Intentional Bounded Rationality Methodology to Assess the Quality of Decision-making Approaches with Latent Alternative Performances

Carlos Sáenz-Royo*[1], Francisco Chiclana[2,3], Enrique Herrera-Viedma[3]

[1] Centro Universitario de la Defensa, Zaragoza, Spain

[2] Institute of Artificial Intelligence, School of Computer Science and Informatics, De Montfort University, Leicester, UK

[3] Andalusian Research Institute on Data Science and Computational Intelligence (DaSCI), University of Granada, Granada, Spain

*Corresponding author: Academia General Militar, carretera Huesca s/n, 50090 Zaragoza, Spain; +34 976 739 830; csaenz@unizar.es

Abstract

Expert's judgments have been crucial in the development of decision theory; however, what criterion to use in the selection of experts remains an issue to address. Decision support techniques proposed to improve the quality of expert judgment decision making consider a demonstrated inconsistency of the judgments expressed by an expert as a criterion of exclusion in the decision-making process of such expert. Although consistency appears to be a desirable condition to qualify as "expert", little is known about the quality of the decisions made imposing consistency as the expert qualifying condition. This paper proposes a simulation methodology, based on an automaton programmed to make decisions in an intended but bounded rational way, to assess the cost-benefit of different aspects of decision support techniques. Within this methodology, the imposition of the consistency condition in the selection of experts is studied. In particular, the paper shows with a case study example that the Analytical Hierarchy Process (AHP) decision support technique expected payoff is at most 5% higher when implementing Saaty's consistency criterion of the expert's judgments than when the consistency criterion is not considered.

1. Introduction

Relying on people's judgments is a common way of making decisions when the information available on the consequences of the alternatives is diffuse, and/or when the decision criteria require weighting multiple attributes whose relationships cannot be formally established [1–4]. A key aspect when ordering alternatives using people's judgments is the choice of the expert person or persons to entrust with the mission of proposing an ordering (from the most to the least preferred) of the alternatives for the achievement of the decision goal(s). Assuming that experts have complete rationality is unrealistic [5,6] and this could generate biases in the analysis. Jones [7] shows how bounded rationality is a mechanism superior to conventional rationality for the analysis of human judgments, and presents better prediction results.

Academic researchers have developed different decision support techniques based on experts' judgments that improve the results of human decisions and endow the treatment of the expressed judgments with scientific rigor [8,9]. The experts can be individually evaluated a priori, when prior information to the judgments of the analysed problem is used, such as experience (number of previous participations in similar problems) [10], reputation (scientific or professional trajectory) [11], trust (level of social influence) [12]; experts can be individually evaluated a posteriori when the decision support techniques use the properties of the judgments expressed in the analysed problem for expert evaluation, such as hesitation (lack of confidence due to lack of knowledge), interest (a high degree of interest in a criterion must show that the expert clearly identifies the best alternative in that criterion), preference (clearly distinguishing between the various criteria) [13,14], or consistency (judgments show a precise logical relationship) [15,16]. The evaluation criteria listed can be used individually or combined to weigh the judgment of the experts in the search for consensus [17], or they can establish guarantee systems, so that when an expert does not obtain the required level in the criterion all expert's judgments are eliminated in the selection process.

There exist a wide variety of approaches to eliminate human judgments that contain inaccuracies or errors [18–20]. Inconsistency in judgments is used in the literature as the main indicator of possible errors and, therefore, of the quality of the decision-maker, which in practice means that the following transitivity property is not satisfied: if alternative A is twice preferred to alternative B, and alternative B is three times preferred to alternative C, then the alternative A should be six times more preferred than alternative C [15,21,22]. The objective of this work is to assess inconsistency as a criterion to exclude experts in decision-making based on judgments, proposing a new methodology that allows building as benchmark the performance observed in a simulated decision situation where a hypothetical expert (automaton) makes decisions in an intended but bounded rational way [23].

Decision support techniques establish a set of computations and logical structures that allow evaluating the judgments of experts to detect and avoid erroneous judgments; once these filters are overcome, a ratio scale can be built to establish priorities associated with the alternatives of the problem, that is to derive a priority vector of alternatives (solution). When the judgments of the decision-maker are accurate, then all techniques give the same solution, and the criterion used to choose the appropriate technique should be that of simplicity. However, in the case of erroneous judgments, different techniques may lead to different solutions [24]. Many aspects of decision support techniques have been studied [25], among them the way to represent the priority scale (a review can be seen in [26]), the group error distribution versus individual error distribution [19], and the decomposition of alternatives into attributes to reduce error [20]. As far as we know, the value contributed by the consistency of the judgments of the decision-maker has not been specifically studied.

A critical element in the evaluation of decision support techniques is the treatment of the decision-maker’s opinions to reach a final solution. If decision-makers do not make errors in their judgments, then the said judgments are consistent; however, the fact that judgments are consistent does not necessarily imply that they are error-free [18,19]. The problem is that due to the characteristics of the situation, where the help of one or more experts is needed in making decisions, knowing the true quality of the experts’ decisions is not straightforward. With this premise in mind, this work proposes a simulation methodology to compare the performance of expert-based decision making with a different criterion to select the participating experts. The key element of the methodology is an “artificial” expert, an automaton that is programmed to make decisions in an intended but bounded rational way. The intended rationality is incorporated into the automaton assuming that for a given level of expertise, alternatives that have higher payoffs will be ranked higher than those with lower payoffs, but the possibility of error cannot be excluded, in line with discrete choice models [27]. For given payoffs of the alternatives compared, the automata can be programmed to allow for different degrees of expertise that result in lower or higher probability of error.

In addition to a detailed explanation of the form of representation of the automaton and the probabilistic decision-making process, the paper illustratively compares the "quality" of decisions applying the AHP technique with the requirement of consistency in the judgments of the decision-maker as the different criterion to select the participating experts, and the "quality" when the AHP technique is applied without imposing the consistency criterion. The results show that the automaton achieves at most 5% higher expected payoff (quality criterion of the automaton decisions) when the inconsistencies observed in the judgments are excluded than when included.

The paper makes different contributions to the literature on decisions based on experts' judgments. First, it proposes an automaton with intentional but bounded rationality, in Simon's sense [28], as a laboratory that allows developing a methodology to evaluate and compare the performance of different decision support techniques. Second, it proposes a classification of errors that a decision support technique can make when generating a priority vector. Third, the methodology establishes a relationship between the manifest inconsistency of the automaton and the quality of the final decisions, proving the consistency restriction quality of the decisions measured with their expected performance. Fourth, the proposed method opens future lines of research such as the comparison between the expected performance and the necessary resources of different decision support techniques. Currently, comparisons between techniques in the literature limit their analysis to describe in detail the differences [21,29–31] and empirically analyse their use [32].

The work is structured as follows: Section 2 introduces a new framework of intentional bounded rationality as a mechanism for choosing a simulated expert (automaton), in which the level of error depends on the relative performance of the alternative being judged and the expertise of the automaton. Section 3 studies the judgments through peer-to-peer analysis (pairwise comparison) and its implications in decision support techniques. Section 4 analyses the relationship between error and consistency. Section 5 describes the intentional bounded rationality automaton methodology and illustrates, through a case study, how much the quality of AHP with the consistency criterion to select experts outperforms the quality of AHP without the consistency criterion. Finally, the last section presents the most relevant conclusions of this research study and possible future research lines.

2. Intentional Bounded Rationality

Decision theory in its prescriptive aspect has provided people methodologies and techniques to improve their decisions. However, there is a gap between this approach and the mechanisms governing human cognition. It seems obvious that expert decision analysis must have a solid foundation to link human behaviour with decision support techniques. Intentional bounded rationality [33] tries to close this gap by proposing a solid functional representation of human behaviour. Given that the concept of intentional bounded rationality is a central part of the methodology proposed in this work, some of the properties of the functional form that, from our point of view, are relevant as a guarantee of its sturdiness are covered herein.

This work considers an automaton as a representation of human behaviour with intentional bounded rationality, recognizing the possibility of making mistakes in his judgments given the human limitations regarding the available time and processing of information in a complex and changing reality [23]. To overcome this, the decision-maker must use resources that facilitate the decision, bringing the problem closer to the limited aspects where he has a deep knowledge, where he is considered an expert, and where his solution is most likely to be close to the best.

There are numerous papers describing and comparing methods that incorporate decision-maker's subjective judgments in the search for the optimal decision. A basic problem discussed throughout this literature has been the possibility of judgments containing serious inaccuracies and inconsistencies [34–36]. In the present paper, we analyse the behaviour of an automaton in a situation in which we know the payoffs of the alternatives. The automaton behaves as a person who does not know these performances, which allows us to evaluate the performance contribution of the mechanisms established in the decision support techniques. The functional forms that characterize our automaton are those that best represent human behaviour. The automaton represents an individual with a level of reliability, herein represented by a parameter β, which is reflected in the probability of making a right or wrong decision. The fallibility of people is not the result of noisy signals of information as in [19], [20], or [37], but the consequence of a well-meaning but bounded rationality, in the sense that the decision-maker intends to choose the best option but with limitations in processing complex information that prevents it. Intentionality is reflected in the existence of an inverse relationship between the difference in latent performance of the alternatives $(V_j - V_i)$ and the error made on their appreciation $(p_{\beta i})$. The smaller the latent performance difference between the alternatives the more accurate the analysis or judgment should be, more information must be processed so as not to make a mistake and the decision is more difficult. This relationship between error and difference in latent performance of alternatives has been used as a screening function of decision-makers [37–40]. However, not all decision-makers have the same precision; this depends on their expertise. Several methods have been used in the literature to determine the expertise of a decision-maker [34,41,42]. What is decisive here is to characterize the automaton and its capacity according to its probability of error; this specification of the bounded, but intentional, rationality allows a priori evaluation of decision support techniques.

