An incentive mechanism based minimum adjustment consensus model under dynamic trust relationship

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***Abstract*—In traditional group decision making, the inconsistent experts are usually forced to make compromises towards the group opinion to increase the group consensus level. However, the strategy of reaching group consensus via an incentive mechanism encouraging adjustment of preferences is more effective than forcing, which is the aim of this article. Specifically, this paper establishes a novel incentive mechanism to support group consensus under dynamic trust relationship. First, the supremum and infimum incentives based rule driven by trust relationship is defined. Based on the assumption that if incentive conditions are met, then experts will be willing to adjust their preferences, the incentive behavior driven minimum adjustment consensus model is developed to generate optimal incentive based recommendation preferences. Thus, the proposed incentive mechanism can effectively reduce the preference adjustment cost and promote group consensus reaching. Third, the updated trust relationships between experts are shown to be strengthen by the proposed incentive driven preference revision. Consequently, the optimization model based on trust interaction relationship is constructed to obtain the final group preference matrix. Finally, a supplier selection case of high-end medical equipment is provided to illustrate the proposed method, and to show the rationality and advantages of the proposed methodology with both a sensitivity analysis and a comparison analysis.**

***Index Terms*—Group decision making, Consensus, Incentive behaviour, Social network, Dynamic trust relationship**

I. Introduction

GROUP decision making (GDM), by which the given preferences of multiple experts on alternatives are summarised via an aggregation process into group preference from which the best alternative is selected [1]–[5], is widely used in daily life [6]–[10]. The rapid development of social media means that experts making a decision may be socially connected. As a response to this decision making framework, Wu et al. [11] proposed some social network group decision-making (SN-GDM) methods that regard trust relationship as a useful resource to derive recommendation opinions for inconsistent experts, which has been widely adopted and extended by other scholars [12]–[15]. In SN-GDM, experts with different social backgrounds and professional knowledge may have certain conflicts [16]–[19], which may lead to group inconsistency issues [20]–[22]. In reaching group consensus, the proposed models implement communication and feedback rounds where the inconsistent experts are invariably forced to make greater sacrifices regarding the adjustment of their preferences [23], [24]. This approach negative impact on experts' self-esteem may lead to the rejection of the adjustment of their preference advices [25]–[28]. A more sensible strategy is to encourage experts to revise their preferences using advices based on psychological behaviors such as trust relationship.

Recently, Ma et al. [29] proposed a reputation and interaction relationship based optimization methodology to derive experts' comprehensive weights. However, it is assumed that social trust relationship is static in nature. Subsequently, Xing et al. [30] proposed a maximum entropy approach to model dynamic trust relationship driven by experts' conflict degrees. Some dynamic trust methods have been studied in depth by other scholars [25]–[27], [34]–[38]. For example, Tan et al. [25] established the retention ratio model with experts' conflict degree to update trust relationships. A dynamic trust model considering experts' grey relationship coefficient is explored in [27]. Hao et al. [35] studied dynamic trust methods considering the expert's own influence and interpersonal relationships. Zhang et al. [36] built trust evolution models considering social influence. Chen et al. [37] also studied dynamic trust models with value-based social influence. Li et al. [38] proposed a dynamic trust risk management mechanism based on third-party supervision. The characteristics of these methods are that the social trust relationship between experts changes dynamically by the experts' conflict degree or the interpersonal relationship. However, according to the contact hypothesis [31], experts are more willing to trust experts with similar opinions [39]–[41].

Then, the first research issue of this article is how to model trust relationships between experts in dynamic environmen-t. To do that, the trust relationships are updated with the similarity degrees between experts in the 2-tuple linguistic context, and then, the updated trust relationships are strengthen by group experts tending to reach consensus. Hence, the advantage of this dynamic trust model is that it considers not only the explicit trust of experts (i.e., the social trust relationship), but also the implicit trust of experts (i.e., the opinion similarity). Therefore, the dynamic trust model can be used to calculate the interaction weights of experts.

The psychological behaviors of experts have been widely

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studied in GDM [42]–[46]. Ben-Arieh et al. [47] proposed the concept of minimum adjustment cost consensus model, which was later extended by Dong et al. to a linguistic framework [48]. Subsequently, Wu et al. [12] studied a minimum adjustment consensus model based on twofold personalized feedback mechanism, which utilizes trust relationship to generate recommendation preferences for inconsistent experts. Other scholars have studied consensus researching based on bounded confidence [34], [49]–[52]. For example, Liu et al. [34] investigated consensus considering self-confident behavior for GDM. Recently, Xing et al. [30] proposed a bargaining based feedback mechanism in which both the inconsistent experts and the the consistent experts can revise their preferences. Zhang et al. [53] designed the minimum cost consensus model considering limited compromise and tolerant behavior, which enables experts to adjust their preferences within a certain range. Zou et al. [41] also explored minimum adjustment consensus models with consideration of dynamic compromise behavior. Meng et al. [54] established a minimum adjustment consensus model based on Nash bargaining game, which not only punishes these non-cooperative experts, but also ensures the fairness and efficiency of the experts' modifica-tion. However, most of these work merely use strategies to force inconsistent experts to adopt group opinion, which may sacrifice individual interests of inconsistent experts.

Therefore, the second research issue of this article is how to design a reasonable interaction mechanism between ex-perts. To do so, it establishes an incentive mechanism by encouraging experts to adjust their preferences to efficiently promote consensus reaching. Incentive behavior is primarily the psychological process of motivating experts, after which they are willing to consider how to adjust their preference. Meanwhile, if experts modify preferences by the following an incentive process based on trust relationships, then the experts become more and more familiar with each other and, as a consequence, the trust relationships between experts become stronger. In other words, the change of trust relationship between experts and the feedback mechanism considering incentive behavior driven by trust relationship complement each other.

Generally, this article aims to develop a novel incentive mechanism to support group consensus under dynamic trust relationship. Specifically, the innovation of this article is as follows:

1) A dynamic trust model is investigated by combining the social trust relationship and the opinion similarity relationship. It is used to calculate the interaction weights of experts. After the individual weights are calculated by the incentive feedback mechanism, the comprehensive weights can be determined by combining the interac-tion weights and individual weights. Consequently, the innovation of this proposed comprehensive weights can reflect both the dynamic trust relationship and their own reputation simultaneously.
2) An incentive mechanism in dynamic trust relationship is proposed. The supremum and infimum incentive rule driven by trust relationship is first developed. If incentive conditions are satisfied, then experts are willing to adjust their preferences. Then, the incentive behavior driven minimum adjustment consensus model is established to generate recommendation advices to effectively reduce the adjustment cost and feedback rounds in reaching group consensus processes.

The remainder of the article is structured as follows. Required concepts on social network, the 2-tuple linguistic model and the Choquet integral are recalled in Section II. Section III reports the novel consensus reaching framework with incentive behavior in SN-GDM, including (1) the design of a new comprehensive weight model of experts; (2) the proposal of an incentive mechanism in dynamic trust relationship; and (3) the construction of the optimization model based on trust relationship. A case study on supplier selection of high-end medical equipment is provided in Section IV. Conclusions and future works are drawn in Section V.

II. Preliminaries

An undirected network between a set of experts, $E = \{E_1, E_2, \cdots, E_q\}$, is represented with a graph, $G(E, \mathbb{E})$, with vertices/nodes representing the experts E and edges $\mathbb{E}$ connecting pairs of nodes. Algebraically, an undirected network $G(E,\ \mathbb{E})$ is represented with a so-called adjacency matrix $T_G = (f_{hk})_{q\times q}$, where $f_{hk} = 1$ if $e_{hk} \in \mathbb{E}$; $f_{hk} = 0$ otherwise.

It assumes trust relationships between experts as social network (SN) analysis [55], [56] is advocated to use for exploiting social trust relationship, reputation and network relationship characteristics among experts. Herein, trust is modelled via a trust function [57], which associated a binary pair $\xi = (t, d)$ of trust degree, t, and distrust degree, d, to each pair of socially connected experts. The trust score of a given binary pair $\xi = (t, d)$ is $TS(\xi) = (t - d + 1)/2$.

In group decision making (GDM), experts may express their preferences using terms from a set of ordinal linguistic terms: $\mathbb{S} = \{s_0, s_1, \cdots, s_r\}$ (r even) such that $s_0 < s_1 < \cdots < s_r$, $s_{\frac{r}{2}}$ represents indifference of preference, and negator operator $neg(s_a) = s_{r-a}$ [58]–[60]. The 2-tuple linguistic model was introduced in [61] to manage the aggregation of a number of ordinal linguistic terms. This model relies on the so-called symbolic translation, $\psi \in [0, r]$, defined as per the mappings $\Delta(\psi) = (s_a, \omega_a)$, for $s_a \in \mathbb{S}, \omega_a \in [-0.5, 0.5)$, with inverse $\Delta^{-1}(s_a, \omega_a) = a + \omega_a = \psi$.

