{V} = [\overline{v}_{ij}]_{n \times m}$: Standardized decision matrix;

S: The set of attribute weight vectors;

$D(x_i)$: The evaluation value of the alternative x_i;

$r_w(x_k)$: The ranking of the alternative x_k under the attribute weight vector w;

$\underline{r}(x_k)$: The best ranking of the alternative x_k;

$\bar{r}(x_k)$: The worst ranking of the alternative x_k;

$R(x_k) = [\underline{r}(x_k), \bar{r}(x_k)]$: Ranking range of the alternative x_k;

$R_{WA}(x_k) = [\underline{r}_{WA}(x_k), \bar{r}_{WA}(x_k)]$: Ranking range under the WA operator;

$R_{OWA}(x_k) = [\underline{r}_{OWA}(x_k), \bar{r}_{OWA}(x_k)]$: Ranking range under the OWA operator;

$C(x_k) = [\underline{c}(x_k), \bar{c}(x_k)]$: Attribute ranking range of the alternative x_k.

3. Ranking range

The ranking range of an alternative is used to provide the best and worst ranking of the alternative, which is a basis for strategically setting the attribute weights in MADM problems. In this section, we present mixed 0-1 linear programming models to obtain the ranking range of an alternative, and show several desired properties of the ranking range of an alternative.

3.1 Obtaining the ranking range via a mixed 0-1 linear programming

Let $y_i \in \{0,1\}$, M a large enough number, and $D(x_i)$ be defined as per Eq. (3). Then, we can easily obtain the following results.

(1) $x_i \succ x_k$ if and only if $y_i = 1$ under the conditions $D(x_i) > D(x_k) - (1 - y_i)M$ and $D(x_i) \le D(x_k) + y_i M$.

(2) $x_i \precsim x_k$ if and only if $y_i = 0$ under the conditions $D(x_i) \le D(x_k) + y_i M$ and $D(x_i) > D(x_k) - (1 - y_i)M$.

Based on the above results, Theorems 1 and 2 to obtain the ranking range $R(x_k) = [\underline{r}(x_k), \bar{r}(x_k)]$ of the alternative x_k under the WA and OWA operators are presented.

Theorem 1: Let $R_{WA}(x_k) = [\underline{r}_{WA}(x_k), \bar{r}_{WA}(x_k)]$ be the ranking range of alternative x_k when the WA operator F is used to compute the decision evaluation function as per Eq. (4). Then,

(1) The best ranking of alternative x_k, $\underline{r}_{WA}(x_k)$ can be obtained via the mixed 0-1 linear programming models (8)-(13).

$$\begin{cases} \underline{r}_{WA}(x_k) = \min \sum_{i=1}^{n} y_i + 1 & (8) \\ s.t. \begin{cases} \sum_{j=1}^{m} w_j \bar{v}_{ij} > \sum_{j=1}^{m} w_j \bar{v}_{kj} - (1 - y_i)M, \quad (i = 1,2,...,n) & (9) \\ \sum_{j=1}^{m} w_j \bar{v}_{ij} \le \sum_{j=1}^{m} w_j \bar{v}_{kj} + y_i M, \quad (i = 1,2,...,n) & (10) \\ \sum_{j=1}^{m} w_j = 1 & (11) \\ 0 \le w_j \le 1, \quad (j = 1,2,...,m) & (12) \\ y_i = 1 \ \ or \ \ 0, \quad (i = 1,2,...,n) & (13) \end{cases} \end{cases}$$

(2) In models (8)-(13), replace the objective function (8) by

$$\overline{r}_{WA}(x_k) = \max \sum_{i=1}^{n} y_i + 1 \tag{14}$$

Then, the worst ranking of alternative x_k, $\overline{r}_{WA}(x_k)$, can be obtained via the mixed 0-1 linear programming models (9)-(14).

The proof of Theorem 1 is provided in Appendix A.

To simplify the notation, models (8)-(13) and models (9)-(14) are both called P_1 in this paper.

Theorem 2: Let $R_{OWA}(x_k) = [\underline{r}_{OWA}(x_k), \overline{r}_{OWA}(x_k)]$ be the ranking range of alternative x_k when the OWA operator F is used to compute the decision evaluation function as per Eq. (5). Then,

(1)The best ranking of alternative x_k, $\underline{r}_{OWA}(x_k)$ can be obtained via the 0-1 linear programming models (15)-(20).

$$\begin{cases} \underline{r}_{OWA}(x_k) = \min \sum_{i=1}^{n} y_i + 1 & (15) \\ s.t. \begin{cases} \sum_{j=1}^{m} w_j \overline{v}_{i(j)} > \sum_{j=1}^{m} w_j \overline{v}_{k(j)} - (1 - y_i) M, \quad (i = 1, 2, \ldots, n) & (16) \\ \sum_{j=1}^{m} w_j \overline{v}_{i(j)} \le \sum_{j=1}^{m} w_j \overline{v}_{k(j)} + y_i M, \quad (i = 1, 2, \ldots, n) & (17) \\ \sum_{j=1}^{m} w_j = 1 & (18) \\ 0 \le w_j \le 1, \quad (j = 1, 2, \ldots, m) & (19) \\ y_i = 1 \ \ or \ \ 0, \quad (i = 1, 2, \ldots, n) & (20) \end{cases} \end{cases}$$

(2) In models (15)-(20), replace the objective function (15) by

$$\overline{r}_{OWA}(x_k) = \max \sum_{i=1}^{n} y_i + 1 \tag{21}$$

Then, the worst ranking of alternative x_k, $\overline{r}_{OWA}(x_k)$, can be obtained via the mixed 0-1 linear programming models (16)-(21).

The proof of Theorem 2 is provided in Appendix A.

To simplify the notation, models (15)-(20) and models (16)-(21) are both called P_2 in this paper. In both models P_1 and P_2, $w_j\,(j = 1,2,\ldots,m)$ and $y_i\,(i = 1,2,\ldots,n)$ are decision variables.

3.2 Desirable properties of ranking range

In this subsection, we present some desired properties of the ranking range of the alternatives based on the concept of attribute ranking and attribute ranking range.

Let $[\underline{r}_{WA}(x_k), \overline{r}_{WA}(x_k)]$, $[\underline{r}_{OWA}(x_k), \overline{r}_{OWA}(x_k)]$ and $[\underline{c}(x_k), \overline{c}(x_k)]$ be as defined above. The following properties hold:

Property 1: $[\underline{c}(x_k), \overline{c}(x_k)] \subseteq [\underline{r}_{WA}(x_k), \overline{r}_{WA}(x_k)]$ for any $x_k \in X$.

The proof of Property 1 is provided in Appendix A. This property shows that the attribute ranking range of an alternative is contained in the ranking range of the alternative under the WA operator.

Let $O_{I_k}(x_k) = \{x_i | \overline{v}_{ij} > \overline{v}_{kj},\ j=1,2,\ldots,m\}$ be the set of the alternatives whose decision evaluation value is greater than that of the alternative x_k for all attributes, and $|O_{I_k}(x_k)|$ be its cardinality. Let $\overline{O}_{I_k}(x_k) = \{x_i | \overline{v}_{ij} \leq \overline{v}_{kj},\ j=1,2,\ldots,m\}$ be the set of alternatives whose decision evaluation value is not greater than that of the alternative x_k for all attributes, and $|\overline{O}_{I_k}(x_k)|$ be its cardinality. The following property holds:

Property 2: (i) $\underline{r}_{WA}(x_k) \in [|O_{I_k}(x_k)| + 1, \underline{c}(x_k)]$ and (ii) $\overline{r}_{WA}(x_k) \in [\overline{c}(x_k),\ n - |\overline{O}_{I_k}(x_k)|]$.