The probability that an automaton chooses the alternative A_i with performance V_i combines the ability to process information and the decision difficulty:

$$p_{\beta i} = p_\beta(A_i) = \frac{\mathrm{e}^{\beta \frac{V_i}{\sum_{k=1}^{n} V_k}}}{\sum_{j=1}^{n} \mathrm{e}^{\beta \frac{V_j}{\sum_{k=1}^{n} V_k}}} = \frac{1}{1 + \sum_{j \neq i}^{n} \mathrm{e}^{\frac{\beta (V_j - V_i)}{\sum_{k=1}^{n} V_k}}}. \quad (1)$$

This function is like the functions used in probabilistic choice theory by Luce [43], (see also [44–46]). Puranam et al. [28] relate probability (1) to Simon's bounded rationality. The complementary probability, $1 - p_i$, is the probability that any of the other alternatives will be chosen.

Decision support techniques seek to improve performance by providing rational treatment to subjective judgments of people [8], with the aim to improve the quality of the decision-making process by giving procedural rationality to the outcome [9]. However, this approach has the limitation of being able to simply know the inconsistency of the decision-maker in the judgments manifested, limiting the evaluation of the method. The intentional bounded rationality of automaton presented here makes it possible to establish a certain starting situation, from which the decision-maker shows a level of error depending on the difference in the latent performance of the alternatives $(V_j - V_i)$ and his/her reliability (β). This approach allows to evaluate a priori the results obtained from any decision support technique beyond the internal coherence of the decision-maker. It shows a constant struggle in the search for the best option facing a blurred reality between the right decision and the wrong decision. However, the human being (intentional bounded rational) is attracted to the right decision to a greater degree than to the wrong, as shown by the fact that in equation (1) the best performance alternative is the one with highest probability of being chosen. This property of intentional bounded rationality ensures that decision support techniques add value to the search for the optimal option.

As mentioned above, the parameter β, interpreted as non-negative, is a measure of the decision processing skills, and its value reflects the specific knowledge that the automaton has about the comparative alternatives. When β decreases, the dispersion increases as for possible performances considered by the automaton, increasing the weight of the probability queues, increasingly admitting the possibility of accepting extreme differences between the performance of the alternatives. The limit case value $\beta = 0$ implies that the probability of choosing one of the alternatives does not depend on the relative value of each of them, being possible any performance for any alternative. In other words, decisions are made purely randomly. An "intentional bounded rationality" automaton value $\beta > 0$ is therefore most realistic for modelling decision-makers. Higher positive values of β mean that the expert makes the decision less binding to the difference in performance values, with the limit case of β tending to infinity implying that the probability of error converges to 0, even for small relative differences in performance values. This is expanded more formally below using probabilities with normalised alternative performance values[1] $v_i = \frac{V_i}{\sum_{j=1}^{n} V_j}$: $p_{\beta i} = \frac{e^{\beta v_i}}{\sum_{j=1}^{n} e^{\beta v_j}} = \frac{1}{\sum_{j=1}^{n} e^{\beta\left(v_j - v_i\right)}}$, $i = 1, \dots, n$.

1. The case of the reliability parameter $\beta = 0$. In this case, it is

$$\forall i = 1, 2, \dots, n\colon\ e^{\beta v_i} = 1 \implies p_{0i} = \frac{1}{n}.$$

[1] The normalization is introduced so that the comparison between the latent performances is independent of the scale.

This is in agreement with the interpretation of the reliability parameter as a measure of the decision processing skills, i.e., the specific knowledge that the automaton has about the alternatives to compare. Indeed, when there is a lack of knowledge about how to process a comparison of the alternatives' performance values, all alternatives are equally treated by the automaton.

2. The case of the reliability parameter $\beta > 0$.

2.1. We start by analysing first the case of all alternative performance values being positive and different. Without loss of generality, it can be assumed that $v_1 > v_2 > \cdots > v_n$ (> 0). Therefore, it is:

$$v_i > v_k \Leftrightarrow v_j - v_i < v_j - v_k \ \forall j \ \Leftrightarrow \ \beta(v_j - v_i) < \beta(v_j - v_k) \ \forall j \ \Leftrightarrow$$
$$e^{\beta(v_j - v_i)} < e^{\beta(v_j - v_k)} \ \forall j \ \Leftrightarrow \sum_{j=1}^{n} e^{\beta(v_j - v_i)} < \sum_{j=1}^{n} e^{\beta(v_j - v_k)} \ \Leftrightarrow \ p_{\beta i} > p_{\beta k}$$

Thus, in this case, $v_1 > v_2 > \cdots > v_n$ $(> 0) \Leftrightarrow p_{\beta 1} > p_{\beta 2} > \cdots > p_{\beta n}$.

2.2. To the above positivity assumption, it is added that some of the performance values are the same. It is sufficient to analyse only the case of two consecutive performance values being equal in the above ordering. Let us denote these as $k, k+1$, i.e., $v_1 > v_2 > \cdots > v_k = v_{k+1} > v_{k+2} > \cdots > v_n$ (> 0). Therefore, it is

$$v_k = v_{k+1} \Leftrightarrow \sum_{j=1}^{n} e^{\beta(v_j - v_k)} = \sum_{j=1}^{n} e^{\beta(v_j - v_{k+1})} \ \Leftrightarrow \ p_{\beta k} = p_{\beta k+1} \ ,$$

and $p_{\beta 1} > p_{\beta 2} > \cdots > p_{\beta k} = p_{\beta k+1} > p_{\beta k+2} > \cdots > p_{\beta n}$.

2.3. The case of some performance values being equal to zero is analysed. This assumption implies the existence of a value $k > 1$ such that $v_k = 0$. Then for $k_1 > k$, it is also $v_{k_1} = 0$ and

$$p_{\beta k_1} = \frac{e^{\beta v_{k_1}}}{\sum_{j=1}^{n} e^{\beta v_j}} = \frac{1}{\sum_{j=1}^{k-1} e^{\beta v_j} + (n-k+1)} \ \forall k_1 \geq k,$$

which means that $p_{\beta 1} > p_{\beta 2} > \cdots > p_{\beta k} = \cdots = p_{\beta n}$.

Summarising, a positive reliability parameter ($\beta > 0$) leads to the following association between alternative performance values and their probabilities of being chosen by an automaton:

$$v_1 \geq v_2 \geq \cdots \geq v_n \ (\geq 0) \Leftrightarrow \ p_{\beta 1} \geq p_{\beta 2} \geq \cdots \geq p_{\beta n} \, ,$$

with corresponding strict inequalities in both statements of the above equivalence.

3. Finally, the reliability parameter limit case $\beta \rightarrow \infty$ is analysed. First, we assume that the maximum value of the set of performance values is unique, i.e., $v_1 > v_2$. Since $\beta(v_j - v_1) < 0 \ \forall j > 1$, it is

$$\beta(v_j - v_1) \xrightarrow{\beta \rightarrow \infty} -\infty \ \forall j > 1 \Leftrightarrow e^{\beta(v_j - v_1)} \xrightarrow{\beta \rightarrow \infty} 0 \ \forall j > 1 \ \Leftrightarrow \ \sum_{j>1} e^{\beta(v_j - v_1)} \xrightarrow{\beta \rightarrow \infty} 0.$$

Therefore, it is

$$p_{\beta 1} = \frac{1}{1 + \sum_{j>1} e^{\beta(v_j - v_1)}} \xrightarrow{\beta \rightarrow \infty} 1.$$

3. Judgments Through Peer-to-peer Analysis

The use of peer-to-peer comparisons, rather than direct score allocation, originates from psychological studies [47]. Psychologists argue that it is easier and more accurate to make a judgment on two alternatives than simultaneously on all the alternatives. Some examples that use these techniques in the hope of obtaining the “best” solution are: choice of the “best” supplier [48,49]; search for the “best” investment projects [50]; selection and evaluation issues in the area of engineering and personnel [51]. One advantage of this method is that decision-makers do not need to evaluate the specific values of the alternative performances $\{V_i > 0; i = 1, \ldots, n\}$; it is sufficient to estimate the comparison of each alternative with respect to another by means of a judgment relationship $\left\{a_{ij} = \frac{V_i}{V_j}; i \neq j\right\}$, which avoids scale problems. Thus, the decision-maker compares the intensity of the performance of a pair of alternatives, a_{ij}, against its inverse, a_{ji}, wondering which is greater.