The weighted average of 2-tuple linguistic terms $\{(s_1, \omega_1), (s_2, \omega_2), \cdots (s_q, \omega_q)\}$ with normalized weights $\{\lambda_1, \lambda_{2,} \cdots, \lambda_q | \lambda_i \geq 0\, \forall i \wedge \sum_{i=1}^{q} \lambda_i = 1\}$ is [62]:

$$(\overline{s}, \overline{\omega}) = \Delta\left(\sum_{i=1}^{q} \lambda_i \Delta^{-1}(s_i, \omega_i)\right) \tag{1}$$

The arithmetic means is derived with all weights equal to $1/q$.

The Choquet integral [63] is a useful method for preferences aggregation in GDM. Recall that a fuzzy measure μ on a finite set $\mathbb{O} = \{o_1, o_2, \cdots, o_q\}$ is a $[0, 1]$-valued mapping on the set of all sub-coalitions of $\mathbb{O}$, $P(\mathbb{O})$, satisfying (i) $\mu(\emptyset) = 0$; $\mu(\mathbb{O}) = 1$; and (ii) $\phi \subseteq \varphi \Rightarrow \mu(\phi) \leq \mu(\varphi)$. Given a a fuzzy measure μ on $\mathbb{O}$, its Möbius transform is [64], [65]:

$$M(\mathbb{T}) = \sum_{\mathbb{U}\subseteq\mathbb{T}} (-1)^{\#\mathbb{T}-\#\mathbb{U}}\mu(\mathbb{U}), \quad \mathbb{T}\in P(\mathbb{O}) \tag{2}$$

Given a Möbius transform M, its fuzzy measure μ is [66]:

$$\mu(\mathbb{T}) = \sum_{\mathbb{U}\subseteq\mathbb{T}} M(\mathbb{U}), \quad \mathbb{T}\in P(\mathbb{O}). \tag{3}$$

Let f and μ be a positive real valued function and a fuzzy measure defined on $\mathbb{O}$, respectively. The Choquet integral CI_μ of $\{f(o_1), f(o_2), \cdots, f(o_q)\}$ is

$$CI_\mu(f(o_1), f(o_2), \cdots, f(o_q)) = \sum_{i=1}^{q} [\mu(\zeta_{\sigma(i)}) - \mu(\zeta_{\sigma(i+1)})] f(o_{\sigma(i)}) \tag{4}$$

where permutation $\sigma\colon Q \to Q$, being $Q = \{1, 2, \ldots, q\}$, verifies $f(o_{\sigma(1)}) \leq f(o_{\sigma(2)}) \leq \cdots \leq f(o_{\sigma(q)})$, $\zeta_{\sigma(i)} = \{o_{\sigma(i)}, o_{\sigma(i+1)}, \cdots, o_{\sigma(q)}\}$ and $\zeta_{\sigma(q+1)} = \emptyset$.

If the q elements in $\mathbb{O}$ interact only in pairs, then defining a fuzzy measure μ would require $\frac{q(q-1)}{2}$ pairs of interaction coefficients in addition to the q individual coefficients, while its corresponding Möbius transform would satisfy the below set of properties [67], [68]:

$$\begin{cases} M(\emptyset) = 0, \\ M(\{o_i\}) \geq 0, \forall o_i \in \mathbb{O}, \\ \sum_{i\in Q} M(\{o_i\}) + \sum_{i,j\in Q} M(\{o_i, o_j\}) = 1, \\ M(\{o_i\}) + \sum_{j\in U} M(\{o_i, o_j\}) \geq 0, \forall U \subseteq Q\backslash\{i\}. \end{cases} \tag{5}$$

The 2-additive CI_μ for Möbius transform (5) is [66]:

$$CI_\mu(f(o_1), f(o_2), \cdots, f(o_q)) = \sum_{i\in Q} M(\{o_i\}) f(o_i) + \sum_{i,j\in Q} M(\{o_i, o_j\}) min\{f(o_i), f(o_j)\} \tag{6}$$

When there is no interaction between the elements in $\mathbb{O}$, i.e. $M(\{o_i, o_j\}) = 0$, $\forall i, j$, the 2-additive CI_μ becomes the weighted average $\sum_{i\in Q} M(\{o_i\}) f(o_i)$.

The minimum cost consensus model was first proposed by Ben-Arieh et al. [47], while Dong et al. [69] established the minimum adjustment consensus model, which is described as follows.

Assume that $\{E_1, E_2, \cdots, E_q\}$, $\{f(o_1), f(o_2), \cdots, f(o_q)\}$ and $\{\bar{f}(o_1), \bar{f}(o_2), \cdots, \bar{f}(o_q)\}$ denote experts set, experts' initial opinions set and experts' revised opinions set, respectively. The consensus threshold is set to ε. Thus, the basic minimum adjustment consensus model is described:

$$\textbf{Min} \quad \sum_{i=1}^{q} |f(o_i) - \bar{f}(o_i)|$$
$$s.t. \begin{cases} |\bar{f}(o_i) - \bar{f}^c| \leq \varepsilon, (i = 1, 2, \cdots, q) \\ \bar{f}^c = F(\bar{f}(o_1), \bar{f}(o_2), \cdots, \bar{f}(o_q)) \end{cases}$$

where $\bar{f}^c$ is a revised group opinion obtained through utilizing the aggregation operator $F(\bar{f}(o_1), \bar{f}(o_2), \cdots, \bar{f}(o_q))$. This basic minimum adjustment consensus model aims to minimize group adjustment when all experts reach an acceptable consensus threshold ε.

III. Consensus reaching framework with incentive behavior in dynamic trust relationships

This study proposes an incentive behaviour mechanism to promote consensus for dynamic trust social network group decision making (SN-GDM). The corresponding framework consists of the following stages: (A) Expert's comprehensive weight construction; (B) Feedback element identification; (C) Incentive behaviour mechanism; (D) Incentive driven minimum adjustment consensus model; (E) Selection process.

A. Experts' comprehensive weight construction

Recently, Ma et al. [29] proposed a reputation and interaction relationship based optimization methodology to derive experts' comprehensive weights. However, it is assumed that social trust relationship is static in nature. Subsequently, Xing et al. [30] proposed a maximum entropy approach to model dynamic trust relationship driven by experts' conflict degrees. However, in SN-GDM processes experts are more willing to trust the experts with similar opinions [26]. Hence, inspired by the above literature and to better reflect the trust interaction relationships between experts and effectively promote consensus reaching, this section proposes a methodology to derive experts' comprehensive weights based on dynamic social trust relationship and opinion similarity trust relationship.

It is assumed that a set of experts E are to select the best alternative from a set $A = \{A_1, A_2, \cdots, A_m\}$ with respect to a set of criteria $\vartheta = \{\vartheta_1, \vartheta_2, \cdots, \vartheta_n\}$ using their opinions represented by a set of 2-tuple linguistic preference relations/matrices $\{L^h = (l_{ij}^h)_{m\times n} = ((s_{ij}^h, \omega_{ij}^h))_{m\times n}; h = 1, \cdots, q\}$. The similarity degree score matrix, $SD = (SD_{hk})$, between pairs of experts' preferences is computed:

$$SD_{hk} = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(1 - d\left(l_{ij}^h, l_{ij}^k\right)\right) \tag{7}$$

where $d\left(l_{ij}^h, l_{ij}^k\right) = \frac{\left|\Delta^{-1}\left(l_{ij}^h\right) - \Delta^{-1}\left(l_{ij}^k\right)\right|}{r}$, and $\Delta^{-1}\left(l_{ij}^h\right) = \Delta^{-1}\left(s_{ij}^h, \omega_{ij}^h\right)$ and $\Delta^{-1}\left(l_{ij}^k\right) = \Delta^{-1}\left(s_{ij}^k, \omega_{ij}^k\right)$.

In the process of building trust relationships, experts are more likely to trust familiar experts as per the below theory of contact hypothesis in [31]:

- Familiar experts are more likely to obtain information and have more opportunities to communicate opinions, attitudes, values and beliefs between them. This information can help experts better judge the trustworthiness of the other party, and then can help establish a stronger trust relationship between experts [32].
- Familiar experts are more likely to establish common ground such as common interests, experiences, circles of friends, etc. These commonalities can enhance their sense of closeness and trust with each other, which is beneficial to get understanding and support. Therefore, experts are more likely to trust people with things in common. For example, links and comments in Social Network Wechat' moments lead to an increase in the trust relationship.

similar opinions [26], [32]. Therefore, experts are more likely to trust experts with

- Familiar experts are more likely to gain social recognition in social networks by experts they are familiar with. This indicates that when an expert is socially recognized that expert becomes more familiar with the friends of familiar experts, and is more likely to trust the friends of familiar experts.

Therefore, this article assumes that experts are more likely to trust familiar experts. In turn, to facilitate consensus reaching, an expert modifies his/her opinions based on the opinions of the experts he/she trusts [32], [33]. Based on this, the following undirected trust relationship model based on the social trust relationship and the opinion similarity trust relationship is constructed.