The proof of Property 2 is provided in Appendix A. This property provides an estimation for the ranking range of the alternative x_k under the WA operator. The following examples show that Properties 1 and 2 do not hold in the case of the OWA operator.

Example 2: Assume five alternatives $\{x_1, x_2, x_3, x_4, x_5\}$ and four attributes $\{a_1, a_2, a_3, a_4\}$ with the following standardized decision matrix $\overline{V} = [\overline{v}_{ij}]_{5\times 4}$ is:

$$\overline{V} = \begin{bmatrix} 0.95 & 0.9 & 0.73 & 0.66 \\ 0.8 & 0.59 & 1 & 0.7 \\ 0.8 & 0.9 & 0.65 & 0.65 \\ 0.6 & 0.66 & 0.9 & 0.71 \\ 0.5 & 0.6 & 0.7 & 0.8 \end{bmatrix}$$

Based on Definition 2,

$[\underline{c}(x_1), \overline{c}(x_1)] = [1, 4]$, $[\underline{c}(x_2), \overline{c}(x_2)] = [1, 5]$, $[\underline{c}(x_3), \overline{c}(x_3)] = [1, 5]$, $[\underline{c}(x_4), \overline{c}(x_4)] = [2, 4]$, and $[\underline{c}(x_5), \overline{c}(x_5)] = [1, 5]$.

Meanwhile, solving P_2 using the software package LINGO, we have,

$[\underline{r}_{OWA}(x_1),\bar{r}_{OWA}(x_1)] = [1, 2]$, $[\underline{r}_{OWA}(x_2),\bar{r}_{OWA}(x_2)] = [1, 4]$, $[\underline{r}_{OWA}(x_3),\bar{r}_{OWA}(x_3)] = [2, 4]$, $[\underline{r}_{OWA}(x_4),\bar{r}_{OWA}(x_4)] = [3, 4]$, and $[\underline{r}_{OWA}(x_5),\bar{r}_{OWA}(x_5)] = [5, 5]$.

Then, it is obvious that $[\underline{c}(x_k),\bar{c}(x_k)] \not\subseteq [\underline{r}_{OWA}(x_k),\bar{r}_{OWA}(x_k)]$ $(k = 1,2,3,4,5)$, which means that Property 1 is not true when the OWA operator is used.

Example 3: Assume three alternatives $\{x_1, x_2, x_3\}$ and four attributes $\{a_1, a_2, a_3, a_4\}$ with the following standardized decision matrix $\bar{V} = [\bar{v}_{ij}]_{3\times 4}$ is:

$$\bar{V} = \begin{bmatrix} 0.59 & 1 & 0.8 & 0.63 \\ 0.6 & 0.7 & 0.9 & 0.46 \\ 0.8 & 0.5 & 0.4 & 0.6 \end{bmatrix}$$

Based on Definition 2,

$\{\underline{c}(x_1),\underline{c}(x_2),\underline{c}(x_3)\} = \{1,1,1\}$,

and

$\{\bar{c}(x_1),\bar{c}(x_2),\bar{c}(x_3)\} = \{3,3,3\}$.

Meanwhile, we have $|O_{I_1}(x_1)| + 1 = 1$, $|O_{I_2}(x_2)| + 1 = 1$, $|O_{I_3}(x_3)| + 1 = 1$, $3 - |\bar{O}_{I_1}(x_1)| = 3$, $3 - |\bar{O}_{I_2}(x_2)| = 3$, and $3 - |\bar{O}_{I_3}(x_3)| = 3$.

Solving P_2 with the software package LINGO, we have

$\{\underline{r}_{OWA}(x_1),\underline{r}_{OWA}(x_2),\underline{r}_{OWA}(x_3)\} = \{1,2,3\}$,

and

$\{\bar{r}_{OWA}(x_1),\bar{r}_{OWA}(x_2),\bar{r}_{OWA}(x_3)\} = \{1,2,3\}$.

Clearly, it is $\underline{r}_{OWA}(x_2) \notin \left[|O_{I_2}(x_2)| + 1,\underline{c}(x_2)\right]$, $\underline{r}_{OWA}(x_3) \notin \left[|O_{I_3}(x_3)| + 1,\underline{c}(x_3)\right]$, $\bar{r}_{OWA}(x_1) \notin \left[\bar{c}(x_1),3 - |\bar{O}_{I_1}(x_1)|\right]$, and $\bar{r}_{OWA}(x_2) \notin \left[\bar{c}(x_2),3 - |\bar{O}_{I_2}(x_2)|\right]$, and consequently Property 2 does not hold in the case of the OWA operator being used.

4. Strategic weight manipulation

In MADM problems, the decision maker can strategically set the attribute weights to obtain her/his desired ranking of the alternative(s). In this section, we continue to use mixed 0-1 linear programming models to set a strategic weight to manipulate the ranking of alternative(s) under different aggregation operators.

Let $w_0 = (w_1^0, w_2^0, \dots, w_m^0)$ be the objective weight vector of the attributes in a MADM problem. Without loss of generality, the decision maker wishes to manipulate the ranking of the alternatives $\{x_1, x_2, \dots, x_l\}$.

Let $w = (w_1, w_2, \dots, w_m)$ be the decision maker's strategic weight vector to manipulate the alternatives $\{x_1, x_2, \dots, x_l\}$. It is natural that the decision maker wishes to minimize the difference between the objective and strategic weight vector, i.e.,

$$\min \sum_{j=1}^{m} \left| w_j - w_j^0 \right| \tag{22}$$

Without loss of generality, if the decision maker's desired ranking of the alternatives $\{x_1, x_2, \dots, x_l\}$ is $\{r^*(x_1), r^*(x_2), \dots, r^*(x_l)\}$, then we have

$$r_w(x_k) = r^*(x_k) \quad (k = 1, 2, \dots, l) \tag{23}$$

Based on Eqs. (22) and (23), an optimization-based model to find out the decision maker's strategic weight is presented as follows.

$$\begin{cases} \min \sum_{j=1}^{m} \left| w_j - w_j^0 \right| \\ s.t. \quad r_w(x_k) = r^*(x_k) \quad (k = 1, 2, \dots, l) \end{cases} \tag{24}$$

In order to obtain the optimum solution to model (24), in the following it is shown that model (24) can be transformed into mixed 0-1 linear programming models.

Lemma 1: Let F be the WA operator as per Eq. (4). If there exists $w^* = (w_1^*, w_2^*, \dots, w_m^*)$ satisfying the constraint conditions (25)-(30) below

$$\begin{cases} \sum_{j=1}^{m} w_j^* \overline{v}_{ij} > \sum_{j=1}^{m} w_j^* \overline{v}_{kj} - (1 - y_{ik}) M, \quad (i = 1, 2, \dots, n; \quad k = 1, 2, \dots, l) & (25) \\ \sum_{j=1}^{m} w_j^* \overline{v}_{ij} \le \sum_{j=1}^{m} w_j^* \overline{v}_{kj} + y_{ik} M, \quad (i = 1, 2, \dots, n; \quad k = 1, 2, \dots, l) & (26) \\ \sum_{i=1}^{n} y_{ik} + 1 = r^*(x_k), \quad (i = 1, 2, \dots, n; \quad k = 1, 2, \dots, l) & (27) \\ \sum_{j=1}^{m} w_j^* = 1 & (28) \\ 0 \le w_j^* \le 1, \quad (j = 1, 2, \dots, m) & (29) \\ y_{ik} = 1 \quad or \quad 0, \quad (k = 1, 2, \dots, l) & (30) \end{cases}$$

then, $r_{w^*}(x_k) = r^*(x_k) \quad (k = 1, 2, \dots, l)$.