Within the intentional bounded rationality methodology, in the case of conducting pairwise comparison of alternatives, the following probability of choosing alternative A_i over alternative A_j by the automaton with reliability β, $p_{\beta ij}$, is derived:

$$p_{\beta ij} = \frac{e^{\beta \frac{V_i}{V_j}}}{e^{\beta \frac{V_i}{V_j}} + e^{\beta \frac{V_j}{V_i}}} = \frac{1}{e^{\beta\left(\frac{V_j}{V_i} - \frac{V_i}{V_j}\right)} + 1} = \frac{1}{e^{\beta\left(a_{ji} - a_{ij}\right)} + 1}. \quad (2)$$

Expression (2) is obtained by implementing the preference modelling framework methodology developed by Herrera et al. [52] to derive a multiplicative preference relation $A = (a_{ij})$ from a set of ratio scale performance values $\{V_i > 0; i = 1, \dots, n\}$ associated to a set of alternatives $\{A_1, A_2, \dots, A_n\}$. Herrera et al. [52] proved that

$$a_{ij} = \left(\frac{V_i}{V_j}\right)^c, \ c > 0, \quad (3)$$

where c is a fit parameter that satisfies the properties required in multiplicative preference ordering.

The probability of choosing alternative A_i over alternative A_j, p_{ij}, based on their performance values can be defined in terms of the corresponding intensity of preference values as

$$p_{ij} = \frac{a_{ij}}{a_{ij} + a_{ji}}. \quad (4)$$

Following [53], the reliability value $\beta (\geq 0)$ can be considered as an indication of the "power or importance [of the individual the automaton represents] in the decision, the higher the number the more important". In other words, there is an increasing function $f_\beta \colon [0, \infty) \to [0, \infty)$ such that

$$a_{\beta ij} = f_\beta(a_{ij}), \quad (5)$$

and

$$p_{\beta ij} = \frac{a_{\beta ij}}{a_{\beta ij} + a_{\beta ji}}. \quad (6)$$

In a fuzzy context, the methodology to implement importance values associated to decision makers is usually done via a t-norm operator, in particular the product t-norm. However, the power implementation of importance values is superior to the multiplication implementation, since in this last case the reliability value does not play any role in determining the probability values. Indeed, if we were to use the multiplication approach to implement importance values, then $f_\beta(x) = \beta f(x)$ ($\beta > 0$) and

$$p_{\beta ij} = \frac{a_{\beta ij}}{a_{\beta ij}+a_{\beta ji}} = \frac{f_\beta(a_{ij})}{f_\beta(a_{ij})+f_\beta(a_{jj})} = \frac{\beta f(a_{ij})}{\beta f(a_{ij})+\beta f(a_{ji})} = \frac{f(a_{ij})}{f(a_{ij})+f(a_{ji})}. \qquad (7)$$

Hence, the probabilities would not depend on the reliability value of the individual making the decision, which is not what we expect. Thus, it will be $f_\beta(x) = f(x)^\beta$ ($\beta \geq 0$) with $f: [0, \infty) \to [0, \infty)$ increasing. Notice that when $f(x) \in [0,1)$ ($\forall x$), we would have that $f(x)^\beta \xrightarrow{\beta \to 0} 1$ and $f(x)^\beta \xrightarrow{\beta \to \infty} 0$. In the first case, as the reliability value decreases towards the value $\beta = 0$, the probabilities $p_{\beta ij}$ and $p_{\beta ji}$ will approach to the common value $\frac{1}{2}$. Although, this limit case is expected (lack of decision processing skills translates into treating alternative equally no matter their performance), the second limit case is counterintuitive, since unlimited decision processing skills means that the automaton would be able to differentiate the alternatives no matter what performance values they have, i.e. the automaton would be able to achieve different probability values for different performance values. To avoid this, function f range could be assumed to be $[1, \infty)$, i.e., $f: [0, \infty) \to [1, \infty)$, which in turn avoids as well zero denominator in the expression of $p_{\beta ij}$. In any case, without loss of generality, it can be assumed the boundary condition $f(0) = 1$; and function $f(x) = e^x$, or more general $f(x) = e^{g(x)}$ with $g: [0, \infty) \to [0, \infty)$ an increasing function verifying $g(0) = 0$. Thus, $f_\beta(x) = f(x)^\beta = e^{\beta x}$ ($\beta \geq 0$), and the probability of choosing alternative A_i over alternative A_j by the automaton with reliability β, $p_{\beta ij}$, would be:

$$p_{\beta ij} = \frac{e^{\beta\left(\frac{V_i}{V_j}\right)^c}}{e^{\beta\left(\frac{V_i}{V_j}\right)^c} + e^{\beta\left(\frac{V_j}{V_i}\right)^c}}, \; c > 0, \qquad (8)$$

In particular, and for computation efficiency, we consider the value $c = 1$:

$$p_{\beta ij} = \frac{e^{\beta\frac{V_i}{V_j}}}{e^{\beta\frac{V_i}{V_j}} + e^{\beta\frac{V_j}{V_i}}} = \frac{1}{1+e^{\beta\left(\frac{V_j}{V_i}-\frac{V_i}{V_j}\right)}} = \frac{1}{1+e^{\beta\frac{(V_j-V_i)(V_j+V_i)}{V_iV_j}}}. \qquad (9)$$

When the difference between the performance values of the compared alternatives increases in absolute value, the difference between the probabilities of the compared alternatives increases in absolute value as well.

A higher relative performance of the alternative A_i (V_i) increases the likelihood that such alternative will be preferred in paired comparisons. The probability of expressing preference for the relative value $a_{ij}\left(=\frac{V_i}{V_j}\right)$ over the relative value $a_{ji}\left(=\frac{V_j}{V_i}\right)$, $p_{\beta ij}$, depends on the relative difference of their valuations $\left(\frac{V_j}{V_i}-\frac{V_i}{V_j}\right)$. When ($V_i > V_j$) the probability of making the mistake of showing preference for the relative value that presents a lower relative performance is $1 - p_{\beta ij} = p_{\beta ji}$. A rational person with unlimited computational capability ($\beta = \infty$) would show preference with probability 1 when $V_i > V_j$. Figure 1 illustrates the probability values of (2) as a function of $\left(\frac{V_j}{V_i}-\frac{V_i}{V_j}\right)$ with parameter value $\beta = 1$. The probabilities of making a mistake decrease when the difference between V_i and V_j increases and are higher when the performance of V_i and V_j are relatively close. The function represents the hypothesis that decision-maker are rationally bounded (make mistakes) and intend to be rational. Thus, it is concluded that in paired judgments, equation (2) of intentional bounded rationality shows how the automaton is not entirely in darkness: it is attracted to the higher-performance option, so that the probability of choosing the best option is always higher than 0.5 if $\beta > 0$. This allows to establish decision support techniques that improve individual performance by approaching the “best” option.

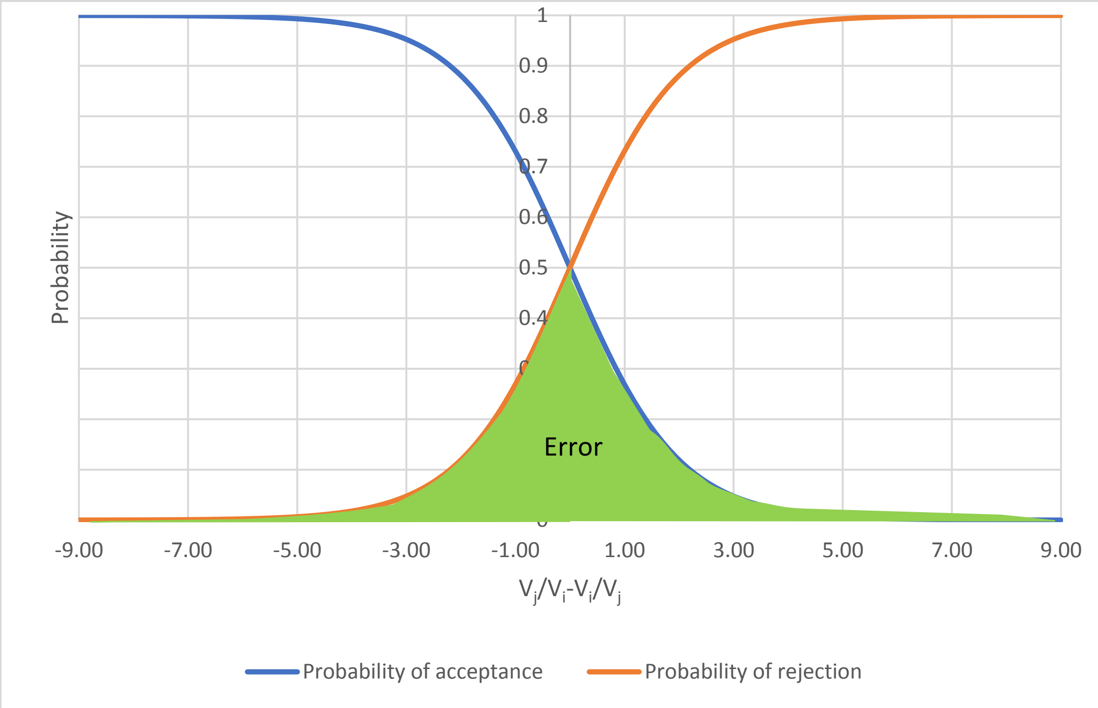

Figure 1. Probability to prefer A_i and Probability to prefer A_j with performance from $\frac{V_j}{V_i} - \frac{V_i}{V_j}$ in (1) with $\beta = 1$

Peer-to-peer comparison is intended to obtain the decision-maker judgement regarding the performance of two alternatives. Intentional bounded rationality introduces a probabilistic approach in which the reliability of the automaton when choosing each pair of alternatives (A_i, A_j), $p_{\beta ij}$, is adequately known. Intensity judgment, also known as intensity of preference, is the decision-maker's assessment of how many times the performance of one alternative A_i is greater than the performance of another alternative A_j, $\hat{a}_{ij}$.