Let $T = (\xi_{hk})_{q\times q} = ((t_{hk}, d_{hk}))_{q\times q}$ be the assumed undirected trust sociomatrix between between experts E, with corresponding trust score between experts $TS = (TS_{hk})_{q\times q}$. Using $\theta \in [0,1]$ as a predetermined parameter to control the importance of experts' social trust relationship and opinion similarity degree, the undirected social trust-opinion similarity degree score (SOT) matrix, $SOTM = (SOT_{hk})_{q\times q}$, between experts is defined as

$$SOT_{hk} = \theta TS_{hk} + (1-\theta)SD_{hk}, \tag{8}$$

with the following incentive process based on the trust relationship: if the opinions between experts E_h and E_k are modified to become closer, then their similarity degree, SD_{hk} at the next consensus iteration will be higher than their current consensus iteration similarity degree since their opinions will be closer. This has an effect on the experts' trust relationship: since their opinions are more similar, the experts become more familiar with each other, and their trust relationship SOT_{hk} will be updated as per Eq.(8) at the next consensus round resulting in a strengthened/increased trust relationship between them. Thus, the trust relationship between experts in this process leads dynamic.

The normalized undirected social trust-opinion similarity degree score matrix, $NSOTM = (NSOT_{hk})_{q\times q}$, between experts is

$$NSOT_{hk} = \frac{SOT_{hk}}{\sum\limits_{\substack{h,k=1\\h\neq k}}^{q} SOT_{hk}}. \tag{9}$$

As per [29], [30], $NSOT_{hk} = NSOT_{kh}\,(h \neq k, h, k = 1, 2, \cdots, q)$ is the interaction weight between experts E_h and E_k, i.e. $M(\{E_h, E_k\}) = NSOT_{hk}$, while $\sum_{k=1,k\neq h}^{q} NSOT_{hk}$ is the interaction weight of expert E_h. And interaction weight $\sum_{k=1,k\neq h}^{q} NSOT_{hk}$ is not only used to reflect the trust interaction relationship between expert E_k and other remaining experts in the group, but also to represent the part that preference interacts with other remaining experts in the aggregated group preference. Thus, Eq. (5) allows to propose the following method to derive expert's comprehensive weights:

***Definition* 1.** *Expert's comprehensive weights, which include: individual weights, $\{M(\{E_h\}) \geq 0; h = 1, \cdots, q\}$, and interaction weights, $M(\{E_h, E_k\}) = NSOT_{hk}(h \neq k, h, k = 1, 2, \cdots, q)$, in an undirected trust social network with preference similarity, verify the conditions*

$$M(\{E_h\}) + \sum_{\substack{k=1\\k\neq h}}^{q} NSOT_{hk} \geq 0, \tag{10}$$

$$\sum_{h\in Q} M(\{E_h\}) + \sum_{h,k\in Q} NSOT_{hk} = 1. \tag{11}$$

The individual weights $\{M(\{E_h\}) \geq 0; h = 1, \cdots, q\}$, which denote the individual experts' own reputation, are unknown decision variables and are determined through the incentive-driven minimum adjustment consensus model below (see Model (19)). The interaction weights $M(\{E_h, E_k\}) = NSOT_{hk}(h \neq k, h, k = 1, 2, \cdots, q)$, which represent the trust interaction relationship between pairs of experts E_h and E_k, are known and obtained by Eq. (8) and (9) above.

The group 2-tuple linguistic preference matrix $L^c = (l^c_{ij})_{m\times n}$, computed using expert's comprehensive weights, is derived with the corresponding 2-additive CI_μ as per Eq. (6):

$$\begin{aligned} l^c_{ij} =& \Delta\left(\sum_{h\in Q} M(\{E_h\})\Delta^{-1}\left(l^h_{ij}\right)\right. \\ &\left. + \sum_{h,k\in Q} NSOT_{hk} \min\left\{\Delta^{-1}\left(l^h_{ij}\right), \Delta^{-1}(l^k_{ij})\right\}\right) \end{aligned} \tag{12}$$

Notice that this proposal extends the classical framework with no social interaction at all between experts, $NSOT_{hk} = 0\,\forall h, k$. Indeed, in this case, the group preference becomes Eq. (1): $l^c_{ij} = \Delta\left(\sum_{h=1}^{q} M(\{E_h\})\Delta^{-1}\left(l^h_{ij}\right)\right)$.

B. Feedback element identification

Consensus level is an effective tool to identify inconsistent experts in GDM [30]. Consensus levels of experts are measured at the three different hierarchical levels of the preference matrix [30]:

$$Elements: \qquad ACDE^h_{ij} = \frac{1}{q-1}\sum_{\substack{k=1\\k\neq h}}^{q}\left(1 - d\left(l^h_{ij}, l^k_{ij}\right)\right)$$

$$Alternatives: \quad ACDA^h_i = \frac{1}{n}\sum_{j=1}^{n} ACDE^h_{ij}$$

$$Matrix: \qquad ACDPM^h = \frac{1}{m}\sum_{i=1}^{m} ACDA^h_i$$

A consensus threshold is set to ε $(\varepsilon \geq 0.5)$ [60] to identify the following hierarchical feedback sets of inconsistent experts, alternatives and elements [30]:

$$IES = \left\{h | ACDPM^h < \varepsilon\right\}$$
$$IEAS = \left\{(h,i) | h \in IES \wedge ACDA^h_i < \varepsilon\right\}$$
$$IEES = \left\{(h,i,j) | (h,i) \in IEAS \wedge ACDE^h_{ij} < \varepsilon\right\}$$

It deals with the incentive behavior among all experts to revise their preferences. Thus, the set of feedback elements that require incentive behavior and preference revision among all experts is:

$$FES = \{(i,j) | \exists h: (h,i,j) \in IEES\} \tag{13}$$

C. Incentive mechanism in dynamic trust relationship

In traditional SN-GDM, inconsistent experts are requested to adjust their preference; if they don't adjust, the moderator penalizes them [26]. However, this is not reasonable without an effective incentive mechanism to motivate an expert to adjust preferences. Zhang et al. [70] proposed a dynamic dual-incentive evaluation mechanism to select the best alternative to adjust. Inspired by this, the below pair of dual-incentive control values are defined.

Recall that the ordering of any two 2-tuple linguistic preferences corresponds to the ordering of their respective symbolic translations obtained using Δ^{-1}. Thus, for each $(i,j) \in FES$, the arithmetic mean and median and of the 2-tuple linguistic preferences of the group of experts are computed, $Aver_{ij}$ and Med_{ij}, to define the following dual-incentive control values:

$$g_{ij}^{sup} = \max\{Aver_{ij}, Med_{ij}\} \tag{14}$$

$$g_{ij}^{inf} = \min\{Aver_{ij}, Med_{ij}\}. \tag{15}$$

The dual-incentive control values are the supremum and infimum incentive quantities in the proposed incentive process between experts.

Definition 2. *For $(i,j) \in FES$. The supremum and infimum incentive quantities in the incentive behavior process between any expert E_h and the other experts of the group driven by dynamic trust relationship are*

$$I_{h,ij}^{sup} = \begin{cases} \eta^{sup}\left(\Delta^{-1}\left(l_{ij}^{h}\right) - \Delta^{-1}\left(l_{ij}^{c}\right)\right) & \text{if } \Delta^{-1}\left(l_{ij}^{h}\right) > g_{ij}^{sup} \\ 0 & \text{otherwise.} \end{cases} \tag{16}$$

$$I_{h,ij}^{inf} = \begin{cases} \eta^{inf}\left(\Delta^{-1}\left(l_{ij}^{c}\right) - \Delta^{-1}\left(l_{ij}^{h}\right)\right) & \text{if } \Delta^{-1}\left(l_{ij}^{h}\right) < g_{ij}^{inf} \\ 0 & \text{otherwise.} \end{cases} \tag{17}$$

where $\eta^{sup}, \eta^{inf} \in [0,1]$ are the supremum and infimum incentive factors, respectively.

After obtaining the supremum and infimum incentive quantities of expert E_h $(h = 1,2,\cdots,q)$ by Eqs.(16)-(17), the feedback/revised recommendation opinion for expert E_h regarding $(i,j) \in FES$ would be:

$$\Delta^{-1}\left(\bar{l}_{ij}^{h}\right) = \Delta^{-1}\left(l_{ij}^{h}\right) - I_{h,ij}^{sup} + I_{h,ij}^{inf}. \tag{18}$$

Since $I_{h,ij}^{sup} I_{h,ij}^{inf} = 0,\ (h = 1,2,\cdots,q)$, Eq.(18) above results in the following three cases:

(a) If $\Delta^{-1}\left(l_{ij}^{h}\right) < g_{ij}^{inf}$ for $(i,j) \in FES$, then E_h would receive a feedback/revised opinion:

$$\Delta^{-1}\left(\bar{l}_{ij}^{h}\right) = \left(1 - \eta^{inf}\right)\Delta^{-1}\left(l_{ij}^{h}\right) + \eta^{inf}\Delta^{-1}\left(l_{ij}^{c}\right).$$

(b) If $g_{ij}^{inf} \le \Delta^{-1}\left(l_{ij}^{h}\right) \le g_{ij}^{sup}$ for $(i,j) \in FES$, then expert E_h is not provided with feedback/revised opinion.