The proof of Lemma 1 is provided in Appendix A.

Lemma 2: Let F be the OWA operator as per Eq. (5). If there exists $w^* = (w_1^*, w_2^*, \ldots, w_m^*)$ satisfying the constraint conditions (27)-(32) below

$$\begin{cases} \sum_{j=1}^{m} w_j^* \bar{v}_{i(j)} > \sum_{j=1}^{m} w_j^* \bar{v}_{k(j)} - (1 - y_{ik})M, \quad (i = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, l) & (31) \\ \sum_{j=1}^{m} w_j^* \bar{v}_{i(j)} \le \sum_{j=1}^{m} w_j^* \bar{v}_{k(j)} + y_{ik}M, \quad (i = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, l) & (32) \end{cases}$$

Then, $r_{w^*}(x_k) = r^*(x_k) \quad (k = 1, 2, \ldots, l)$.

The proof of Lemma 2 is provided in Appendix A.

Based on Lemmas 1 and 2, Theorem 3 is obtained.

Theorem 3: Let $b_j = w_j - w_j^0$ and $g_j = |w_j - w_j^0|$.

(1) Let F be the WA operator as per Eq. (4), model (24) can be equivalently transformed into the following mixed 0-1 linear programming models (33)-(42)

$$\begin{cases} \min \sum_{j=1}^{m} g_j & (33) \\ s.t. \begin{cases} \sum_{j=1}^{m} w_j \bar{v}_{ij} > \sum_{j=1}^{m} w_j \bar{v}_{kj} - (1 - y_{ik})M, \quad (i = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, l) & (34) \\ \sum_{j=1}^{m} w_j \bar{v}_{ij} \le \sum_{j=1}^{m} w_j \bar{v}_{kj} + y_{ik}M, \quad (i = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, l) & (35) \\ b_j = w_j - w_j^0, \quad (j = 1, 2, \ldots, m) & (36) \\ b_j \le g_j, \quad (j = 1, 2, \ldots, m) & (37) \\ -b_j \le g_j, \quad (j = 1, 2, \ldots, m) & (38) \\ \sum_{i=1}^{n} y_{ik} + 1 = r^*(x_k), \quad (k = 1, 2, \ldots, l) & (39) \\ \sum_{j=1}^{m} w_j = 1 & (40) \\ 0 \le w_j \le 1, \quad (j = 1, 2, \ldots, m) & (41) \\ y_{ik} = 1 \quad or \quad 0, \quad (k = 1, 2, \ldots, l) & (42) \end{cases} \end{cases}$$

(2) In models (33)-(42), replace the constraints (34)-(35) by the constraints (43)-(44)

$$\begin{cases} \sum_{j=1}^{m} w_j \bar{v}_{i(j)} > \sum_{j=1}^{m} w_j \bar{v}_{k(j)} - (1 - y_{ik})M, \quad (i = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, l) & (43) \\ \sum_{j=1}^{m} w_j \bar{v}_{i(j)} \le \sum_{j=1}^{m} w_j \bar{v}_{k(j)} + y_{ik}M, \quad (i = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, l) & (44) \end{cases}$$

Let F be the OWA operator as per Eq. (5) and model (24) can be equivalently transformed into the mixed 0-1 linear programming models (33), (36)-(44).

The proof of Theorem 3 is provided in Appendix A.

In this paper, we denote the models (33)-(42) as P_3, and denote the models (33), (36)-(44) as P_4. In both models P_3 and P_4, w_j $(j = 1,2,...,m)$ and y_{ik} $(i = 1,2,...,n;\ k = 1,2,...,l)$ are decision variables.

A decision maker can manipulate a strategic weight to obtain her/his desired ranking of the alternatives $\{x_1,x_2,...,x_l\}$ if the optimum solution to P_3 or P_4 exists. Otherwise, it is not possible to obtain her/his desired ranking of the alternatives by manipulating a strategic weight.

Finally, in this section, the existence of solution to models P_3 and P_4 is discussed in Properties 3-5.

Property 3: There exist $\{r^*(x_1),r^*(x_2),...,r^*(x_l)\}$ $(r^*\{x_k\} \in [\underline{r}(x_k),\bar{r}(x_k)],\ k = 1,2,...,l)$ that satisfy the following conditions: (a) $r^*(x_k) \le r_{w_0}(x_k)$ for any $k \in \{1,2,...,l\}$, and (b) $\exists f \in \{1,2,...,l\}$ such that $r^*(x_f) < r_{w_0}(x_f)$. Then, models P_3 and P_4 have feasible solutions.

The proof of Property 3 is provided in Appendix A.

Property 3 provides the condition under which a decision maker can manipulate a strategic weight to obtain a better ranking for the alternatives $\{x_1,x_2,...,x_l\}$.

Property 4: Let $r^*(x_k) = \underline{r}(x_k)$ for any $k \in \{1,2,...,l\}$. Then, the solution of models P_3 and P_4 does not exist under the following two conditions:

(1) there exists b such that $b = |\{x_i|\underline{r}(x_i) = \underline{r}(x_k),\ i = 1,2,...,l\}|$ and $b<l$;

(2) there exists $h \in \{1,2,...,l\}$ such that $\underline{r}(x_k) < \underline{r}(x_h) < \underline{r}(x_k) + b$.

The proof of Property 4 is provided in Appendix A.

Property 4 provides conditions under which a decision maker can not manipulate a strategic weight to obtain her/his desired ranking for the alternatives $\{x_1,x_2,...,x_l\}$ for both WA and OWA operators.

Property 5: When l=1, we have that (1) the optimal solution to P_3 exists if and only if $\underline{r}_{WA}(x_k) \le r^*(x_k) \le \bar{r}_{WA}(x_k)$; (2) the optimal solution to P_4 exists if and only if

$\underline{r}_{OWA}(x_k) \le r^*(x_k) \le \bar{r}_{OWA}(x_k)$.

The proof of Property 5 is provided in Appendix A.

Property 5 provides condition that make possible for a decision maker to manipulate a strategic weight to obtain any desired ranking within the ranking range of an alternative.

5. Numerical analysis and simulation experiments

In this section, an example with real data (provided in Appendix B) taken from the Academic Ranking of World Universitie (ARWU; http://www.arwu.org /) is used to illustrate how the proposed MADM strategic weight manipulation model works. Moreover, simulation experiments comparing the performances of the WA and OWA operators in defending against strategic weight manipulation are also included.

5.1 Numerical analysis

Let us consider 50 Universities taken from ARWU as the set of alternatives $\{x_1,x_2,\ldots,x_{50}\}$, and the following 6 attributes $\{a_1,a_2,\ldots,a_6\}$ to rank them:

a_1: Quality of Education (Alumni: Alumni of an institution winning Nobel Prizes and Fields Medals);

a_2: Quality of Faculty 1 (Award: Staff of an institution winning Nobel Prizes and Fields Medals);

a_3: Quality of Faculty 2 (HiCi: Highly cited researchers in 21 broad subject categories);

a_4: Papers published in Nature and Science (N&S);

a_5: Papers indexed in Science Citation Index-expanded and Social Science Citation Index (PUB);

a_6: Per capita academic performance of an institution (PCP).

First, the data of the 50 universities over the 6 attributes is normalized into a standardized decision matrix $\bar{V} = [\bar{v}_{ij}]_{50\times 6}$. Then, using models P_1 and P_2, the ranking range of the alternatives $\{x_1,x_2,\ldots,x_{50}\}$, R_{WA} and R_{OWA}, are obtained and listed in Table 1.