Intensity judgment is therefore provided with respect to a default discrete preference scale. Saaty [18] provides a discrete scale ($\hat{a}_{ij,k} = k$) to classify the verbal judgments of decision-makers. In this case, the decision-maker shows that V_i is k times V_j measured in a discrete way, $\hat{a}_{ij,k} = k \in \{1, \dots, K\}$, to quantify the probabilities of each intensity segment that determines the intentional bounded rationality, $p_{ij,k}$. Thus, the critical points of each section of the chosen scale (k) must be obtained. Each critical point represents the value from which the decision maker must change the segment, i.e., if the segments are those of Saaty's scale (there are 18 segments, for $a_{ij,k} = 1, 2, \dots, 9$) when the decision-maker judges an intensity somewhat below 1.5 (for example 1.4) he will choose the intensity range $a_{ij,1} = 1$ and if decision-maker judges the intensity somewhat above 1.5 (for example 1.6) he will choose the intensity range $a_{ij,2} = 2$, then 1.5 is a critical point on Saaty's scale. For the automaton, the critical point compares the minimum required intensity (Z) with the mean of the distribution at (2), obtaining the probability that the automaton will consider that the relative performance $\frac{V_i}{V_j}$ is at least Z.

$$p_{\beta ij}(Z) = \frac{1}{\mathrm{e}^{\beta\left(\left(Z-\frac{1}{Z}\right)-\left(\frac{V_i}{V_j}-\frac{V_j}{V_i}\right)\right)}+1} \qquad (10)$$

The probability with which the automaton manifests an intensity of preference k, delimited by a higher critical point $Z_{k,h}$ and a lower critical point $Z_{k,l}$, is equal to the difference between the probability that the automaton considers that the intensity is greater than the lower critical point $Z_{k,l}$ and the probability that it considers the intensity to be greater than the higher critical point $Z_{k,h}$, i.e., $p_{ij}(\hat{a}_{ij,k} = k) = p_{ij}(Z_{k,l}) - p_{ij}(Z_{k,h})$. Using (10), we have that

$$p_{ij}(k) = \frac{1}{\mathrm{e}^{\beta\left(\left(Z_{k,l}-\frac{1}{Z_{k,l}}\right)-\left(\frac{V_i}{V_j}-\frac{V_j}{V_i}\right)\right)}+1} - \frac{1}{\mathrm{e}^{\beta\left(\left(Z_{k,h}-\frac{1}{Z_{k,h}}\right)-\left(\frac{V_i}{V_j}-\frac{V_j}{V_i}\right)\right)}+1} \qquad (11)$$

4. Error and Inconsistency

The intensity judgment adds information to a simple preference judgment and is a central element of decision support techniques such as AHP. From the decision-maker manifested intensity judgments on a set of n alternatives, the AHP technique builds an array of preference comparisons, a matrix A of dimension $n \times n$, that is eventually solved algebraically [15].

The intensity of the decision-maker's judgment $\hat{a}_{ij}$ shows the number of times that the performance assigned to the alternative A_i is greater than the performance of the alternative A_j:

$$A = \begin{pmatrix} \hat{a}_{11} & \cdots & \hat{a}_{1n} \\ \vdots & \ddots & \vdots \\ \hat{a}_{n1} & \cdots & \hat{a}_{nn} \end{pmatrix}.$$

When intensity of preference evaluates tangible quantifiable criteria, preference values $\hat{a}_{ij}$ are obtained directly from the measured information, for example weights (in kilograms) or prices (in euros). However, it is not always possible to determine precisely the intensity numerically, either because it is complex information or because it refers to intangible aspects. In these cases, the cardinal information $\hat{a}_{ij}$ is provided with respect to a default discrete preference scale. A set of verbal judgments was proposed by Saaty [18] to provide a scale, which has been widely used in various applications [12] despite some critics [54,55]. Intensity judgements are defined for each possible comparison and the decision-maker is required to have a minimum of coherence, so that $\hat{a}_{ij} = \frac{1}{\hat{a}_{ji}}$. If $\hat{a}_{ij} > 1$, then the decision-maker thinks that $V_i > V_j$ and A_i is therefore preferred to A_j. From the reciprocal matrix constructed $A = \left(\hat{a}_{ij}\right)_{n\text{x}n}$, a vector of priorities $\widehat{w}$ is derived such that $\hat{a}_{ij} = \frac{\widehat{w}_i}{\widehat{w}_j}$ and $\sum_i^n \widehat{w}_i = 1$. If there are no errors and the decision-maker is totally rational ($\beta = \infty$ in (2)), then $\hat{a}_{ij} = \frac{V_i}{V_j}$.

When the decision-maker has manifested the intensity judgments, the matrix A is cardinally consistent (transitive) when the following property is verified:

$$\hat{a}_{ik} = \hat{a}_{ij}\hat{a}_{jk}\left(\frac{V_i}{V_k} = \frac{V_i}{V_j}\frac{V_j}{V_k}\right) \quad \forall i \neq j \neq k. \tag{12}$$

A consistent matrix priority vector, $\widehat{w}$, obtained by the AHP method is the solution to the decision problem. Since consistent matrix A has range 1, this takes the form $A\widehat{w} = n\widehat{w}$, which can be written as $(A - nI)\widehat{w} = 0$. This equation has a solution if and only if n is an eigenvalue (λ_i) of A. Since A has range 1, all the eigenvalues are equal to zero ($\lambda_i = 0$) except one λ_{max} that will be equal to n. Although any column of consistent matrix A is a solution to the equation $(A - nI)\widehat{w} = 0$, the standardized solution is unique, i.e., the vector $\widehat{w}$ whose components verify $\frac{\widehat{w}_i}{\widehat{w}_j} = \frac{V_i}{V_j}$, what has been specifically called the priority vector. Thus, matrix A is fully consistent if and only if $\lambda_{max} = n$ and it is not consistent when $\lambda_{max} > n$. Saaty [15] defines the consistency index of a matrix A as:

$$CI_A = \frac{\lambda_{max} - n}{n-1} \tag{13}$$

where $\lambda_{max} = \sum_{i=1}^{n} \hat{a}_{ij} \frac{\hat{w}_j}{\hat{w}_i}$, indicates the cardinal difference between the decision-maker valuation and the inverse of its corresponding prioritization value. This formulation is tremendously intuitive: the closer each paired judgment is to the inverse of its estimate (the more consistent the judgments will be), the closer to 1 each addend will be, and the closer λ_{max} will be to n and CI_A to 0 (total consistency). On the contrary, the further the judgments of the decision-maker are from the inverse of their estimates (the more disparate the intensities of the judgments), the greater λ_{max} and CI_A will be. A unique measure of consistency that does not depend on the dimensions of matrices A, known as the consistency ratio (CR), was proposed by Saaty [15] by computing random consistency index (RI) values for sets of randomly generated matrices (for different dimension values n):

$$CR_A = \frac{CI_A}{RI}. \tag{14}$$

Saaty [15] established a consistency criterion of acceptability of decision-makers' judgments based on the CR_A being less than 0.1, which has been widely used (as can be seen in [37]).

Not all problems have the same difficulty, nor all errors have the same consequences. Suppose that the automaton knows fully the performance of all the alternatives assuming an infinite value of β in equation (2). This implies that the automaton can manifest an error-free decision matrix since it is able to compare all the alternatives correctly. This automaton rationality implies that the decision matrix is positive, reciprocal, and consistent. However, the opposite is not true, i.e., the fact that a decision matrix is reciprocal positive and consistent does not imply that it is error-free. Recall that consistency measures the logic of the decision-maker's judgments but does not measure validity (proximity to the optimal solution). Indeed, we can have consistent matrices that lead to the wrong ordering of the alternatives, which can happen when the starting hypotheses are false but the logical structures of the relationship between the judgments are correct.