(c) If $\Delta^{-1}\left(l_{ij}^{h}\right) > g_{ij}^{sup}$ for $(i,j) \in FES$, then E_h would receive a feedback/revised opinion:

$$\Delta^{-1}\left(\bar{l}_{ij}^{h}\right) = \left(1 - \eta^{sup}\right)\Delta^{-1}(l_{ij}^{h}) + \eta^{sup}\Delta^{-1}\left(l_{ij}^{c}\right).$$

D. Incentive driven minimum adjustment consensus model

Dong et al. [71] proposed the minimum adjusted consensus model to minimize the distance between the original and modified preference of experts while ensuring that the consensus levels of all decision makers are acceptable. Dong et al. [71] proposed a minimum adjustment consensus model to supervise experts' weights, so as to improve the consensus level of experts. However, the experts they consider are independent of each other, and they do not consider the psychological behaviors between experts, such as trust relationships and incentive behaviors. That is to say, on the one hand, whether the expert is influenced by the experts he trusts when adjusting his preferences, and on the other hand, whether experts are willing to adjust their preferences is ignored. This is addressed herein with Model (19) by considering the above proposed incentive mechanism to generate feedback/revised recommendation opinion for experts, with the experts's individual weights $\{M(\{E_h\}); h = 1,2,\cdots,q\}$ being its decision variables and the interaction weight between experts obtained using their previously defined normalized undirected social trust-opinion similarity degrees. The nonlinear objective function and constraint (19.1) of Model (19) can be linearized, by introducing corresponding positive real valued variables $\{a_{ij}, b_{ij}; i = 1,2,\ldots,m; j = 1,2,\ldots,n\}$, to become the equivalent linear optimization Model (20)

In model (20), the parameter ε represents an acceptable threshold level of consensus, and it is fixed by the group of experts. The larger the value of ε, the smaller the feasible region of model (20). In addition, for a given threshold value ε, small values of the incentive factors η^{sup} and η^{inf} may imply that not every individual expert's consensus levels will verify constraint (20.2), and the model feasible region will be the empty set, and no optimal solution will exist. Thus, the following supplement incentive driven minimum adjustment consensus model (21), which is driven by the dynamic trust relationship between experts, is established to analyze the solution of model (20).

E. Selection process

a) Group preference acquisition: When the consensus levels of all experts reach the consensus threshold, the final group preferences, from which the best alternative is selected, are derived by aggregating the final modified preference of all individual experts derived using the corresponding 2-additive CI_μ of the optimal solution of Model (22) using the final updated normalized undirected social-trust opinion similarity degrees. As before, the introduction of auxiliary real valued variables $\{c_{ij}; i = 1,2,\ldots,m; j = 1,2,\ldots,n\}$ permits to transform model (22) equivalently into the linear model (23).

b) Alternative score: Assuming that the criteria weights are $\{\lambda_j; j = 1,2,\cdots,n\}$, each alternative can associated the final score:

$$\bar{l}_{i}^{c} = \Delta\left(\sum_{j=1}^{n}\lambda_j\Delta^{-1}(\bar{l}_{ij}^{c})\right) \tag{24}$$

$$\textbf{Min} \quad \sum_{h=1}^{q} \sum_{(i,j)\in FES} \left|\Delta^{-1}\left(\bar{l}_{ij}^{h}\right) - \Delta^{-1}\left(l_{ij}^{h}\right)\right| \tag{19}$$

$$s.t. \begin{cases} \overline{ACDPM}^{h} = \dfrac{1}{mn(q-1)} \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{\substack{k=1\\h\neq k}}^{q} \left(1 - d\left(\bar{l}_{ij}^{h}, \bar{l}_{ij}^{k}\right)\right) \geq \varepsilon & (19.1)\\ \Delta^{-1}\left(\bar{l}_{ij}^{h}\right) = \begin{cases} \left(1-\eta^{inf}\right)\Delta^{-1}\left(l_{ij}^{h}\right) + \eta^{inf}\Delta^{-1}\left(l_{ij}^{c}\right) & \text{if } (i,j)\in FES \wedge \Delta^{-1}\left(l_{ij}^{h}\right) < g_{ij}^{inf} \\ \left(1-\eta^{sup}\right)\Delta^{-1}(l_{ij}^{h}) + \eta^{sup}\Delta^{-1}\left(l_{ij}^{c}\right) & \text{if } (i,j)\in FES \wedge \Delta^{-1}\left(l_{ij}^{h}\right) > g_{ij}^{sup} \\ \Delta^{-1}(l_{ij}^{h}) & \text{otherwise.} \end{cases} & (19.2)\\ g_{ij}^{sup} = \max\left\{Aver_{ij}, Med_{ij}\right\}; \quad g_{ij}^{inf} = \min\left\{Aver_{ij}, Med_{ij}\right\} & (19.3)\\ \Delta^{-1}\left(l_{ij}^{c}\right) = \displaystyle\sum_{h=1}^{q} M(\{E_h\})\Delta^{-1}\left(l_{ij}^{h}\right) + \sum_{h,k\in Q} NSOT_{hk}\min\{\Delta^{-1}\left(l_{ij}^{h}\right), \Delta^{-1}\left(l_{ij}^{k}\right)\} & (19.4)\\ M(\{E_h\}) \geq 0 & (19.5)\\ M(\{E_h\}) + \displaystyle\sum_{k=1,k\neq h}^{q} NSOT_{hk} \geq 0 & (19.6)\\ \displaystyle\sum_{h=1}^{q} M(\{E_h\}) + \sum_{h,k\in Q} NSOT_{hk} = 1 & (19.7)\\ NSOT_{hk} = NSOT_{kh} = \dfrac{\theta TS_{hk} + (1-\theta)SD_{hk}}{\displaystyle\sum_{\substack{h,k=1\\h\neq k}}^{q}\left(\theta TS_{hk} + (1-\theta)SD_{hk}\right)} & (19.8)\\ \theta, \eta^{sup}, \eta^{inf} \in [0,1], \varepsilon \in [0.5,1], E_h \in E, h,k \in Q & (19.9) \end{cases}$$

$$\textbf{Min} \quad \sum_{h=1}^{q} \sum_{(i,j)\in FES} a_{ij}^{h} \tag{20}$$

$$s.t. \begin{cases} -a_{ij}^{h} \leq \Delta^{-1}\left(\bar{l}_{ij}^{h}\right) - \Delta^{-1}\left(l_{ij}^{h}\right) \leq a_{ij}^{h}, \quad \forall (i,j)\in FES & (20.1)\\ \overline{ACDPM}^{h} = \dfrac{1}{mn(q-1)} \displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{\substack{k=1\\h\neq k}}^{q} \left(1 - \dfrac{b_{ij}^{hk}}{r}\right) \geq \varepsilon & (20.2)\\ -b_{ij}^{hk} \leq \Delta^{-1}\left(\bar{l}_{ij}^{h}\right) - \Delta^{-1}\left(\bar{l}_{ij}^{k}\right) \leq b_{ij}^{hk}, \quad \forall i,j & (20.3)\\ \Delta^{-1}\left(\bar{l}_{ij}^{h}\right) = \begin{cases} \left(1-\eta^{inf}\right)\Delta^{-1}\left(l_{ij}^{h}\right) + \eta^{inf}\Delta^{-1}\left(l_{ij}^{c}\right) & \text{if } (i,j)\in FES \wedge \Delta^{-1}\left(l_{ij}^{h}\right) < g_{ij}^{inf} \\ \left(1-\eta^{sup}\right)\Delta^{-1}(l_{ij}^{h}) + \eta^{sup}\Delta^{-1}\left(l_{ij}^{c}\right) & \text{if } (i,j)\in FES \wedge \Delta^{-1}\left(l_{ij}^{h}\right) > g_{ij}^{sup} \\ \Delta^{-1}(l_{ij}^{h}) & \text{otherwise.} \end{cases} & (20.4)\\ g_{ij}^{sup} = \max\left\{Aver_{ij}, Med_{ij}\right\}; \quad g_{ij}^{inf} = \min\left\{Aver_{ij}, Med_{ij}\right\} & (20.5)\\ \Delta^{-1}\left(l_{ij}^{c}\right) = \displaystyle\sum_{h=1}^{q} M(\{E_h\})\Delta^{-1}\left(l_{ij}^{h}\right) + \sum_{h,k\in Q} NSOT_{hk}\min\{\Delta^{-1}\left(l_{ij}^{h}\right), \Delta^{-1}\left(l_{ij}^{k}\right)\} & (20.6)\\ M(\{E_h\}) \geq 0 & (20.7)\\ M(\{E_h\}) + \displaystyle\sum_{k=1,k\neq h}^{q} NSOT_{hk} \geq 0 & (20.8)\\ \displaystyle\sum_{h=1}^{q} M(\{E_h\}) + \sum_{h,k\in Q} NSOT_{hk} = 1 & (20.9)\\ NSOT_{hk} = NSOT_{kh} = \dfrac{\theta TS_{hk} + (1-\theta)SD_{hk}}{\displaystyle\sum_{\substack{h,k=1\\h\neq k}}^{q}\left(\theta TS_{hk} + (1-\theta)SD_{hk}\right)} & (20.10)\\ \theta, \eta^{sup}, \eta^{inf} \in [0,1], \varepsilon \in [0.5,1], E_h \in E, h,k \in Q & (20.11) \end{cases}$$

c) Ranking of alternatives: The alternatives final ranking is produced using their values $\{\Delta^{-1}(\bar{l}_{i}^{C}); i = 1, 2, \cdots, n\}$, decreasing with the best alternative being the one with highest value.