Table 1: The ranking range R_{WA} and R_{OWA} for the 50 universities

x_i	R_{WA}	R_{OWA}	x_i	R_{WA}	R_{OWA}	x_i	R_{WA}	R_{OWA}

x_1	[1,2]	[1,2]	x_2	[2,12]	[2,9]	x_3	[2,25]	[2,8]
x_4	[2,12]	[2,8]	x_5	[2,15]	[2,5]	x_6	[2,47]	[4,10]
x_7	[1,47]	[1,10]	x_8	[3,24]	[6,17]	x_9	[3,42]	[6,14]
x_{10}	[4,12]	[5,13]	x_{11}	[5,19]	[8,23]	x_{12}	[6,25]	[10,22]
x_{13}	[9,27]	[10,30]	x_{14}	[5,35]	[9,29]	x_{15}	[4,34]	[10,28]
x_{16}	[8,31]	[16,25]	x_{17}	[9,28]	[11,23]	x_{18}	[6,32]	[11,29]
x_{19}	[9,50]	[13,50]	x_{20}	[8,34]	[13,38]	x_{21}	[8,49]	[14,43]
x_{22}	[3,50]	[9,50]	x_{23}	[13,45]	[14,43]	x_{24}	[16,44]	[18,33]
x_{25}	[3,45]	[9,45]	x_{26}	[15,48]	[19,40]	x_{27}	[20,48]	[20,38]
x_{28}	[18,44]	[18,42]	x_{29}	[17,40]	[17,40]	x_{30}	[15,48]	[20,41]
x_{31}	[16,50]	[18,50]	x_{32}	[20,48]	[19,49]	x_{33}	[9,50]	[22,46]
x_{34}	[20,47]	[18,48]	x_{35}	[10,49]	[21,42]	x_{36}	[14,50]	[20,47]
x_{37}	[27,50]	[28,50]	x_{38}	[13,50]	[17,49]	x_{39}	[24,50]	[28,50]
x_{40}	[18,50]	[22,50]	x_{41}	[17,49]	[29,50]	x_{42}	[11,50]	[21,49]
x_{43}	[32,50]	[28,50]	x_{44}	[14,50]	[19,50]	x_{45}	[24,50]	[25,50]
x_{46}	[20,50]	[27,49]	x_{47}	[25,50]	[26,50]	x_{48}	[16,50]	[23,50]
x_{49}	[28,50]	[39,50]	x_{50}	[30,50]	[28,50]			

Existing approaches to attribute weights setting [1-5, 8, 11, 14, 20, 23, 26, 28, 29, 40, 41] assume that decision makers honest, and aim to set attribute weights to get an optimal ranking of alternatives. However, a decision maker might be dishonest, and she/he would aspire to strategically set attribute weights to achieve her/his purpose. Next, based on the data in Table 1, we assume that the decision maker aims to strategically set attribute weights in the Academic Ranking of World Universities, illustrating the use of our model in the MADM strategic weight manipulation.

Let w_0 be the objective weight vector of attributes, and r_0 be the ranking of the corresponding alternative(s) under the attribute weight vector w_0. In the example, we set $w_0 = (1/6, 1/6, 1/6, 1/6, 1/6, 1/6)$. We set different manipulated alternatives and the desired rankings of these manipulated alternatives, r^*. Afterwards, models P_3 and P_4 are applied to get the manipulated strategic weight vector w^* corresponding to the desired ranking of the manipulated alternatives. For example,

(1) Let x_3 be the manipulated alternative. Clearly, $r_0(x_3) = 2$. Meanwhile, the

desired ranking of alternative x_3 for the decision maker is set to $r^*(x_3) = 8$. In other words, the decision maker plans to dishonestly depress the ranking of the university x_3. Then, let F be the OWA operator as per Eq. (5), P_4 is used to obtain the strategic weight vector $w^* = (0.548, 0, 0.167, 0, 0.12, 0.273)$ to achieve the above purpose;

(2) Let $\{x_8, x_{13}, x_{14}, x_{15}\}$ be the manipulated alternatives. Clearly, $r_0 = \{8,13,12,14\}$. Meanwhile, the desired ranking of alternatives $\{x_8, x_{13}, x_{14}, x_{15}\}$ for the decision maker is $r^* = \{6,12,10,9\}$. In other words, the decision maker plans to dishonestly improve the ranking of the universities $\{x_8, x_{13}, x_{14}, x_{15}\}$. Then, let F be the WA operator as per Eq. (4), P_3 is used to obtain the strategic weight vector $w^* = (0.37, 0.1, 0.232, 0.308, 0.323, 0)$ to achieve the *targeted ranking*;

(3) Let $\{x_9, x_{10}, x_{11}, x_{12}\}$ be the manipulated alternatives. Clearly, $r_0 = \{9,10,11,13\}$. Meanwhile, the desired ranking of alternatives $\{x_9, x_{10}, x_{11}, x_{12}\}$ for the decision maker is $r^* = \{3,4,5,6\}$. In other words, the decision maker plans to dishonestly improve the ranking of the universities $\{x_9, x_{10}, x_{11}, x_{12}\}$. Then, let F be the WA operator as per Eq. (4), P_3 does not have a solution, which means that it is not possible to strategically set a attribute weight vector to achieve the *desired ranking*.

Table 2 shows the strategic weight vector w^* under different manipulated alternatives and the corresponding desired ranking r^*.

Table 2: The strategic weight vector w^* under different manipulated alternatives and the desired ranking r^*.

Manipulated alternative	F	r_0	r^*	w^*
x_3	WA	2	25	$(0, 0, 0, 0.05, 0, 0.995)$
	OWA	2	8	$(0.548, 0, 0.167, 0, 0.12, 0.273)$
x_6	WA	7	2	$(0.167, 0.457, 0, 0, 0, 0.376)$
	OWA	7	4	$(0.586, 0.167, 0, 0, 0.081, 0.167)$
x_{20}	WA	18	8	$(0, 0, 0.009, 0.167, 0.748, 0.076)$
	OWA	18	13	$(0, 0.029, 0.167, 0.167, 0.471, 0.167)$
$\{x_8, x_{13}, x_{14}, x_{15}\}$	WA	{8,13,12,14}	{3,9,5,4}	No solution
	OWA	{8,13,12,14}	{6,10,9,11}	No solution

	WA	{8,13,12,14}	{6,12,10,9}	(0.37, 0.1, 0.232, 0308, 0.323, 0)
	OWA	{8,13,12,14}	{6,13,9,11}	(0, 0, 1, 0, 0, 0)
$\{x_9, x_{10}, x_{11}, x_{12}\}$	WA	{9,10,11,13}	{3,4,5,6}	No solution
	OWA	{9,10,11,13}	{6,5,8,10}	No solution
	WA	{9,10,11,13}	{17,12,13,15}	(0.06, 0, 0.783, 0.089, 0, 0.068)
	OWA	{9,10,11,13}	{10,12,17,14}	(0.439, 0, 0.561, 0, 0, 0)
$\{x_{20}, x_{23}, x_{25}, x_{27}\}$	WA	{18,23,25,29}	{8,13,3,20}	No solution
	OWA	{18,23,25,29}	{13,14,9,20}	No solution
	WA	{18,23,25,29}	{17,23,24,28}	(0.124, 0.165, 0.043, 0.107, 0.561, 0)
	OWA	{18,23,25,29}	{14,22,23,24}	(0.06, 0, 0.32, 0.332, 0, 0.288)

This illustrative example also highlights the main difference between the existing approaches to attribute weights setting and the proposed models in this study, which consists in the assumption made in the proposed model in this regarding the decision maker as dishonest, and aiming to find which the strategically setting of attribute weights to allow her/him to achieve the desired/targeted ranking of interest.