It is very common to see how in the real world decision-makers qualified as "experts" show inconsistencies in their judgments, demonstrating the fallibility of human nature. This error can result from an incorrect way to ask the peer comparison question, or it can be an error in the scale used. Saaty [18] states that it is practically impossible to find decision-makers who provide fully consistent peer comparison matrices. Recall that Saaty defines consistency using a cardinal criterion: to be consistent, judgments in intensity of preference in the pairwise comparison of the alternatives must verify $\hat{a}_{ik} = \hat{a}_{ij}\hat{a}_{jk}$. Consistency requires all judgments to show a unique ordering of alternatives. However, decision-makers may choose a correct ordering of alternatives but incorrect intensities of preference in the pairwise comparison of the alternatives [56]. If the decision-maker is the only one able to know the performance values assigned to the alternatives, then invisible errors might exist because it is not possible to check these values with reality since the inconsistency only shows the existence of errors when the judgments present internal incoherence of the decision-maker (their lack of precision when comparing elements separately). For example, a balance can give similar values in different weight measurements, which shows it has great consistency in its measurements, i.e., it is quite precise; however, the balance may show a systematic bias (underestimating or overestimating weights) and therefore its lack of validity could not be perceived. The same can happen with human judgments. Lack of validity will be observable only when the alternative performances are known; when the performances are latent this lack of validity cannot be perceived. Thus, consistency may hide issues in decision support techniques, and limit their validity to just one part of the problem: the decision maker's internal coherence (accuracy). If the decision-maker evaluates badly one alternative only, $\hat{V}_i = V_i e_i$ (where e_i represents the judgment error), then the errors in the comparisons will not lead to a consistency issue since $\hat{a}_{ij} = \frac{V_i}{V_j} e_i \ (\forall j)$ and all comparisons will verify the consistency property $\hat{a}_{ik} = \hat{a}_{ij}\hat{a}_{jk}$, i.e., $\frac{V_i}{V_k} e_i = \frac{V_i}{V_j} e_i \frac{V_j}{V_k}$, and the matrix A will be completely consistent. However, this type of error can lead to an incorrect ordering of the decision alternatives. Thus, if the performances of the alternatives are only known through the decision-maker's judgments and the judgements matrix of such decision-maker is consistent, then we might think that it is an error-free matrix when it may not be. Let us show this with an example. Suppose we want to evaluate three vehicles of three different brands X, Y and Z. The latent performance of each of these vehicles are $V_x = 10$, $V_y = 20$ and $V_z = 30$, values that must be estimated by an expert. The selected expert has a notable predilection for brand X (he tends to estimate the performance of X by four times its value), is neutral with respect to brand Y and has a strong aversion to brand Z

(he tends to estimate the performance of Z as one-third of its value), that is, $\widehat{V_x} = 4V_x$, $\widehat{V_y} = V_y$, and $\widehat{V_z} = \frac{1}{3}V_z$. The security shown by the expert is maximum and his estimates have hardly any variance, although they show the above bias. The paired judgments that this expert will display are $\hat{a}_{xy} = 2\ [= 4 \cdot \frac{10}{20}]$, $\hat{a}_{yz} = 2\ [= \frac{20}{10}]$, and $\hat{a}_{xz} = 4\ [= (4 \cdot 10)/(30/3)]$. The expert's ordering is wrong since the first alternative should be last and the last should be first, however the expert's judgments are completely consistent: $\hat{a}_{xy} \cdot \hat{a}_{yz} = \hat{a}_{xz}\ [2 \cdot 2 = 4]$; $\hat{a}_{yz} \cdot \hat{a}_{zx} = \hat{a}_{yz}\ [2 \cdot 1/4 = 1/2]$, this is true in all relations between alternatives. From the point of view of consistency, judgments of this expert are of high quality given their consistency, however, in a practical way they are unacceptable.

It is true that in reality decision-making requires relativism; however, this does not negate the existence of absolute optimal alternatives. If no initial hypothesis is specified about the fallibility of decision-makers, improving the coherence of a group of people does not mean getting closer to the "optimal" solution but making their provided judgments, as a collection of samples, closer to being logically related than being chosen at random [18]. On the other hand, there may be significant errors in the perception of intensity that may lead to the decision support technique to reject a set of judgments. However, these errors may not affect the ordering of alternatives, and therefore the decision support technique may reject judgments that allow achieving the maximum possible performance. The relevant question is whether consistency always helps detect errors. Establishing a priori conditions that can be verified is very important to design 'good' decision support techniques. Methodology base on intentional bounded rationality allows assessing a priori the error percentages of ways to collect the judgments of a decision-maker. This is discussed and shown in the next section with the analysis of the results from an illustrative case study.

5. Intentional Bounded Rationality Methodology. Illustrative Case

The proposed methodology of intentional bounded rationality states that an automaton can manifest different judgments (intensities) with different probabilities, allowing to evaluate both orderings of alternatives and elections of the best alternative. Thus, the proposed methodology analyses all the possible results of a situation with performances, known a priori, of an automaton that behaves like an expert that cannot accurately anticipate the performances of the alternatives. When the alternatives studied are not mutually exclusive and it is possible to elect more than one (i.e., carry out investment projects until the budget constraint is reached), it is useful to assess the quality of a decision support technique by computing the probability of the complete ordering of alternatives derived with such technique. When the alternatives are mutually exclusive and only one can be chosen, the quality evaluation will reduce to computing the probability of the elected alternative, with errors in the ordering of the other alternatives being not relevant. Thus, the following steps of intentional bounded rationality methodology are carried out to evaluate a decision support technique:

1. All possible combinations of judgments that the automaton can manifest (a matrix A for each one) and their probabilities are listed.
2. The priority vector and its validity parameters (consistency) for each combination of judgments in step 1 is obtained by applying the decision support technique to evaluate.
3. Orderings are classified according to the impact of the error they entail.
4. Priority vectors with same ordering of alternatives or elected alternative are clustered together. The probability of an ordering/elected alternative is obtained as the sum of the probabilities of all priority vectors in its corresponding cluster.
5. The probability of choosing the correct ordering or the correct best alternative by the decision support technique is analysed and the (expected) performance of decision the support techniques can be computed.

This methodology allows to evaluate the a priori help of a decision support technique by incorporating the possibility that the automaton is wrong. The results obtained will depend on how the decision support technique manages the automaton's errors. The intentional bounded rationality methodology will be applied to Saaty's AHP decision support technique on the set of possible judgement matrices constructed as combinations of intensities the automaton can manifest. In particular, the performance values will be compared for two versions of the AHP technique: with the application of Saaty's consistency criterion (AHPwC) and without the application of such criterion (AHPwoC).

For the case scenario, it is assumed that the bounded rationality automaton probabilities are obtained with a reliability parameter value $\beta = 0.7$, which entails a certain lack of expertise on the part of the decision-maker that the automaton is representing. In addition, and for simplicity, it is assumed three alternatives $\{A_1, A_2, A_3\}$ with latent performance values $V_1 = 62.5$, $V_2 = 25$, and $V_3 = 10$, respectively. According to the proposed methodology, the automaton may manifest in each paired comparison different intensities of preference, $\frac{V_i}{V_j}$, with different probabilities, $p_{\beta ij}$. In what follows, we will drop the subscript 0.7 for β when referring to such probabilities[2]. To simplify as much as possible, using $\frac{V_1}{V_3}$ and $\frac{V_2}{V_3}$ as guiding values, the below four-stage intensity scale judgments is established as a simplification of the scale proposed by Saaty [18][3]:

1. *Much higher (the performance of the alternative A_i is at least 4.5 times bigger than the performance of alternative A_j)*: In this case, the value $\hat{a}_{ij} = \frac{25}{4}$ is assigned;
2. *Higher (the performance of the alternative A_i is bigger than the performance of alternative A_j but less than 4.5 times)*: In this case, the value $\hat{a}_{ij} = \frac{10}{4}$ is assigned;
3. *Lower (the performance of the alternative A_j is higher than the performance of alternative A_i but less than 4.5 times)*: In this case, the value $\hat{a}_{ij} = \frac{4}{10}$ is assigned;
4. *Much lower (the performance of the alternative A_j is at least 4.5 times bigger than the performance of alternative A_i)*: In this case, the value $\hat{a}_{ij} = \frac{4}{25}$ is assigned.

This scale requires 3 critical points (Z), which are 4.5, 1, and -4.5. From these values, using (10) and (11), we can obtain the probability that the automaton will show a preference ($1, 2, 3, or\ 4$) in each paired judgment. For example, the probability that the automaton values $\hat{a}_{ij} = \frac{25}{4}$ is obtained as the probability that the automaton believes that V_i is much higher than V_j, which applying (11) leads to

$$p_{ij}\left(\frac{25}{4}\right) = \frac{1}{e^{0.7\left(\left(4.5-\frac{1}{4.5}\right)-\left(\frac{V_i}{V_j}-\frac{V_j}{V_i}\right)\right)}+1} = \frac{1}{e^{0.7\left(4.28-\left(\frac{V_i}{V_j}-\frac{V_j}{V_i}\right)\right)}+1}.$$

[2] The value $\beta = 0.7$ for the level of reliability was selected because it presents the maximum expected performance difference between the requirement and the non-requirement of consistency as Figure 2 illustrates.

[3]To simplify, Saaty's categories 1, 2, 3, 4 have been grouped in the segment whose assigned value is 2.5 (10/4) while Saaty's categories 5, 6, 7, 8, 9 have been grouped in the segment whose assigned value is 6.25 (25/4). The critical point between the two segments is 4.5. The segment simplification and the laten performances of the alternatives were consciously chosen to allow for fully consistent matrices.