$$\mathbf{Min} \quad \sum_{h=1}^{q} \sum_{(i,j)\in FES} a_{ij}^{h} + \frac{1}{mnq(q-1)} \sum_{h=1}^{q} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1,k\neq h}^{q} \frac{b_{ij}^{hk}}{r} \tag{21}$$

$$s.t. \begin{cases} -a_{ij}^{h} \leq \Delta^{-1}\left(\bar{l}_{ij}^{h}\right) - \Delta^{-1}\left(l_{ij}^{h}\right) \leq a_{ij}^{h}, \quad \forall (i,j)\in FES & (21.1) \\ -b_{ij}^{hk} \leq \Delta^{-1}\left(\bar{l}_{ij}^{h}\right) - \Delta^{-1}\left(\bar{l}_{ij}^{k}\right) \leq b_{ij}^{hk}, \quad \forall i,j & (21.2) \\ \Delta^{-1}\left(\bar{l}_{ij}^{h}\right) = \begin{cases} \left(1-\eta^{inf}\right)\Delta^{-1}\left(l_{ij}^{h}\right) + \eta^{inf}\Delta^{-1}\left(l_{ij}^{c}\right) & \text{if } (i,j)\in FES \wedge \Delta^{-1}\left(l_{ij}^{h}\right) < g_{ij}^{inf} \\ \left(1-\eta^{sup}\right)\Delta^{-1}(l_{ij}^{h}) + \eta^{sup}\Delta^{-1}\left(l_{ij}^{c}\right) & \text{if } (i,j)\in FES \wedge \Delta^{-1}\left(l_{ij}^{h}\right) > g_{ij}^{sup} \\ \Delta^{-1}(l_{ij}^{h}) & \text{otherwise.} \end{cases} & (21.3) \\ g_{ij}^{sup} = \max\{Aver_{ij}, Med_{ij}\}; \quad g_{ij}^{inf} = \min\{Aver_{ij}, Med_{ij}\} & (21.4) \\ \Delta^{-1}\left(l_{ij}^{c}\right) = \sum_{h=1}^{q} M(\{E_h\})\Delta^{-1}\left(l_{ij}^{h}\right) + \sum_{h,k\in Q} NSOT_{hk} \min\{\Delta^{-1}\left(l_{ij}^{h}\right), \Delta^{-1}\left(l_{ij}^{k}\right)\} & (21.5) \\ M(\{E_h\}) \geq 0 & (21.6) \\ M(\{E_h\}) + \sum_{k=1,k\neq h}^{q} NSOT_{hk} \geq 0 & (21.7) \\ \sum_{h=1}^{q} M(\{E_h\}) + \sum_{h,k\in Q} NSOT_{hk} = 1 & (21.8) \\ NSOT_{hk} = NSOT_{kh} = \dfrac{\theta TS_{hk} + (1-\theta)SD_{hk}}{\sum\limits_{\substack{h,k=1 \\ h\neq k}}^{q} (\theta TS_{hk} + (1-\theta)SD_{hk})} & (21.9) \\ \theta, \eta^{sup}, \eta^{inf} \in [0,1], \varepsilon \in [0.5,1], E_h \in E, h,k \in Q & (21.10) \end{cases}$$

IV. Illustrative Example and Discussion

The in vitro diagnostic products (IVD) industry is an important branch of medical device manufacturing industry. The demand for diagnostic tests caused by the COVID-19 pandemic has fueled the rapid growth of the IVD industry. A medical equipment manufacturing company in Shanghai, company B, is using new energy work system and planning to produce a batch of COVID-19 antigen detection kits. Company B needs to purchase a batch of NC (nitrocellulosefilter) membrane from three identified potential sustainable suppliers $A = (A_1, A_2, A_3)$, using expert members $E = (E_1, E_2, E_3, E_4, E_5)$ from different departments of the company (production; research and development department; raw material procurement; sales; and finance) to negotiate and communicate according to the internal and external environment of company, and detailed sustainable evaluation criteria are considered as the following three dimensions: (1) *Economic dimension:* the quality and safety of NC membrane (ϑ_1); the price of NC membrane (ϑ_2); (2) *Social dimension:* the safety and work attitude of employees (ϑ_3); (3) *Environmental dimension:* green technology (ϑ_4).

Since experts come from different departments with different knowledge backgrounds, they may have different views and cognition on the evaluation of suppliers, which may lead to certain conflicts and hinders the consensus. In addition, it is assumed the existence of a social trust relationship between them (Fig. 1), and that incentive behavior in the process of negotiation and communication is acceptable. Hence, it is crucial for Company B to reach a consensus and select the best sustainable supplier through motivated communication among all experts driven by their dynamic trust relationship.

Assume that preferences of expert on about (A_i, ϑ_j) $(i = 1,2,3; j = 1,2,3,4)$ using the linguistic term set $\mathbb{S} = \{s_0$ =extremely poor, s_1 =very poor, s_2 =poor, s_3 =fair, s_4 =good, s_5 =very good, s_6 =extremely good$\}$ [11] are L^1, L^2, L^3, L^4 and L^5 given below.

$$L^1 = \begin{pmatrix} (s_2,0) & (s_1,0) & (s_3,0) & (s_5,0) \\ (s_4,0) & (s_3,0) & (s_2,0) & (s_6,0) \\ (s_2,0) & (s_4,0) & (s_2,0) & (s_4,0) \end{pmatrix},$$
$$L^2 = \begin{pmatrix} (s_4,0) & (s_1,0) & (s_2,0) & (s_6,0) \\ (s_6,0) & (s_2,0) & (s_3,0) & (s_6,0) \\ (s_2,0) & (s_4,0) & (s_1,0) & (s_4,0) \end{pmatrix},$$
$$L^3 = \begin{pmatrix} (s_3,0) & (s_1,0) & (s_3,0) & (s_6,0) \\ (s_3,0) & (s_6,0) & (s_3,0) & (s_1,0) \\ (s_1,0) & (s_4,0) & (s_5,0) & (s_1,0) \end{pmatrix},$$
$$L^4 = \begin{pmatrix} (s_5,0) & (s_1,0) & (s_2,0) & (s_6,0) \\ (s_5,0) & (s_2,0) & (s_3,0) & (s_6,0) \\ (s_3,0) & (s_4,0) & (s_2,0) & (s_4,0) \end{pmatrix},$$
$$L^5 = \begin{pmatrix} (s_4,0) & (s_1,0) & (s_6,0) & (s_3,0) \\ (s_0,0) & (s_4,0) & (s_3,0) & (s_5,0) \\ (s_1,0) & (s_4,0) & (s_2,0) & (s_6,0) \end{pmatrix}.$$

The sociometric of the undirected graph of the trust relationship between expert members is assumed given as T_G. Let aggregation parameter $\theta = 0.4$, the consensus threshold $\varepsilon = 0.8$, $\eta^{sup} = 0.5$ and $\eta^{inf} = 0.2$.

$$T_G = \begin{pmatrix} - & [0.7,0.9] & [0.5,0.9] & [0.4,0.6] & - \\ [0.7,0.9] & - & - & - & [0.8,0.5] \\ [0.5,0.9] & - & - & [0.9,0.7] & [0.2,0.8] \\ [0.4,0.6] & - & [0.9,0.7] & - & [0.6,0.9] \\ - & [0.8,0.5] & [0.2,0.8] & [0.6,0.9] & - \end{pmatrix}$$

$$\textbf{Min} \quad \frac{1}{mnq}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{h=1}^{q}\frac{|\Delta^{-1}\left(\bar{l}_{ij}^{h}\right)-\Delta^{-1}(\bar{l}_{ij}^{c})|}{r} \tag{22}$$