5.2 Simulation experiments

In MADM problems, the WA operator and the OWA operator are both frequently used to aggregate the associated attribute preference values to rank the alternatives. Therefore, a challenge for analysts is how to compare the performances of the WA and OWA operators in defending against the MADM strategic weight manipulation. In this subsection, we design simulation experiments to deal with this challenge.

Let $r_w^{WA}(x_k)$ and $r_w^{OWA}(x_k)$ be the rankings of x_k under the attribute weight vector $w = (w_1, w_2, \ldots, w_m)$ when setting F to be the WA and OWA operators, respectively. As stated previously, $r_w^{WA}(x_k)$ and $r_w^{OWA}(x_k)$ will vary for different weight vector $w = (w_1, w_2, \ldots, w_m)$. Next, we design simulation experiment I to show the fluctuation of both rankings of the alternatives as the attribute weight vector changes.

Simulation experiment I:

Step 1: We randomly generate a standardized decision matrix $\overline{V} = [\overline{v}_{ij}]_{50 \times 6}$, where $\bar{v}_{ij} \in [0,1]$.

Step 2: We randomly generate 1000 attribute weight vectors, $w_i = (w_{i,1}, w_{i,2}, \ldots, w_{i,m})$ $(i = 1,2,\ldots,1000)$. Based on Eqs. (4) and (5), and the standardized decision matrix $\bar{V} = [\bar{v}_{ij}]_{50\times 6}$ obtained in Step 1. We obtain the ranking of alternative x_k under the WA and OWA operators, $r_{w_i}^{WA}(x_k)$ and $r_{w_i}^{OWA}(x_k)$ $(k = 1,2,\ldots,50;\ i = 1,2,\ldots,1000)$, respectively.

Figure 1 shows the average values of $r_{w_i}^{WA}(x_k)$ and $r_{w_i}^{OWA}(x_k)$ $(k \in \{6, 12, 20, 35, 46, 50\};\ i = 1, 2,\ldots, 1000)$ in Simulation experiment I.

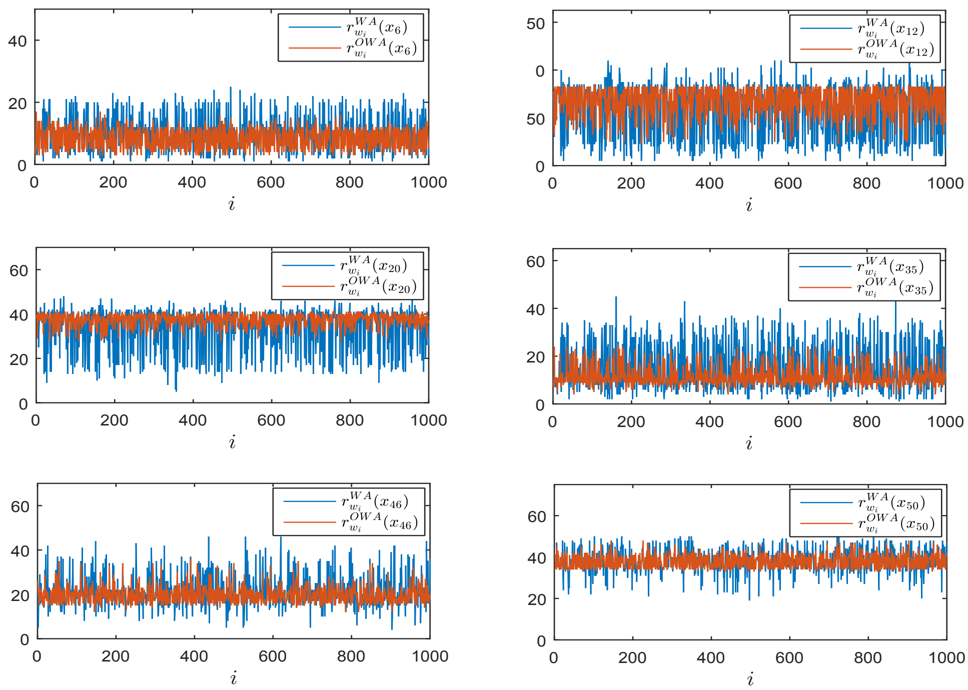

Figure 1: The average values of $r_{w_i}(x_k)$ for the alternatives under the WA and OWA operators.

Figure 1 clearly shows that the fluctuation of the rankings of the alternatives in the WA case is much larger than the rankings in the OWA case. Notably, we ran Simulation experiment I many times, and the obtained observations coincide. Generally, with the change of the attribute weight vector w, a larger fluctuation of the rankings of the alternatives implies a higher possibility to manipulate a strategic attribute weight vector to obtain a desired ranking. In other words, the larger the fluctuation of the rankings of the alternatives the worse performance in defending against strategic weight manipulation.

Further, we use the ranking range of the alternative x_k to measure the fluctuation degree of $r_w^{WA}(x_k)$ and $r_w^{OWA}(x_k)$. Let $R_{WA} = [\underline{r}_{WA}(x_k), \overline{r}_{WA}(x_k)]$ and $R_{OWA} = [\underline{r}_{OWA}(x_k), \overline{r}_{OWA}(x_k)]$ be as defined before in the paper.

Figure 2 shows the ranking range of the alternatives under the WA and OWA operators in the example presented in Section 5.1.

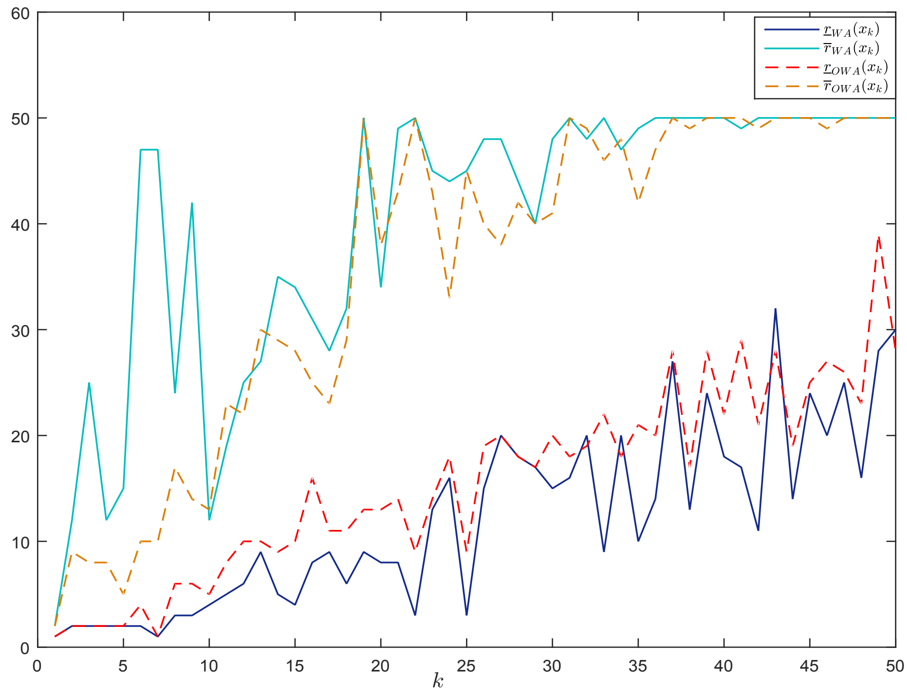

Figure 2: The ranking range of the alternatives in Example 5.1

A simulation experiment II is designed to analyse the average ranking range of the alternatives under the WA and OWA operators, respectively.