Previously to the calculations of the first two stages of the intentional bounded rationality methodology for the example are presented below the automaton's intensity judgment probabilities are calculated. The probability with which the automaton with intentional bounded rationality will choose each intensity in each peer-to-peer comparison allows for situations in which it is possible to get right which of the two compared alternative is best but with not correct intensity. For example, the probability of choosing rightly alternative A_1 in the comparison $A_1\ vs\ A_2$ with intensity $\hat{a}_{12} = 10/4$ will lead to

$$p_{12}\left(\frac{10}{4}\right) = \frac{1}{e^{0.7\left(\left(1-\frac{1}{1}\right)-\left(\frac{V_1}{V_2}-\frac{V_2}{V_1}\right)\right)}+1} - \frac{1}{e^{0.7\left(\left(4.5-\frac{1}{4.5}\right)-\left(\frac{V_1}{V_2}-\frac{V_2}{V_1}\right)\right)}+1} = 0.6342;$$

while with intensity $\hat{a}_{12} = \frac{25}{4}$ will lead to

$$p_{12}\left(\frac{25}{4}\right) = \frac{1}{e^{0.7\left(\left(4.5-\frac{1}{4.5}\right)-\left(\frac{V_1}{V_2}-\frac{V_2}{V_1}\right)\right)}+1} = 0.1788.$$

The complete set of probability values of peer-to-peer comparison are:

$\hat{a}_{12} = \frac{25}{4}$; $p_{12}\left(\frac{25}{4}\right) = 0.1788$; $\hat{a}_{12} = \frac{10}{4}$; $p_{12}\left(\frac{10}{4}\right) = 0.6342$;

$\hat{a}_{12} = \frac{4}{10}$; $p_{12}\left(\frac{4}{10}\right) = 0.1756$; $\hat{a}_{12} = \frac{4}{25}$; $p_{12}\left(\frac{4}{25}\right) = 0.0114$;

$\hat{a}_{13} = \frac{25}{4}$; $p_{13}\left(\frac{25}{4}\right) = 0.7805$; $\hat{a}_{13} = \frac{10}{4}$; $p_{13}\left(\frac{10}{4}\right) = 0.2056$;

$\hat{a}_{13} = \frac{4}{10}$; $p_{13}\left(\frac{4}{10}\right) = 0.0132$; $\hat{a}_{13} = \frac{4}{25}$; $p_{13}\left(\frac{4}{25}\right) = 0.0007$;

$\hat{a}_{23} = \frac{25}{4}$; $p_{23}\left(\frac{25}{4}\right) = 0.1788$; $\hat{a}_{23} = \frac{10}{4}$; $p_{13}\left(\frac{10}{4}\right) = 0.6342$;

$\hat{a}_{23} = \frac{4}{10}$; $p_{23}\left(\frac{4}{10}\right) = 0.1756$; $\hat{a}_{23} = \frac{4}{25}$; $p_{13}\left(\frac{4}{25}\right) = 0.0114$;

STEP 1. All possible combinations of judgments that the automaton can manifest (a matrix A for each one) and their probabilities are listed.

The total number of different combinations of possible judgments (matrices) the automaton can manifest is 64 $(= 4^3)$:

$$\mathrm{A}(1) = \begin{pmatrix} - & \frac{25}{4} & \frac{25}{4} \\ \frac{4}{25} & - & \frac{25}{4} \\ \frac{4}{25} & \frac{4}{25} & - \end{pmatrix} \quad \mathrm{A}(2) = \begin{pmatrix} - & \frac{25}{4} & \frac{25}{4} \\ \frac{4}{25} & - & \frac{10}{4} \\ \frac{4}{25} & \frac{4}{10} & - \end{pmatrix} \ldots \mathrm{A}(64) = \begin{pmatrix} - & \frac{4}{25} & \frac{4}{25} \\ \frac{25}{4} & - & \frac{4}{25} \\ \frac{25}{4} & \frac{25}{4} & - \end{pmatrix}.$$

a) The probabilities of each matrix are listed.

$$p(\mathrm{A}(1)) = p_{12}\left(\frac{25}{4}\right) * p_{13}\left(\frac{25}{4}\right) * p_{23}\left(\frac{25}{4}\right) = 0.0250$$

$$p(\mathrm{A}(2)) = p_{12}\left(\frac{25}{4}\right) * p_{13}\left(\frac{25}{4}\right) * p_{23}\left(\frac{10}{4}\right) = 0.0885$$

…

$$p(\mathrm{A}(64)) = p_{12}\left(\frac{4}{25}\right) * p_{13}\left(\frac{4}{25}\right) * p_{23}\left(\frac{4}{25}\right) = 9.12E-08$$

STEP 2. The priority vector and its validity parameters (consistency) for each combination of judgments in step 1 is obtained by applying the decision support technique to evaluate.

The AHP method allows obtaining the priority vector, $\widehat{w}$, from the matrix of A:

$$A = \begin{pmatrix} \hat{a}_{11} & \cdots & \hat{a}_{1n} \\ \vdots & \ddots & \vdots \\ \hat{a}_{n1} & \cdots & \hat{a}_{nn} \end{pmatrix}$$

Multiplying matrix A by itself as many times as necessary so that between one step and the next there are no variations in the elements of the priority vector $\widehat{w}$.

$\mathrm{A}(1) \rightarrow \widehat{w}(\widehat{w}_1, \widehat{w}_2, \widehat{w}_3) = (0.6880, 0.2397, 0.0723); CR = 0.3539$

$\mathrm{A}(2) \rightarrow \widehat{w}(\widehat{w}_1, \widehat{w}_2, \widehat{w}_3) = (0.7385, 0.1694, 0.0920); CR = 0.0829$

…

$A(64) \rightarrow \widehat{w}(\widehat{w}_1, \widehat{w}_2, \widehat{w}_3) = (0.0723, 0.2397, 0.6880); CR = 0.3539$

STEP 3. Orderings are classified according to the impact of the error they entail.

Regardless of the intensity values, there is only one correct ordering of alternatives: $A_1 \succ A_2 \succ A_3$. Herein, this ordering is called Error Free and it is denoted O_{123}. Using this notation, wrong ordering O_{132} chooses rightly alternative A_1, and it is called Right-Soft Error. Ordering O_{213} is called Medium-Soft Error; O_{231} is called Medium-Hard Error; O_{312} is called Extreme-Soft Error; while O_{321} is called Extreme-Hard Error. The priority vector leading to the three alternatives being equivalent, $A_1 = A_2 = A_3$, is denoted $O_{1=2=3}$, and called Total Error. Any of the error orderings but Total Error can be obtained from a fully consistent matrix. For example, the fully consistent matrix

$$\begin{pmatrix} - & \frac{10}{4} & \frac{4}{10} \\ \frac{4}{10} & - & \frac{4}{25} \\ \frac{10}{4} & \frac{25}{4} & - \end{pmatrix}$$

has a priority vector leading to the Extreme-Soft Error ordering, while the fully consistent matrix

$$\begin{pmatrix} - & \frac{4}{10} & \frac{4}{10} \\ \frac{10}{4} & - & \frac{4}{25} \\ \frac{10}{4} & \frac{25}{4} & - \end{pmatrix}$$

has a priority vector leading to the Extreme-Hard Error ordering. Notice that there are inconsistent matrices

$$\begin{pmatrix} - & \frac{10}{4} & \frac{10}{4} \\ \frac{4}{10} & - & \frac{25}{4} \\ \frac{4}{10} & \frac{4}{25} & - \end{pmatrix} \text{and} \begin{pmatrix} - & \frac{25}{4} & \frac{10}{4} \\ \frac{4}{25} & - & \frac{25}{4} \\ \frac{4}{10} & \frac{4}{25} & - \end{pmatrix},$$

that have a priority vector leading to a correct ordering of alternatives.

