Graph model for conflict resolution with internal consensus reaching and external game

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Response to Editor and Reviewers to the Report on SMCA-23-07-2107

Title of the paper: Graph model for conflict resolution with internal consensus reaching and external game

First, we would like to express our sincere thanks to the editor and reviewers for their comments and suggestions. We have revised the paper according to their constructive feedback.

Although there exists some overlap in our responses to the editor and reviewers, we have fully addressed all comments individually without the need to refer to other parts of the response letter.

The changes made in the paper are written in blue font, and the important changes are directly mentioned in our responses to the editor and reviewers below.

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Response to Editor:

General comment: The paper was reviewed by five reviewers. They all find merit in the paper but also point out numerous issues, of which I summarize the main ones. The authors are invited to improve considerably the paper addressing the above issues and all the issues raised by the reviewers.

Response: Thank you very much for your time and efforts in reading and dealing with our manuscript! With your help, we have revised and improved the paper according to the following comments and reviewers' comments.

Comment 1: Lack of consideration of recent references in the field.

Response: Thank you very much for this comment! In the revised paper, we have updated the references by incorporating more recent papers related to graph model and consensus reaching process (see Ref. [30, 31, 40, 43, 48, 49]).

Comment 2: Lack of clarity in the use and definitions of many technical words.

Response: Thank you very much for this insightful comment! As per your comment, we have revised and added some texts to provide further clarity regarding the use and definitions of many technical words. In particular, we have elucidated the concept of "game consensus" and its differences and connections with consensus reaching process, alongside delineating the concepts of "composite conflicting party" and "dual conflict decision-making problems". Please refer to our responses to reviewers for detailed instructions.

Comment 3: Lack of discussion on how to set the many parameters of the framework.

Response: Thank you very much for this important comment! This paper predominantly established two optimization models: minimum deviation model (model (7)) and consensus-reaching model grounded in minimizing preference information loss (model (16)). The parameter involved in these models is the consensus threshold θ, which is utilized in model (16). Upon analyzing model (16), we have observed that the consensus threshold θ significantly affects its objective function. Specifically, as the θ increases, there is a clear upward trend in the objective function's value. Moreover, how to set this parameter is an open problem in real conflict decision-making scenarios. Thus, in the revised version of our paper, we added additional text to discuss this parameter setting issue:

It is obvious that with the rising θ, the objective function's value exhibits an increasing trend. Moreover, the setting of consensus threshold is an open problem in consensus reaching process. Notably, preference learning method is a useful tool to estimate parameters in the field of decision making [33]. Thus, it would be a promising direction to apply this method to tackle consensus threshold configuration challenge.

Please refer to page 6.

Comment 4: Lack of discussion on the practical applicability of the assumptions of the method.

Response: Thank you very much for this useful comment! In the revised paper, we mainly took the following two actions to clarify practical applicability of our proposal.

(1) We added the following texts to elucidate the practical utility of our proposal in managing conflicts within dual-channel supply chains:

The proposed enhanced graph model, which incorporates internal consensus reaching and external game processes, can effectively address practical dual conflict decision-making problems. To demonstrate the efficacy of the proposed approach, an illustrative case study pertaining to dual-channel supply chain conflicts is presented in this section.

Please refer to page 8.

(2) We have expanded upon other potential applications of our proposal:

The proposal of this study can provide the decision support to help composite conflicting party manage dual conflict decision-making problems, and this *capability will be instrumental in addressing supply chain dual conflicts, trade negotiations dual conflicts and environmental dual conflicts.*

Please refer to page 12.

Comment 5: Low quality of the English presentation.

Response: Thank you for this comment! We have polished the language of our paper systematically.

Response to Reviewer 1:

General comment: This paper integrates a graph model and a consensus model to address dual-conflict decision-making problems. Firstly, the best-worst method with comparative linguistic expressions is used to elicit individual preferences regarding game states. Secondly, a preference information loss-based consensus-reaching model is devised to manage preference conflicts. Thirdly, the concept of 'game consensus' with diverse behaviors is introduced to resolve game conflicts among external decision-makers. From my perspective, the manuscript makes significant contributions. The writing and justification of the paper are commendable. It can be considered for publication if the following comments of mine are addressed.

Response: Thank you very much for your positive comments! We have revised and improved this paper according to your insightful comments, and the specific revisions are as follows.

Comment 1: The elicitation of individual preferences over game states is a fundamental aspect of the graph model. Therefore, the use of comparative linguistic expressions requires a strong justification. Furthermore, it would be beneficial to explore whether the proposed graph model framework remains effective when utilizing alternative preference information formats.