Simulation experiment II:

Step 1: We randomly generate an $n \times m$ standardized decision matrix $\overline{V} = [\overline{v}_{ij}]_{n \times m}$, with $\overline{v}_{ij} \in [0,1]$. Using model P_1, the ranking range of the alternative x_i, $[\underline{r}_{WA}(x_i), \overline{r}_{WA}(x_i)]$, is computed, and using model P_2, the ranking range of the alternative x_i, $[\underline{r}_{OWA}(x_i), \overline{r}_{OWA}(x_i)]$, is computed as well. Let

$$WR_{WA}(x_i) = \overline{r}_{WA}(x_i) - \underline{r}_{WA}(x_i) \tag{45}$$

and

$$WR_{OWA}(x_i) = \overline{r}_{OWA}(x_i) - \underline{r}_{OWA}(x_i) \tag{46}$$

be the width of the ranking range of the alternative x_i under the WA and OWA operators, respectively.

Step 2: When setting different m and n, we run 100 times Step 1 to obtain the average values of $WR_{WA}(x_i)$ and $WR_{OWA}(x_i)$.

Figure 3 shows the average values of $WR_{WA}(x_i)$ and $WR_{OWA}(x_i)$ in Simulation experiment II.

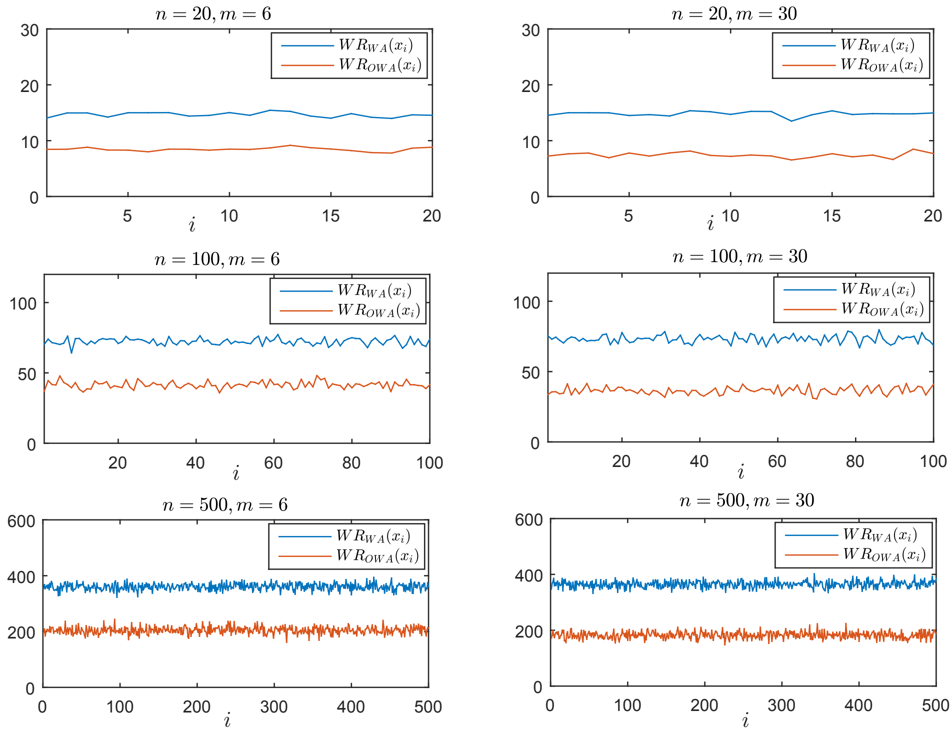

Figure 3: Average values of $WR(x_i)$ in simulation method II under the different parameters.

From Figures 2 and 3, following observations are drawn:

(1) Figure 2 shows that $[\underline{r}_{OWA}(x_i), \overline{r}_{OWA}(x_i)] \subseteq [\underline{r}_{WA}(x_i), \overline{r}_{WA}(x_i)]$ in a vast majority of cases;

(2) Figure 3 shows the average width of the ranking range of the alternatives in the WA case is much larger than the ranking range in the OWA case.

Both observations show a better performance of the OWA operator than the WA operator in defending against strategic weight manipulation.

6. Conclusions

This paper focuses on some issues on the strategic attribute weight used to manipulate the ranking of alternatives. The main contributions presented are as follows:

(1) We define the concept of the ranking range of an alternative in the MADM framework, and propose MLPMs to obtain the ranking range of alternatives under the set of attribute weight W.

(2) We reveal the process of designing a strategic attribute weight vector, and analyze the conditions to manipulate a strategic attribute weight to obtain her/his desired ranking based on the ranking range and the proposed MLPMs.

(3) Simulation experiments are presented that show performance of the OWA operator exceeded that of the WA operator in defending against strategic weight manipulation in MADM problems.

In some MADM problems, a group of decision makers might be involved, and they could provide incomplete attribute weights information (e.g., [17, 19, 33]). In these instances, it will be an interesting future study to analyze the MADM strategic weight manipulation under a group context with incomplete attribute weights information. Other interesting proposal would be to analyze strategic weight manipulation in decision context under trust relationships [34, 35].

Acknowledgments

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Appendix A: Proofs.

Proof of Theorem 1:

The process of proving Theorem 1 is divided into two steps.

Step 1: On the one hand, using WA operator as per Eq. (4), based on the constraints (9)-(13) and result (1), i.e., $x_i \succ x_k$ if and only if $y_i = 1$ under the conditions $D(x_i) > D(x_k) - (1 - y_i)M$ and $D(x_i) \le D(x_k) + y_iM$, we can obtain,

$$\begin{aligned}
\underline{r}(x_k) &= 1 + \min \left| \{x_i \mid D(x_i) > D(x_k),\ \ i = 1,2,...,n\} \right| \\
&= 1 + \min \left\{ \begin{aligned} &\sum_{i=1}^{n} y_i \,\Big|\, D(x_i) > D(x_k) - (1 - y_i)M,\ \ D(x_i) \le D(x_k) + y_iM, \\ &\qquad\qquad\qquad\qquad\qquad\qquad y_i = 1\ \ or\ \ 0,\ \ (i = 1,2,...,n) \end{aligned} \right\} \\
&= 1 + \min \left\{ \begin{aligned} &\sum_{i=1}^{n} y_i \,\Bigg|\, \sum_{j=1}^{m} w_j \bar{v}_{ij} > \sum_{j=1}^{m} w_j \bar{v}_{kj} - (1 - y_i)M,\ \ D(x_i) \le D(x_k) + y_iM, \\ &\qquad\qquad\qquad\qquad\qquad\qquad y_i = 1\ \ or\ \ 0,\ \ (i = 1,2,...,n) \end{aligned} \right\} \\
&= 1 + \min \sum_{i=1}^{n} y_i,\ \ (i = 1,2,...,n)
\end{aligned}$$

where M is a large enough number.

On the other hand, using WA operator as per Eq. (4), $\sum_{j=1}^{m} w_j \bar{v}_{ij} \le \sum_{j=1}^{m} w_j \bar{v}_{kj} + y_iM$, $\sum_{j=1}^{m} w_j \bar{v}_{ij} > \sum_{j=1}^{m} w_j \bar{v}_{kj} - (1 - y_i)M$ when $y_i = 0$, based on result (2), i.e., $x_i \precsim x_k$ if and only if $y_i = 0$ under the conditions $D(x_i) \le D(x_k) + y_iM$ and $D(x_i) > D(x_k) - (1 - y_i)M$. Then, according to the definition of ranking of the alternatives and Eq. (6), we have $\underline{r}(x_k) = 1 + \min \sum_{i=1}^{n} y_i$.