As a summary of the first three steps for a sample of the 64 possible matrices, the probability of their priority vectors, obtained with the AHP technique, CR values and type of errors are provided below:

$A(1) \to \hat{w}(\hat{w}_1, \hat{w}_2, \hat{w}_3) = (0.6880, 0.2397, 0.0723); CR = 0.3539;$
$p(A(1)) = p_{12}(25/4) * p_{13}(25/4) * p_{23}(25/4) = 0.0250$ (O_{123} Error Free)

$A(2) \to \hat{w}(\hat{w}_1, \hat{w}_2, \hat{w}_3) = (0.7385, 0.1694, 0.0920); CR = 0.0829;$
$p(A(2)) = p_{12}(25/4) * p_{13}(25/4) * p_{23}(10/4) = 0.0885$ (O_{123} Error Free)

$A(3) \to \hat{w}(\hat{w}_1, \hat{w}_2, \hat{w}_3) = (0.7385, 0.0920, 0.1694); CR = 0.0829;$
$p(A(3)) = p_{12}(25/4) * p_{13}(25/4) * p_{23}(4/10) = 0.0245$ (O_{132} Right-Soft Error)
…

$A(25) \to \hat{w}(\hat{w}_1, \hat{w}_2, \hat{w}_3) = (0.3306, 0.3976, 0.2718); CR = 1.4987;$
$p(A(25)) = p_{12}(10/4) * p_{13}(4/10) * p_{23}(25/4) = 0.0015$ (O_{213} Medium-Soft Error)
…

$A(29) \to \hat{w}(\hat{w}_1, \hat{w}_2, \hat{w}_3) = (0.2785, 0.3897, 0.3319); CR = 2.5086;$
$p(A(29)) = p_{12}\left(\frac{10}{4}\right) * p_{13}\left(\frac{4}{25}\right) * p_{23}\left(\frac{25}{4}\right) = 0.0001$ (O_{231} Medium-Hard Error)
…

$A(31) \to \hat{w}(\hat{w}_1, \hat{w}_2, \hat{w}_3) = (0.2166, 0.1585, 0.6249); CR = 0.3492;$
$p(A(31)) = p_{12}(10/4) * p_{13}(4/25) * p_{23}(4/10) = 0.0001$ (O_{312} Extreme-Soft Error)
…

$A(43) \to \hat{w}(\hat{w}_1, \hat{w}_2, \hat{w}_3) = (0.1638, 0.2984, 0.5377); CR = 0.0816;$
$p(A(43)) = p_{12}(4/10) * p_{13}(4/10) * p_{23}(4/10) = 0.0004$ (O_{321} Extreme-Hard Error)
…

$A(52) \to \hat{w}(\hat{w}_1, \hat{w}_2, \hat{w}_3) = (0.3333, 0.3333, 0.3333); CR = 3.8017;$
$p(A(52)) = p_{12}(4/25) * p_{13}(25/4) * p_{23}(4/25) = 0.0001$ ($O_{1=2=3}$ Total Error)
…

$A(64) \to \hat{w}(\hat{w}_1, \hat{w}_2, \hat{w}_3) = (0.0723, 0.2397, 0.6880); CR = 0.3539;$
$p(A(64)) = p_{12}(4/25) * p_{13}(4/25) * p_{23}(4/25) = 9.12E - 08$ (O_{321} Extreme-Hard Error)

STEP 4. Priority vectors with same ordering of alternatives or elected alternative are clustered together. The probability of an ordering/elected alternative is obtained as the sum of the probabilities of all priority vectors in its corresponding cluster.

$p(O_{123}) = \sum_{i=1}^{64} p(\mathrm{A}(i)|O_{123})$; $p(O_{132}) = \sum_{i=1}^{64} p(\mathrm{A}(i)|O_{132})$
$p(O_{213}) = \sum_{i=1}^{64} p(\mathrm{A}(i)|O_{213})$; $p(O_{231}) = \sum_{i=1}^{64} p(\mathrm{A}(i)|O_{231})$
$p(O_{312}) = \sum_{i=1}^{64} p(\mathrm{A}(i)|O_{312})$; $p(O_{321}) = \sum_{i=1}^{64} p(\mathrm{A}(i)|O_{321})$
$p(O_{1=2=3}) = \sum_{i=1}^{64} p(\mathrm{A}(i)|O_{1=2=3})$

The probability of orderings as the sum of the probability values of the priority vectors leading to such ordering:
$p(O_{123})$; $p(O_{132})$; $p(O_{213})$; $p(O_{231})$; $p(O_{312})$; $p(O_{321})$; $p(O_{1=2=3})$.

When the alternatives are mutually exclusive, the quality of the decision support technique is measured with its likelihood of choosing the best alternative. The probability of choosing alternative A_i as best is the sum of the probabilities of the orderings that have such alternative as the first option: $p_i = \sum_{j\neq i}^{n} \sum_{k\neq i\neq j}^{n} p\left(O_{ijk}\right)$.
$p_1 = p(O_{123}) + p(O_{132})$; $p_2 = p(O_{213}) + p(O_{231})$;
$p_3 = p(O_{312}) + p(O_{321})$; $p_{1=2=3}$

STEP 5. The probability of choosing the correct ordering or the correct best alternative by the decision support technique is analysed and the (expected) performance of decision the support techniques can be computed.

The expected performance of orderings (when only one alternative should be discarded – $AHPwoC_o$) and of election (when two alternatives must be discarded – $AHPwoC_e$) are detail:

$$E(AHPwoC_o) = \left(p(O_{123}) + p(O_{213})\right)(V_1 + V_2) + \left(p(O_{132}) + p(O_{312})\right)(V_1 + V_3) + \left(p(O_{231}) + p(O_{321})\right)(V_2 + V_3) + p_{1=2=3}\frac{2(V_1+V_2+V_3)}{3}.$$

$$E(AHPwoC_e) = p_1V_1 + p_2V_2 + p_3V_3 + p_{1=2=3}\left(\frac{V_1+V_2+V_3}{3}\right).$$

Finally, these steps must be carried out for all compared decision support techniques. Obtaining the performance of each one for the same case allows us to analyse the differences quantitatively.
In our case we evaluate the contribution of the inconsistency in AHP, therefore the first three steps are common in the two compared techniques, and it will only be necessary to repeat steps 4 and 5 whose results are detailed in subsections 5.1 and 5.2.

5.1. *Performance of AHPwoC*

When consistency is not required to be verified, the 64 comparison matrices are considered. Each one of them having one priority vectors, which are obtained using Saaty's AHP technique.

The probability of an ordering is obtained, which in this case are:

$$p(O_{123}) = 0.6764; p(O_{132}) = 0.1529;\ p(O_{213}) = 0.1529; p(O_{231}) = 0.0026;$$
$$p(O_{312}) = 0.0026; p(O_{321}) = 0.0008; p(O_{1=2=3}) = 0.0118.$$

Thus, for the AHPwoC technique the probability of obtaining an Error Free ordering is 0.6764, much higher than that of any of the other possible error orderings. The probabilities of obtaining a Medium-Soft Error and a Right-Soft Error, respectively, are the same (0.1529) since both suppose an error between two performances that have the same intensity (A_1 is 2.5 times A_2 and A_2 is 2.5 times A_3), while the other error orderings have a probability lower than 0.003.

The probability of election each alternative A_i, in this case, it is:

$$p_1 = p(O_{123}) + p(O_{132}) = 0.8293; p_2 = p(O_{213}) + p(O_{231}) = 0.1555;$$

$$p_3 = p(O_{312}) + p(O_{321}) = 0.0034;\ p_{1=2=3} = 0.0118$$

Thus, the AHPwoC technique probability of choosing A_1 as the best alternative is 0.8293, of choosing A_2 as the best alternative is 0.1555, of choosing A_3 as the best alternative is 0.0034, while the probability of the three alternatives being equivalent is 0.0118.

Once the probabilities of orderings ($AHPwoC_o$) and election ($AHPwoC_e$) are obtained, their expected performances are calculated:

$$E(AHPwoC_o) = \big(p(O_{123}) + p(O_{213})\big)(V_1 + V_2) + \big(p(O_{132}) + p(O_{312})\big)(V_1 + V_3) + \big(p(O_{231}) + p(O_{321})\big)(V_2 + V_3) + p_{1=2=3}\frac{2(V_1+V_2+V_3)}{3} = 84.7239.$$

$$E(AHPwoC_e) = p_1V_1 + p_2V_2 + p_3V_3 + p_{1=2=3}\left(\frac{V_1+V_2+V_3}{3}\right) = 56.1361.$$

An infallible automaton gets $87.5 = V_1 + V_2$ and $62.5 = V_1$, respectively, which means that the AHPwoC technique achieves 96.83% and 89.82% of the maximum possible performance.

5.2. Performance of AHPwC

The AHPwC technique admits a comparison matrix when its consistency ratio verifies $CR < 0.1$ (see section 4). This criterion causes the decision support technique (AHPwC) to reject many priority vectors. In the considered case 24 of the 64 comparison matrices do not verify the consistency criterion, since each comparison matrix has a probability as a function of the probability of the judgments it contains (see STEP 2 above), the probability of rejecting the judgments manifested by the automaton will be the sum of the probabilities of the rejected matrices, which in case study is 0.3141. In other words, the probability that the AHPwC technique will support the judgment expressed by the automaton is 0.6859. From the comparison matrices that pass the consistency criterion, it is obtained:

$$p(O_{123}) = 0.8365; p(O_{132}) = 0.0791; p(O_{213}) = 0.0791;$$

$$p(O_{231}) = 0.0023; p(O_{312}) = 0.0023; p(O_{321}) = 0.0006.$$

It is noticed that the probability of the Error Free ordering experimented an increase of 23.67% while all other error orderings experiment a decrease, which implies that the ordering expected performance of AHPwC technique is superior to that of the AHPwoC technique:

$E(AHPwC_o) = \left(p(O_{123}) + p(O_{213})\right)(V_1 + V_2) + \left(p(O_{132}) + p(O_{312})\right)(V_1 + V_3) + \left(p(O_{231}) + p(O_{321})\right)(V_2 + V_3) = 86.1232.$

Regarding election, the AHPwC technique probability of choosing A_1 as the best alternative is 0.9156, of choosing A_2 as the best alternative is 0.0815, and of choosing A_3 as the best alternative is 0.0030. As with the ordering, the election expected performance with AHPwC is higher than the obtained with the AHPwoC technique:

$$E(AHPwC_e) = p_1V_1 + p_2V_2 + p_3V_3 = 59.2906.$$

The gains in election and ordering of AHPwC in comparison with the AHPwoC amount to just 5.62% and 1.62%, and are due to the very important probability increase of the Error Free ordering achieved by the AHPwC. Indeed, the probability of rejecting the judgment of automaton (0.3141) is distributed as follows: 0.1026 goes to Error Free matrices; 0.0987 to Right-Soft Error matrices; 0.0987 to Medium-Soft Error matrices; 0.0010 to Medium-Hard Error matrices; 0.0010 to Extreme-Soft Error matrices; 0.0004 to Extreme-Hard Error matrices; and 0.0118 to Total Error matrices.