$$s.t.\begin{cases}
\Delta^{-1}(\bar{l}_{ij}^{c})=\sum_{h=1}^{q}\overline{M(\{E_h\})}\Delta^{-1}\left(\bar{l}_{ij}^{h}\right)+\sum_{h,k\in Q}\overline{NSOT}_{hk}\min\{\Delta^{-1}\left(\bar{l}_{ij}^{h}\right),\Delta^{-1}\left(\bar{l}_{ij}^{k}\right)\} & (22.1)\\
\overline{M(\{E_h\})}+\sum_{k=1,k\neq h}^{q}\overline{NSOT}_{hk}\geq 0 & (22.2)\\
\sum_{h=1}^{q}\overline{M(\{E_h\})}+\sum_{h,k\in Q}\overline{NSOT}_{hk}=1 & (22.3)\\
\overline{M(\{E_h\})}\geq 0 & (22.4)\\
\overline{NSOT}_{hk}=\overline{NSOT}_{kh}=\frac{\theta\overline{TS}_{hk}+(1-\theta)\overline{SD}_{hk}}{\sum_{\substack{h,k=1\\h\neq k}}^{q}\left(\theta\overline{TS}_{hk}+(1-\theta)\overline{SD}_{hk}\right)} & (22.5)\\
\theta\in[0,1],E_h\in E,h\in Q,\ i=1,2,\cdots,m,j=1,2,\cdots,n. & (22.6)
\end{cases}$$

$$\textbf{Min} \quad \frac{1}{mnqr}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{h=1}^{q}c_{ij}^{h} \tag{23}$$

$$s.t.\begin{cases}
-c_{ij}^{h}\leq\Delta^{-1}\left(\bar{l}_{ij}^{h}\right)-\Delta^{-1}\left(\bar{l}_{ij}^{c}\right)\leq c_{ij}^{h} & (23.1)\\
\Delta^{-1}(\bar{l}_{ij}^{c})=\sum_{h=1}^{q}\overline{M(\{E_h\})}\Delta^{-1}\left(\bar{l}_{ij}^{h}\right)+\sum_{h,k\in Q}\overline{NSOT}_{hk}\min\{\Delta^{-1}\left(\bar{l}_{ij}^{h}\right),\Delta^{-1}\left(\bar{l}_{ij}^{k}\right)\} & (22.2)\\
\overline{M(\{E_h\})}+\sum_{k=1,k\neq h}^{q}\overline{NSOT}_{hk}\geq 0 & (23.3)\\
\sum_{h=1}^{q}\overline{M(\{E_h\})}+\sum_{h,k\in Q}\overline{NSOT}_{hk}=1 & (23.4)\\
\overline{M(\{E_h\})}\geq 0 & (23.5)\\
\overline{NSOT}_{hk}=\overline{NSOT}_{kh}=\frac{\theta\overline{TS}_{hk}+(1-\theta)\overline{SD}_{hk}}{\sum_{\substack{h,k=1\\h\neq k}}^{q}\left(\theta\overline{TS}_{hk}+(1-\theta)\overline{SD}_{hk}\right)} & (23.6)\\
\theta\in[0,1],E_h\in E,h\in Q,\ i=1,2,\cdots,m,j=1,2,\cdots,n. & (23.7)
\end{cases}$$

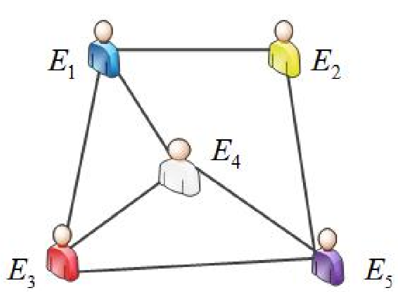


Fig. 1. Trust relationships between expert members

A. Selection of the suitable sustainable supplier

1) ***Initial experts' comprehensive weights and consensus detection process.*** Applying (7)–(9), the expert's undirected social trust-opinion similarity degree score matrix $SOTM_0$ and the corresponding normalized score matrix $NSOTM_0$ are computed:

$$SOTM_0=\begin{pmatrix} - & 0.685 & 0.562 & 0.685 & 0.458\\ 0.685 & - & 0.417 & 0.567 & 0.693\\ 0.562 & 0.417 & - & 0.657 & 0.480\\ 0.685 & 0.567 & 0.657 & - & 0.573\\ 0.458 & 0.693 & 0.480 & 0.573 & - \end{pmatrix},$$

$$NSOTM_0=\begin{pmatrix} - & 0.059 & 0.049 & 0.059 & 0.040\\ 0.059 & - & 0.036 & 0.049 & 0.060\\ 0.049 & 0.036 & - & 0.057 & 0.042\\ 0.059 & 0.049 & 0.057 & - & 0.050\\ 0.040 & 0.060 & 0.042 & 0.050 & - \end{pmatrix}.$$

The consensus levels of all experts are: $ACDPM_0^1=0.8125$, $ACDPM_0^2=0.809$, $ACDPM_0^3=0.6979$, $ACDPM_0^4=0.8090$ and $ACDPM_0^5=0.7188$. Therefore, the inconsistent experts are E_3 and E_5.

2) ***The first incentive behavior-driven feedback mechanism.*** Applying (13), the set of feedback elements that require incentive behaviors among of experts is identified:

$$FES_0 = \{(1,3),(1,4),(2,1),(2,2),(2,4),(3,3),(3,4)\}.$$

Model (20) has no feasible solution and model (21) is solved instead, which results in the following initial experts' comprehensive weights: $M(\{E_1\}) = M(\{E_2\}) = M(\{E_4\}) = 0$, $M(\{E_3\}) = 0.4125$, $M(\{E_5\}) = 0.0875$. These values indicate that the weights of experts E_1, E_2 and E_4 are not affected by their own reputation but by the trust relationships from other experts, while the weights of experts E_3 and E_5 are affected by their own reputation and by the influence of social relationships from other experts.

3) ***Second experts' comprehensive weights and consensus detection process.*** After the first incentive behavior among all experts, their modified preference matrices are derived $L_1^1, L_1^2, L_1^3, L_1^4$ and L_1^5, while the updated social trust-opinions similarity degree score matrix between experts and the corresponding normalized score matrix would become $SOTM_1$ and $NSOTM_1$.

$$L_1^1 = \begin{pmatrix} (s_2,0) & (s_1,0) & (s_3,0) & (s_5,0.0001) \\ (s_4,0) & (s_3,0) & (s_2,0) & (s_6,0) \\ (s_2,0) & (s_4,0) & (s_2,0) & (s_4,0) \end{pmatrix},$$

$$L_1^2 = \begin{pmatrix} (s_4,0) & (s_1,0) & (s_2,0.1791) & (s_6,0) \\ (s_5,0.0277) & (s_2,0.4) & (s_3,0) & (s_6,0) \\ (s_2,0) & (s_4,0) & (s_1,0.4071) & (s_4,0) \end{pmatrix},$$

$$L_1^3 = \begin{pmatrix} (s_3,0) & (s_1,0) & (s_3,0) & (s_6,0) \\ (s_3,0.2111) & (s_5,0) & (s_3,0) & (s_1,0.3572) \\ (s_1,0) & (s_4,0) & (s_4,0.0177) & (s_1,0.2779) \end{pmatrix},$$

$$L_1^4 = \begin{pmatrix} (s_5,0) & (s_1,0) & (s_2,0.1791) & (s_6,0) \\ (s_4,0.5277) & (s_2,0.4) & (s_3,0) & (s_6,0) \\ (s_3,0) & (s_4,0) & (s_2,0) & (s_4,0) \end{pmatrix},$$

$$L_1^5 = \begin{pmatrix} (s_4,0) & (s_1,0) & (s_4,0.4478) & (s_3,0.4001) \\ (s_0,0.8111) & (s_4,0) & (s_3,0) & (s_5,0) \\ (s_1,0) & (s_4,0) & (s_2,0) & (s_4,0.1948) \end{pmatrix},$$

$$SOTM_1 = \begin{pmatrix} - & 0.815 & 0.690 & 0.808 & 0.680 \\ 0.815 & - & 0.623 & 0.801 & 0.765 \\ 0.690 & 0.623 & - & 0.712 & 0.650 \\ 0.808 & 0.801 & 0.712 & - & 0.710 \\ 0.680 & 0.765 & 0.650 & 0.710 & - \end{pmatrix},$$

$$NSOTM_1 = \begin{pmatrix} - & 0.056 & 0.048 & 0.056 & 0.047 \\ 0.056 & - & 0.043 & 0.055 & 0.053 \\ 0.048 & 0.043 & - & 0.049 & 0.045 \\ 0.056 & 0.055 & 0.049 & - & 0.049 \\ 0.047 & 0.053 & 0.045 & 0.049 & - \end{pmatrix}.$$

The consensus levels of experts are: $ACDPM_1^1 = 0.8486$, $ACDPM_1^2 = 0.8583$, $ACDPM_1^3 = 0.7620$, $ACDPM_1^4 = 0.8488$ and $ACDPM_1^5 = 0.8010$. Thus, the inconsistent expert is E_3.

4) ***The second incentive behavior-driven feedback mechanism.*** At this stage it is:

$$FES_1 = \{(2,1),(2,2),(2,4),(3,3),(3,4)\} \subset FES_0.$$

Model (20) has feasible solution: $M(\{E_1\}) = M(\{E_4\}) = M(\{E_5\}) = 0, M(\{E_2\}) = 0.1905$, $M(\{E_3\}) = 0.3095$.