Step 2: On the one hand, based on the constraints (9)-(13) and result (1) in step 1, we can obtain,

$$
\begin{aligned}
\overline{r}(x_k) &= 1+\max\left|\{x_i\,|D(x_i)>D(x_k)\}\right| \\
&= 1+\max\left\{\sum_{i=1}^{n} y_i \,\middle|\, D(x_i)>D(x_k)-(1-y_i)M,\ \ D(x_i)\le D(x_k)+y_iM,\ \ y_i=1\ \ or\ \ 0,\ \ (i=1,2,\dots,n)\right\} \\
&= 1+\max\left\{\sum_{i=1}^{n} y_i \,\middle|\, \sum_{j=1}^{m} w_j\overline{v}_{ij} > \sum_{j=1}^{m} w_j\overline{v}_{kj}-(1-y_i)M,\ \ D(x_i)\le D(x_k)+y_iM,\ \ y_i=1\ \ or\ \ 0,\ \ (i=1,2,\dots,n)\right\} \\
&= 1+\max\sum_{i=1}^{n} y_i,\ \ (i=1,2,\dots,n)
\end{aligned}
$$

where M is a large enough number.

On the other hand, the proof of is similar to the proof of step 1.

Above all, this completes the proof of Theorem 1.

Proof of Theorem 2:

The proof of Theorem 2 is similar to the proof of Theorem 1. Here, we only replace the WA operator $D(x_i)=\sum_{j=1}^{m} w_j\overline{v}_{ij}$ and $D(x_k)=\sum_{j=1}^{m} w_j\overline{v}_{kj}$ by OWA operator $D(x_i)=\sum_{j=1}^{m} w_j\overline{v}_{i(j)}$ and $D(x_k)=\sum_{j=1}^{m} w_j\overline{v}_{k(j)}$ in proof of Theorem 1.

Then, this completes the proof of Theorem 2.

Proof of Property 1:

The proof of Property 1 required proving that given alternative x_k, if its attribute ranking verifies $c_j(x_k)\in[\underline{c}(x_k),\overline{c}(x_k)]$, then it is also $c_j(x_k)\in[\underline{r}_{WA}(x_k),\overline{r}_{WA}(x_k)]$.

Let $c_j(x_k)$ be the attribute ranking of alternative x_k associated with attribute a_j and let the attribute weight vector be $w=(\underbrace{0,\dots,0}_{j-1},1,0,\dots,0)$, based on Eq. (4) and definition of ranking of alternative, we have that $r_{WA}(x_k)=c_j(x_k)$, i.e., $r_{WA}(x_k)\in[\underline{r}_{WA}(x_k),\overline{r}_{WA}(x_k)]$.

This completes the proof of property 1.

Proof of Property 2:

(i) We firstly prove that the best ranking of alternative x_k, $\underline{r}_{WA}(x_k)$, satisfies $\underline{r}_{WA}(x_k)\in[|O_{I_k}(x_k)|+1,\underline{c}(x_k)]$.

On the one hand, let $O_{I_k}(x_k)=\{x_i|\overline{v}_{ij}>\overline{v}_{kj},\ j=1,2,\dots,m\}$ be as previously defined, and $|\overline{O}_{I_k}(x_k)|$ be number of alternatives x_h, satisfying $x_h\in O_{I_k}(x_k)$. Using the WA operator as per Eq. (4), we obtain,

$$D(x_h)-D(x_k)=\sum_{j=1}^{m} w_j\overline{v}_{hj}-\sum_{j=1}^{m} w_j\overline{v}_{kj}=\sum_{j=1}^{m} w_j(\overline{v}_{hj}-\overline{v}_{kj})>0$$
.

According to the definition of ranking of alternatives, we obtain $|O_{I_k}(x_k)|+1\leq \underline{r}_{WA}(x_k)$.

On the other hand, based on property 1, it is obvious that the best ranking of the alternative x_k, $\underline{r}_{WA}(x_k)$, satisfies $\underline{r}_{WA}(x_k)\leq \underline{c}(x_k)$.

(ii) We further prove that the worst ranking of the alternative x_k, $\overline{r}_{WA}(x_k)$, satisfies $\overline{r}_{WA}(x_k)\in[\overline{c}(x_k),\ n-|\overline{O}_{I_k}(x_k)|]$.

On the one hand, let $\overline{O}_{I_k}(x_k)=\{x_i|\overline{v}_{ij}\leq\overline{v}_{kj},\ j=1,2,\ldots,m\}$ be as previously defined, and $|\overline{O}_{I_k}(x_k)|$ be the number of alternatives x_h, satisfying $x_h\in\overline{O}_{I_k}(x_k)$, then there exist at most $n-|\overline{O}_{I_k}(x_k)|-1$ alternatives x_f satisfying $x_f\in O_{I_k}(x_k)$, according to the process of proof in (i), we obtain $\overline{r}_{WA}(x_k)\leq n-|\overline{O}_{I_k}(x_k)|$.

On the other hand, based on property 1, it is obvious that the worst ranking of the alternative x_k, satisfies $\overline{r}_{WA}(x_k)\geq\overline{c}(x_k)$.

This completes the proof of property 2.

Proof of Lemma 1:

(1) Because $w^*=(w_1^*,w_2^*,\ldots,w_m^*)$ satisfies the constraints (25)-(30), then it is:

$$\begin{cases}\sum_{j=1}^{m} w_j^*\overline{v}_{ij}>\sum_{j=1}^{m} w_j^*\overline{v}_{kj}-(1-y_{ik})M, & (i=1,2,\ldots,n;\ \ k=1,2,\ldots,l)\\ \sum_{j=1}^{m} w_j^*\overline{v}_{ij}\leq\sum_{j=1}^{m} w_j^*\overline{v}_{kj}+y_{ik}M, & (i=1,2,\ldots,n;\ \ k=1,2,\ldots,l)\\ \sum_{i=1}^{n} y_{ik}+1=r^*(x_k), & (i=1,2,\ldots,n;\ \ k=1,2,\ldots,l)\\ \sum_{j=1}^{m} w_j^*=1 & \\ 0\leq w_j^*\leq 1, & (j=1,2,\ldots,m)\\ y_{ik}=1\ \ or\ \ 0, & (i=1,2,\ldots,n;\ \ k=1,2,\ldots,l)\end{cases}$$

We have $\sum_{j=1}^{m} w_j^*\overline{v}_{ij}>\sum_{j=1}^{m} w_j^*\overline{v}_{kj}$ $(k=1,2,\ldots,l)$ and $\sum_{j=1}^{m} w_j^*\overline{v}_{ij}\leq\sum_{j=1}^{m} w_j^*\overline{v}_{kj}+1\cdot M$, $(k=1,2,\ldots,l)$ when $y_{ik}=1$, based on result (1) in proof of Lemma 1, we have $x_i\succ x_k$

$(k = 1,2,...,l)$. And, when $y_{ik} = 0$, $\sum_{j=1}^{m} w_j^* \bar{v}_{ij} \leq \sum_{j=1}^{m} w_j^* \bar{v}_{kj}$, $(k = 1,2,...,l)$ and $\sum_{j=1}^{m} w_j^* \bar{v}_{ij} > \sum_{j=1}^{m} w_j^* \bar{v}_{kj} - 1 \cdot M$, $(k = 1,2,...,l)$, based on result (2) in proof of Lemma 1, we have $x_i \precsim x_k$ $(k = 1,2,...,l)$. Based on the condition $\sum_{i=1}^{m} y_{ik} + 1 = r^*(x_k)$ $(i = 1,2,...,n;\ k = 1,2,...,l)$ and the definition of ranking of alternative, we obtain $r_{w^*}(x_k) = r^*(x_k)$ $(k = 1,2,...,l)$.