When the judgments expressed are cardinally inconsistent, those judgments are more likely to be wrong than right, therefore the expected performance of eliminating the inconsistent judgments is positive. However, not all inconsistent judgments are incorrect, nor are all correct judgments consistent. For example, $A(1)$ generates a correct priority vector $(0.6880, 0.2397, 0.0723)$ with probability 0.0250, although it is classed as inconsistent because its $CR = 0.3539$. Therefore, the elimination of inconsistent judgments that are correct could lead to performance losses on certain occasions, and this situation must be assumed.

5.3. Sensitivity analysis between automaton expertise and expected performance of consistency

A sensitivity analyses of the Expected Performance for different levels of expert reliability was carried out, which is illustrated in Figure 2. It is noticed that the maximum expected performance difference provided by the consistency requirement is 5.62% in election and 1.62% in ordering when the expertise level is $\beta = 0.7$. However, these modest increases in expected performance require the rejection of 31.41% of all the possible judgments that can be expressed. The alternative approach, i.e., the reparation of inconsistent matrices by improving their consistency via feedback mechanisms, would involve costs that could be superior to the corresponding gain in expected performance [12,57,58].

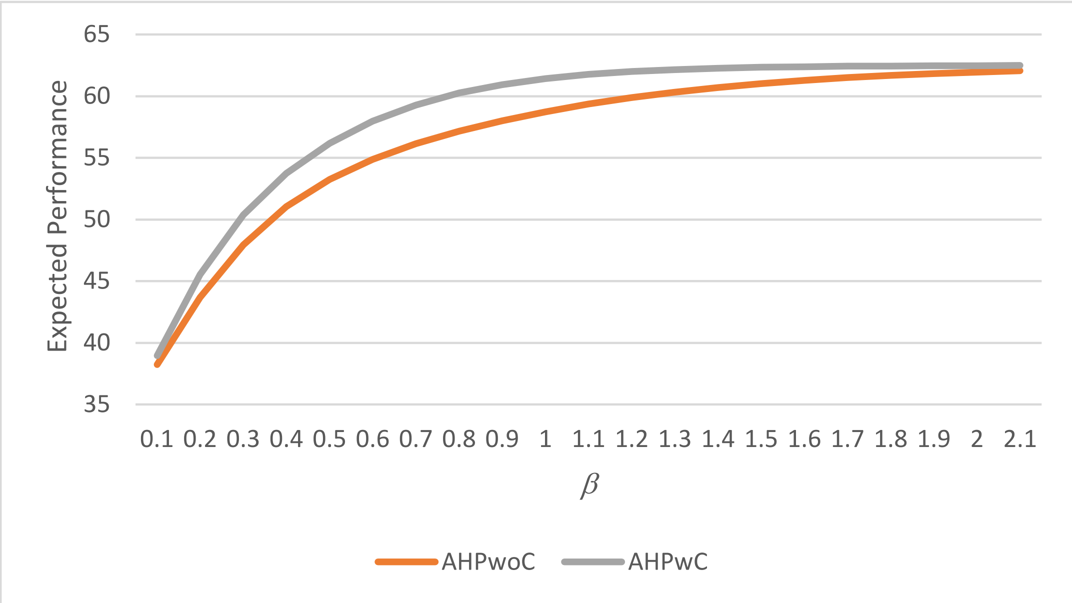

Figure 2: Expected Performance for different levels of reliability (β)

Figure 2 establishes the relationship between the quality of the expert and the contribution of the consistency requirement to decision performance. This result is important because it shows that the performance of the requirement of consistency in judgments depends on the quality of the experts. At the extremes, when the experts are "ignorant" (very low betas, less than 0.1) and when the experts are "sage" (very high betas, greater than 2.1), requiring consistency in judgments does not add performance to the assisting technique to the decision. It could be affirmed that when the a priori selection processes of experts work very well and their wisdom is guaranteed, it is not necessary to demand consistency in their judgments and the same would happen when we are faced with a totally new situation in which there are no experts in the matter.

If the performance of each alternative is unknown, any of the error matrices is indistinguishable from the Error Free matrix, since an ordering with consistency preferences manifestation has traditionally been classified as correct. Only the a priori probability calculations shown in this work allow the evaluation of the decision support technique. Within the probability that the automaton will show a matrix with consistent judgments (proportion of total number of comparison matrices verifying the consistency criterion - in the present case study is 0.6859), the errors are undetectable by the AHP: the probability that the priority vectors supported by this decision support technique are Error Free is 0.8365 $\left(p(O_{123})\right)$, the probability of accepting an ordering that puts A_1 as the best alternative is 0.9156, the probability of electing alternative A_2 is 0.0815, and 0.0030 of electing alternative A_3.

6. Conclusions

Human judgment is a valuable decision-making tool in many fields. This work has addressed problems with latent performance values of alternatives, establishing the difficulties involved in assessing human judgment in this case. To overcome this difficulty, a simulation methodology based on the concept of automaton with intentional bounded rationality is proposed to evaluate a priori decision support techniques. In this conceptual framework, an automaton that can make right or wrong decisions in a controlled way is used to represent the decision maker. The probability of choice by the intentional bounded rationality automaton depends on two factors: the relative difference in performance between alternatives, and its degree of expertise or reliability parameter. The difference in performances capture the complexity of the decision (higher probability of error the closer the performances are), while a higher degree of expertise means that the expert can make more accurate judgments in reaching a decision for the given decision complexity. When a decision support technique is implemented, the correct order of the alternatives cannot be known a priori, but the proposed method allows knowing the a priori reliability provided by the technique used. Without our method, the only thing that can be analysed is the logical level of the relationships expressed by each expert, that is, the level of consistency of each expert.

Decision support techniques seek to analyse the logical relationships that are inferred from the judgments shown by the decision-maker, to detect possible errors, rejecting judgments that do not meet certain pre-established requirements. The results of the case study show how the AHP technique obtains better-expected performances when the consistency proposed by Saaty [15] is required (at most an expected performance of 5.62% higher) but significantly increases the rejection of judgments, which increases the costs of the decision process for its repair (at least 31.41%). This is related to the conclusion of the paper discussion regarding consistency: while a set of free error judgments implies consistency in judgments, consistency in judgments does not imply that the judgements are free error, while inconsistency does not necessarily imply that the judgements are wrong. The methodology proposed makes it possible to detail the probabilities of making different types of error in the two scenarios studied, AHPwoC and AHPwC, by computing the probabilities of orderings being correct and inconsistent, and vice versa. The case study shows how the inconsistency of the AHP technique incorrectly rejects 20.13% of priority vectors that elect the correct best alternative A_1 and correctly rejects 11.28% of the priority vectors that elect another alternative. When the rationality of the decision-maker is different to 0.7 the gains in the expected performance are reduced (as can be seen in Figure 2); when the decision-maker are very expert their judgments are more consistent and the gain in the expected return is reduced, while when the decision makers are very inexpert the rejecting probability increases although it does not imply a greater gain in expected performance. On the other hand, increasing rejections of priority vectors means an increase in decision-making costs; the incorporation of the inconsistency must be compared with the cost that the decision support technique would require. Indeed, rejection of a priority vector supposes further processing to obtain greater reliability of the result: to allow the decision-maker to rectify his judgments (feedback process, change of preferences) or to incorporate another decision-maker into the process.

6.1. Future research and applications

Decision support techniques are a set of mechanisms that help obtain objective judgments from experts and use logical mechanisms that try to provide scientific rigor to treat the individual and collective judgments [8,9]. The intentional bounded rationality method allows evaluating each mechanism of the decision support techniques in an isolated and quantitative way. This level of detail makes it possible to delve into comparisons between decision support techniques, a field of research little studied to date. Most of the comparisons between techniques [21,30–32,54] analyse the differences between them descriptively, and in some cases, empirically illustrate when their application leads to differences in conclusions. The analysis of the expected performances and their costs seems to be a promising line of future research, especially to evaluate novel methods such as the Best-Worst [56] which apparently shows a similar performance to the AHP but with a lower need for resources (judgments of the experts). Another interesting future line of research would be to study automata groups. The methodology would make it possible to analyse the expected performance of different automaton judgment aggregation mechanisms [58–60], as well as the expected gains that would be obtained by adding one more automaton with a given level of expertise (β).

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