5) ***Comprehensive weight construction and consensus detection process after the second incentive behavior among experts.*** After incentive behavior among all experts, their new modified preference matrices are derived $L_2^1, L_2^2, L_2^3, L_2^4$ and L_2^5, while the updated social trust-opinions similarity degree score matrix between experts and the corresponding normalized score matrix would become $SOTM_2$ and $NSOTM_2$.

$$L_2^1 = \begin{pmatrix} (s_2,0) & (s_1,0) & (s_3,0) & (s_5,0.0001) \\ (s_4,0) & (s_3,0) & (s_2,0) & (s_6,0) \\ (s_2,0) & (s_4,0) & (s_2,0) & (s_4,0) \end{pmatrix},$$

$$L_2^2 = \begin{pmatrix} (s_4,0) & (s_1,0) & (s_2,0.1791) & (s_6,0) \\ (s_4,0.1410) & (s_2,0.7200) & (s_3,0) & (s_6,0) \\ (s_2,0) & (s_4,0) & (s_1,0.6034) & (s_4,0) \end{pmatrix},$$

$$L_2^3 = \begin{pmatrix} (s_3,0) & (s_1,0) & (s_3,0) & (s_6,0) \\ (s_3,0.2197) & (s_4,0.5) & (s_3,0) & (s_1,0.7975) \\ (s_1,0) & (s_4,0) & (s_3,0.2032) & (s_1,0.5534) \end{pmatrix},$$

$$L_2^4 = \begin{pmatrix} (s_5,0) & (s_1,0) & (s_2,0.1791) & (s_6,0) \\ (s_3,0.891) & (s_2,0.72) & (s_3,0) & (s_6,0) \\ (s_3,0) & (s_4,0) & (s_2,0) & (s_4,0) \end{pmatrix},$$

$$L_2^5 = \begin{pmatrix} (s_4,0) & (s_1,0) & (s_4,0.4478) & (s_3,0.4001) \\ (s_1,0.2997) & (s_4,0) & (s_3,0) & (s_5,0) \\ (s_1,0) & (s_4,0) & (s_2,0) & (s_3,0.4252) \end{pmatrix}.$$

$$SOTM_2 = \begin{pmatrix} - & 0.879 & 0.758 & 0.863 & 0.769 \\ 0.879 & - & 0.735 & 0.898 & 0.806 \\ 0.758 & 0.735 & - & 0.759 & 0.746 \\ 0.863 & 0.898 & 0.759 & - & 0.773 \\ 0.769 & 0.806 & 0.746 & 0.773 & - \end{pmatrix},$$

$$NSOTM_2 = \begin{pmatrix} - & 0.055 & 0.047 & 0.054 & 0.048 \\ 0.055 & - & 0.046 & 0.056 & 0.050 \\ 0.047 & 0.046 & - & 0.047 & 0.047 \\ 0.054 & 0.056 & 0.047 & - & 0.048 \\ 0.048 & 0.050 & 0.047 & 0.048 & - \end{pmatrix}.$$

The dynamic changes of the trust relationships among experts are shown in Figure 2, where SOT_0 represents the initial undirected social trust-opinion similarity degrees score value obtained by exploiting Eq.(8). Similarly, SOT_1 and SOT_2 denote the updated undirected social trust relationship-opinion similarity degrees score values after the first and second incentive processes, respectively. It can be concluded that the undirected social trust-opinion similarity degrees values scores, SOT_{hk}, between E_h and E_k increase (become stronger) due to the fact that all experts modify their respective preferences through the incentive behavior, which leads to a reduction of conflicts between experts and an increase in familiarity (similarity degrees score/SD_{hk}) of experts with each other.

Since this state consensus levels of all experts are above the threshold value, $ACDPM_2^1 = 0.8635$, $ACDPM_2^2 = 0.8819$, $ACDPM_2^3 = 0.8029$, $ACDPM_2^4 = 0.8670$ and $ACDPM_2^5 = 0.9042$, there is no inconsistent expert and the selection process stage is activated.

6) ***Selection process.*** The solution of model (23) results in the individual weights: $M(\{E_1\}) = M(\{E_2\}) = M(\{E_3\}) = M(\{E_5\}) = 0$ and $M(\{E_4\}) = 0.5$, and final group preference matrix

$$\overline{L}^c = \begin{pmatrix} (s_3,0.9504) & (s_1,0) & (s_2,0.2960) & (s_5,0.3397) \\ (s_3,0.3002) & (s_3,0.4241) & (s_2,0.7953) & (s_5,0.0641) \\ (s_2,0.1652) & (s_4,0) & (s_1,0.9176) & (s_3,0.4562) \end{pmatrix}.$$

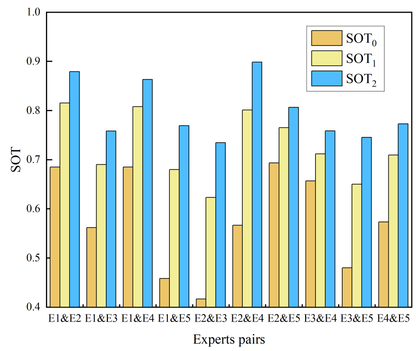


Fig. 2. The dynamic changes of SN trust relationships among experts

The criteria weights $\lambda_1 = 0.2, \lambda_2 = 0.25, \lambda_3 = 0.3, \lambda_4 = 0.25$ leads to the following total values of sustainable supplier $\bar{l}_1^c = (s_3, 0.0638), \bar{l}_2^c = (s_3, 0.6207), \bar{l}_3^c = (s_2, 0.8724)$. Since it has $\Delta^{-1}(\bar{l}_3^c) = 2.8724 < \Delta^{-1}(\bar{l}_1^c) = 3.0638 < \Delta^{-1}(\bar{l}_2^c) = 3.6207$, the sustainable supplier A_2 is selected.

B. Sensitivity analysis

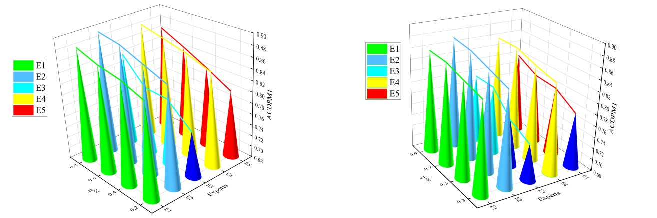
The proposed methodology promotes effectively both the trust relationships between experts and the consensus levels of experts. The change of value of the incentive factors η^{inf} and η^{sup} affects both the experts' consensus level values and the number of incentive behaviors among experts. The following two scenarios of change of value of incentive factors have been carried out for analysis:

S1: For $\eta^{sup} = 0.5$, the $ACDPM$ and sets of feedback element for different values of η^{inf} are obtained (Table I).

S2: For $\eta^{inf} = 0.2$, the $ACDPM$ and sets of feedback element for different values η^{sup} are obtained (Table II).

After the first incentive behaviour process, the $ACDPM_1$ evolution for the different incentive factor values are shown in Figure 3. Both Figure 3a for S1 and Figure 3b for S2 shows that individual experts' consensus level values increase with an increase of the corresponding variable incentive factor when the other incentive factor is fixed. However, some experts may not reach the consensus threshold when the variable incentive factor is small in value (highlights in blue color), which means a further activation of the incentive process with other experts in the group. In such cases, the set of feedback element is always smaller than the previous one, which becomes empty when all individual experts' consensus level values reach the threshold value (see Tables I- II). Thus, it can be concluded that then higher the incentive factor values are, the easier it is to promote the incentive behavior between experts and the more it promotes the consensus between experts, reducing simultaneously the number of rounds of the incentive process between experts.

Table III reports total evaluation values of each sustainable suppliers are computed after all individual experts reach consensus for different incentive factor values combinations.



(a) (b)

Fig. 3. The $ACDPM_1$ evolution process in (a) S1: different η^{inf} values with $\eta^{sup} = 0.5$; (b) S2: different η^{sup} values with $\eta^{inf} = 0.2$.

The robustness of proposed method is evidence by being sustainable supplier A_2 the one with largest evaluation value in all the considered cases.

TABLE III
TOTAL EVALUATION VALUES FOR SUSTAINABLE SUPPLIERS FOR DIFFERENT (η^{inf}, η^{sup}) COMBINATIONS.

(η^{inf}, η^{sup})	$(0.2, 0.5)$	$(0.4, 0.5)$	$(0.6, 0.5)$	$(0.8, 0.5)$	$(0.2, 0.3)$	$(0.2, 0.7)$	$(0.2, 0.9)$
A_1	3.0638	3.0174	3.0170	3.0192	3.0132	3.0177	3.0172
A_2	3.6207	3.4980	3.4979	3.4929	3.6229	3.4990	3.4985
A_3	2.8724	2.8368	2.8373	2.8368	2.8156	2.8364	2.8362

C. Comparison analysis with traditional feedback mechanism

The feedback mechanism without incentive behavior is compared with the proposed feedback mechanism with incentive behavior. The results of the proposed method with $\eta^{sup} = \eta^{inf} = 0.3$ are compared with the feedback mechanism without incentive behavior as per Table IV. The number of feedback rounds for all experts' consensus level to reach the threshold is lower in the proposed methodology (2 versus 3). Additionally, the total adjustment cost is also lower in the proposed methodology (11.2579 versus 12.5562). These differences are due to the fact that in the proposed methodology all experts participate in revising the identified feedback elements, which differs from the feedback methodology without incentive where only the inconsistent experts participate in revising the identified feedback elements.