This completes the proof of Lemma 1.

Proof of Lemma 2:

The proof of Lemma 2 is similar to the proof of Lemma 1. Here, we only replace the WA operator $D(x_i) = \sum_{j=1}^{m} w_j \bar{v}_{ij}$ and $D(x_k) = \sum_{j=1}^{m} w_j \bar{v}_{kj}$ by the OWA operator $D(x_i) = \sum_{j=1}^{m} w_j \bar{v}_{i(j)}$ and $D(x_k) = \sum_{j=1}^{m} w_j \bar{v}_{k(j)}$ in proof of Lemma 1 to obtain $r_{w^*}(x_k) = r^*(x_k)$ ($k = 1,2,...,l$).

This completes the proof of Lemma 2.

Proof of Theorem 3:

Introducing the following two transformed decision variables $b_j = w_j - w_j^0$ and $g_j = |w_j - w_j^0|$, it is $b_j \leq g_j$ and $-b_j \leq g_j$, which guarantee $g_j \geq |b_j| = |w_j - w_j^0|$.

Based on Lemmas 1 and 2, put models (25)-(30) and (27)-(32) into Eq. (24), then the Eq. (24) can be transformed into mixed 0-1 linear programming models (33)-(42) and (35)-(44).

This completes the proof of Theorem 3.

Proof of Property 3:

First, we prove the existence of solution to model P_3 as follows.

Let $r_{w_0} = \{r_{w_0}(x_1), r_{w_0}(x_2), ..., r_{w_0}(x_l)\}$ be the objective ranking of the alternatives $\{x_1, x_2, ..., x_l\}$, let $r^* = \{r^*(x_1), r^*(x_2), ..., r^*(x_l)\}$ be the decision maker's desired rankings of the alternatives $\{x_1, x_2, ..., x_l\}$, and we have $r^*(x_k) \in [\underline{r}_{WA}(x_k), \bar{r}_{WA}(x_k)]$ $(k = 1,2,...,l)$. Based on the continuous of ranking range, using enumeration, let the ranking of alternatives be $r^* = \{r_{w_0}(x_1) - 1, r_{w_0}(x_2), ..., r_{w_0}(x_l)\}$, $\{r_{w_0}(x_1), r_{w_0}(x_2) - 1, ..., r_{w_0}(x_l)\}$, ..., $\{r_{w_0}(x_1), r_{w_0}(x_2), ..., r_{w_0}(x_l) - 1\}$, respectively. Then, there must exist r^*, the feasible solution of model

P_3 exists; otherwise, the objective ranking r_{w_0} is the best ranking of alternatives, which contradicts the assumption in the beginning. Then, we can prove that model P_3 has feasible solution.

Similarly, we can prove the model P_4 has feasible solution.

This completes the proof of Property 3.

Proof of Property 4:

According to the conditions in Property 4, without loss of generality, let $r^* = \{\underline{r}(x_1),\ldots,\underline{r}(x_l)\} = \{\underline{r}(x_1),\ldots,\underbrace{\underline{r}(x_{h-b-1}) = \underline{r}(x_k),\ldots,\underline{r}(x_{h-1}) = \underline{r}(x_k)}_{b+1},\underline{r}(x_h),\ldots,\underline{r}(x_l)\}$ be our desired ranking of the alternatives $\{x_1,x_2,\ldots,x_l\}$, let r^* be denoted as monotonically increasing, then, we have $\underline{r}(x_h) = \underline{r}(x_k) + b + 1$, which contradict the condition $\underline{r}(x_k) < \underline{r}(x_h) < \underline{r}(x_k) + b$ in Property 4.

This completes the proof of Property 4.

Proof of Property 5:

We prove the Property 5 with reduction to absurdity as follows.

When $l = 1$, , we assume that model P_3 has no solution for ranking $r(x_k)$, which satisfies $r(x_k) \in [\underline{r}_{WA}(x_k),\bar{r}_{WA}(x_k)]$, then, there is no attribute weights w such that the alternative x_k can obtain the ranking $r_w(x_k)$, which contradicts the continuous of ranking range, so we obtain that the solution of model P_3 exists.

Similarly, we can prove the existence of solution to model P_4.

This completes the proof of Property 5.

Appendix B: The original data for 50 universities in Section 5.1

x_i	$\bar{v}_{i1}$	$\bar{v}_{i2}$	$\bar{v}_{i3}$	$\bar{v}_{i4}$	$\bar{v}_{i5}$	$\bar{v}_{i6}$
1	100	100.0	100	100	100	76.6
2	40.7	89.6	80.1	70.1	70.6	53.8
3	68.2	80.7	60.6	73.1	61.1	68.0
4	65.1	79.4	66.1	65.6	67.9	56.5
5	77.1	96.6	50.8	55.6	66.4	55.8
6	53.3	93.4	57.1	43.0	42.4	70.3
7	49.5	66.7	49.3	56.4	44.0	100.0
8	63.5	65.9	52.1	51.9	68.8	33.2
9	59.8	86.3	49.0	42.9	49.8	42.0
10	49.7	54.9	52.3	51.9	70.9	43.1
11	47.6	50.4	51.0	58.8	63.0	37.8

12	29.5	47.1	52.3	47.2	70.7	31.6
13	42.0	49.8	50.4	45.3	59.9	40.2
14	19.2	35.5	56.6	55.1	62.9	36.6
15	21.2	31.6	53.0	51.7	71.9	29.3
16	37.7	33.6	44.0	44.9	70.2	28.8
17	31.6	33.8	49.6	39.6	67.7	37.4
18	28.1	36.2	38.5	40.6	71.7	32.7
19	0.0	39.9	46.8	53.5	59.5	34.9
20	29.5	35.5	38.4	45.9	55.7	46.3
21	30.8	14.1	41.9	48.6	70.8	28.8
22	34.4	0.0	56.2	41.3	75.9	25.6
23	14.5	35.8	44.2	34.5	62.0	38.0
24	30.8	34.8	40.2	35.7	62.5	24.6
25	19.9	17.2	38.8	38.6	79.1	29.3
26	31.6	37.2	33.6	32.6	59.0	23.5
27	28.1	31.9	35.2	40.3	56.2	23.3
28	15.4	22.1	50.3	37.0	58.1	28.8
29	29.9	36.2	34.9	32.4	55.5	28.6
30	29.5	16.3	47.3	32.5	64.0	26.2
31	15.4	14.9	50.2	39.0	61.7	23.9
32	22.9	24.9	40.7	41.8	50.5	26.0
33	17.0	59.8	30.3	40.9	19.0	39.6
34	12.6	34.1	37.1	36.0	45.0	34.2
35	21.8	18.8	28.0	34.0	63.2	39.2
36	33.6	27.4	25.8	29.8	59.2	23.9
37	16.2	16.3	38.6	37.5	56.0	26.6
38	14.5	39.1	38.7	27.5	37.3	38.0
39	8.9	16.3	39.8	33.5	61.2	25.6
40	15.4	18.8	32.8	32.0	63.0	24.2
41	18.5	32.6	27.1	26.1	56.2	25.6
42	30.3	54.3	16.8	17.4	47.3	26.6
43	19.2	20.0	33.0	31.6	52.7	26.5
44	17.0	13.3	28.6	25.3	66.9	30.2
45	18.5	34.5	31.1	35.6	35.8	22.4
46	19.9	25.3	23.7	29.4	51.7	34.2
47	20.5	24.9	25.8	30.5	51.9	25.9
48	22.4	26.6	24.8	23.5	50.8	37.4
49	0.0	31.7	35.6	22.1	53.2	20.3
50	0.0	29.3	34.3	28.9	45.3	27.5