D. Discussion

Based on the above Illustrative example analysis, Sensitivity analysis and Comparative analysis, it mainly explores the influence of incentive mechanism on consensus under dynamic trust relationship from three perspectives: dynamic trust relationship, incentive factor and incentive type.

1) From the perspective of dynamic trust relationship, the trust-driven incentive mechanism encourages experts to adjust their preferences instead of being forced to adjust them. In turn, this inter-expert incentive process makes the experts become more and more familiar with each other, (namely, the similarity degree SD_{hk} between E_h and E_k is increased). Furthermore, the trust relationship between experts becomes stronger. (That is, the trust score SOT_{hk} between E_h and E_k is increased (see Figure 2)). In summary, the dynamic trust relationship promotes

TABLE I
THE $ACDPM$ AND FEEDBACK ELEMENT IDENTIFICATION UNDER DIFFERENT η^{inf} WHEN $\eta^{sup} = 0.5$.

	Numbers	$ACDPM$	Feedback element identification
Before	NO	$ACDPM_0^1 = 0.8125, ACDPM_0^2 = 0.8090, ACDPM_0^3 = 0.6979, ACDPM_0^4 = 0.8090, ACDPM_0^5 = 0.7188.$	$FES_0 = \{(1,3),(1,4),(2,1),(2,2),(2,4),(3,3),(3,4)\}$
$\eta^{inf} = 0.2$	First	$ACDPM_1^1 = 0.8486, ACDPM_1^2 = 0.8583, ACDPM_1^3 = 0.7620, ACDPM_1^4 = 0.8488, ACDPM_1^5 = 0.8010.$	$FES_1 = \{(2,1),(2,2),(2,4),(3,3),(3,4)\}$
$\eta^{inf} = 0.2$	Second	$ACDPM_2^1 = 0.8635, ACDPM_2^2 = 0.8819, ACDPM_2^3 = 0.8029, ACDPM_2^4 = 0.8670, ACDPM_2^5 = 0.9042.$	$FES_2 = \emptyset.$
$\eta^{inf} = 0.4$	First	$ACDPM_1^1 = 0.8639, ACDPM_1^2 = 0.8745, ACDPM_1^3 = 0.8000, ACDPM_1^4 = 0.8652, ACDPM_1^5 = 0.8258.$	$FES_1 = \emptyset.$
$\eta^{inf} = 0.6$	First	$ACDPM_1^1 = 0.8714, ACDPM_1^2 = 0.8911, ACDPM_1^3 = 0.8021, ACDPM_1^4 = 0.8761, ACDPM_1^5 = 0.8483.$	$FES_1 = \emptyset.$
$\eta^{inf} = 0.8$	First	$ACDPM_1^1 = 0.8847, ACDPM_1^2 = 0.8985, ACDPM_1^3 = 0.8460, ACDPM_1^4 = 0.8869, ACDPM_1^5 = 0.8690.$	$FES_1 = \emptyset.$

TABLE II
THE $ACDPM$ AND FEEDBACK ELEMENT IDENTIFICATION UNDER DIFFERENT η^{sup} WHEN $\eta^{inf} = 0.2$.

	Numbers	$ACDPM$	Feedback element identification
Before	NO	$ACDPM_0^1 = 0.8125, ACDPM_0^2 = 0.8090, ACDPM_0^3 = 0.6979, ACDPM_0^4 = 0.8090, ACDPM_0^5 = 0.7188.$	$FES_0 = \{(1,3),(1,4),(2,1),(2,2),(2,4),(3,3),(3,4).\}$
$\eta^{sup} = 0.3$	First	$ACDPM_1^1 = 0.8392, ACDPM_1^2 = 0.8461, ACDPM_1^3 = 0.7444, ACDPM_1^4 = 0.8387, ACDPM_1^5 = 0.7776.$	$FES_1 = \{(1,3),(1,4),(2,1),(2,2),(2,4),(3,3),(3,4).\}$
$\eta^{sup} = 0.3$	Second	$ACDPM_2^1 = 0.8601, ACDPM_2^2 = 0.8721, ACDPM_2^3 = 0.7871, ACDPM_2^4 = 0.8620, ACDPM_2^5 = 0.8191.$	$FES_2 = \{(2,1),(2,2),(2,4),(3,3),(3,4).\}$
$\eta^{sup} = 0.3$	Third	$ACDPM_3^1 = 0.8726, ACDPM_3^2 = 0.8890, ACDPM_3^3 = 0.8170, ACDPM_3^4 = 0.8763, ACDPM_3^5 = 0.8401.$	$FES_3 = \emptyset.$
$\eta^{sup} = 0.5$	First	$ACDPM_1^1 = 0.8486, ACDPM_1^2 = 0.8583, ACDPM_1^3 = 0.7620, ACDPM_1^4 = 0.8488, ACDPM_1^5 = 0.8010.$	$FES_1 = \{(2,1),(2,2),(2,4),(3,3),(3,4).\}$
$\eta^{sup} = 0.5$	Second	$ACDPM_2^1 = 0.8635, ACDPM_2^2 = 0.8819, ACDPM_2^3 = 0.8029, ACDPM_2^4 = 0.8670, ACDPM_2^5 = 0.9042.$	$FES_2 = \emptyset.$
$\eta^{sup} = 0.7$	First	$ACDPM_1^1 = 0.8588, ACDPM_1^2 = 0.8706, ACDPM_1^3 = 0.8000, ACDPM_1^4 = 0.8617, ACDPM_1^5 = 0.8040.$	$FES_1 = \emptyset.$
$\eta^{sup} = 0.9$	First	$ACDPM_1^1 = 0.8603, ACDPM_1^2 = 0.8754, ACDPM_1^3 = 0.8000, ACDPM_1^4 = 0.8621, ACDPM_1^5 = 0.8230.$	$FES_1 = \emptyset.$

TABLE IV
COMPARISON ANALYSIS WITH TRADITIONAL FEEDBACK MECHANISM.

	The proposed method	The proposed method without incentive behavior
Feedback mechanism	The incentive	The traditional feedback mechanism
Initial $ACDPM$	$ACDPM_0^1 = 0.8125, ACDPM_0^2 = 0.8090, ACDPM_0^3 = 0.6979, ACDPM_0^4 = 0.8090, ACDPM_0^5 = 0.7188.$	
The number of participants in the revised opinion	All the experts	Inconsistent experts
Number of feedback	2	3
The total adjustment distances cost of all inconsistent experts	11.2579	12.5562

the incentive behavior among experts, which effectively promotes consensus reaching.

2) From the perspective of incentive factors, when incentive factors η^{sup} (η^{inf}) are fixed, the larger the variable incentive factor η^{inf} (η^{sup}) is, the easier the experts reach a consensus (see Figures 3a and 3b). However, some experts have not reached an acceptable consensus (highlights in blue color) and need to continue the incentive process with other experts in the group. This indicates that with a larger incentive factor, it is easier to motivate among experts and reach a consensus quickly, which also reduces the number of incentive rounds.
3) From the perspective of incentive type, the minimum adjustment cost 11.2579 of the proposed method is lower than that 12.5562 of the feedback mechanism without incentive behavior. This proposed technique reduces the interaction cost by 10%. And consensus feedback rounds of the proposed method is also smaller than that of the case without incentive behavior (2 versus 3) (see Table IV). The difference is that the proposed incentive mechanism encourages experts to adjust their preferences, while the feedback mechanism without incentive behavior always forces inconsistent experts to make greater sacrifices to adjust their preferences. This indicates that the proposed technique is beneficial to facilitate consensus reaching with the fastest speed and minimum adjustment cost.

V. CONCLUSION AND FUTURE WORKS

It proposes a new incentive mechanism by dynamic trust relationships in GDM, which the following two research contribution to knowledge:

1) The supremum and infimum incentive rule driven by trust relationship that is based on experts' willingness to adjust their preferences. The incentive behavior driven minimum adjustment consensus model is established to generate experts' recommendation opinions to effectively reduce the preference adjustment cost of inconsistent feedback elements and promote consensus reaching.
2) The dynamic trust relationship is explored by incentive behavior. Then, the optimization model is constructed to obtain the final group preference matrix, which reflects the trust interaction relationship between experts.

There are still some issues that need future research like the implementation of incentive factors dynamism in the model to fit the change of incentive conditions of experts in time. In addition, in the process of SN-GDM, experts should also be aware that there may be some problems when relying too much on familiar experts. For example, some familiar experts may be inconsistently (doubt, betray or cheat), which may lead to a reduction of trust relationship between some pairs of experts. This is an interesting topic for future research. Also, the trust relationship between experts is not symmetric in general, which makes to study the law of dynamic change of the trust relationship between experts when T is a directed sociomatrix an interesting research problem to be addressed.

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