Response: Thank you very much for this valuable comment! On one hand, we added the following texts to clarify the advantage of Comparative Linguistic Expressions (CLEs) in the revised paper:

> *Moreover, various linguistic representation models have been developed to capture the uncertain preferences of decision-makers [19, 28, 44]. Among these, Comparative Linguistic Expressions (CLEs) stand out as a preference modeling tool with enhanced flexibility, finding extensive application in GDM [39, 52].*

Please refer to page 2.

On the other hand, considering the advantage of CLEs, we adopted BWM with CLEs to model individuals' preferences. In the revised paper, we rewrote associated texts to clarify the choice of CLEs:

> *Recognizing the challenges individuals face in expressing precise preferences, this study employs linguistic BWM. To ensure generality, the study integrates BWM with CLEs within the graph model framework. This integration aims to streamline the elicitation process of individuals' preferences regarding game states.*

Please refer to page 2.

Comment 2: Given that the concept of 'game consensus' is a key element in this paper, I recommend that the authors provide additional information and elaboration on this concept.

Response: Thank you very much for your important comment! In the revised paper, we provided the following texts to introduce game consensus in detail:

In the framework of game consensus, conflicting parties opting to stay within the current game state can be attributed to various individual behaviors. This stands in contrast to existing global stability concepts, where conflicting parties remaining in the current game state are ascribed to uniform behavior. In this study, we posit that conflicting parties can manifest four distinct behaviors: Nash, GMR, SMR, and SEQ behaviors. As a result, the concept of game consensus can be viewed as a more comprehensive interpretation of established global stability concepts. Specifically, when each conflicting party's choice to persist in the current game state aligns with Nash, GMR, SMR, or SEQ stability, the game consensus assumes the form of global Nash, GMR, SMR, or SEQ stability, respectively. The interrelations among game consensus, global Nash, GMR, SMR, and SEQ stabilities are illustrated in Fig. 3.

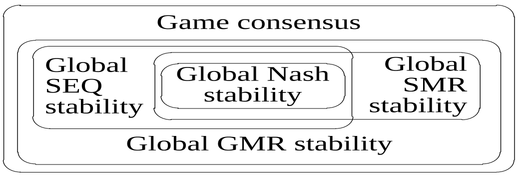

Fig. 3. The interrelationship of different solution concepts within the graph model framework

Please refer to page 7.

Comment 3: In my view, social trust often plays a significant role in practical conflict scenarios, with a substantial impact on the ultimate conflict resolution. Therefore, it is essential to address this aspect when dealing with real-world conflict situations. Regrettably, this paper does not appear to account for the influence of social trust in its analysis.

Response: Thank you for this important comment! In the revised paper, we added some texts to discuss social network:

Social network is a trust relationships structure composed of various individuals, which are widespread in groups, organizations [9, 49]. It is worth *mentioning that dual conflict decision-making problems involve multiple individuals and conflicting parties. Therefore, it is interesting to extend our proposal to the social network context and investigate its impact.*

Please refer to page 11.

Comment 4: I recommend updating the references by incorporating more recent works from the field.

Response: Thank you! In the revised paper, we have updated the references by incorporating recent works regarding to graph model and consensus (see Ref. [30, 31, 40, 43, 48, 49]).

Comment 5: I have observed that the proposed graph model framework relies on various parameters, such as consensus thresholds. However, the paper does not provide guidance on how to set these parameters, which may hinder the practical application of the proposal.

Response: Thank you very much for this important comment! This paper predominantly established two optimization models: minimum deviation model (model (7)) and consensus-reaching model grounded in minimizing preference information loss (model (16)). The parameter involved in these models is the consensus threshold θ, which is utilized in model (16). Upon analyzing model (16), we have observed that the consensus threshold θ significantly affects its objective function. Specifically, as the θ increases, there is a clear upward trend in the objective function's value. Moreover, how to set this parameter is an open problem in real conflict decision-making scenarios. Thus, in the revised version of our paper, we added additional text to discuss this parameter setting issue:

It is obvious that with the rising θ, the objective function's value exhibits an increasing trend. Moreover, the setting of consensus threshold is an open problem in consensus reaching process. Notably, preference learning method is a useful tool to estimate parameters in the field of decision making [33]. Thus, it would be a promising direction to apply this method to tackle consensus threshold configuration challenge.

Please refer to page 6.

Comment 6: The consideration of diverse behaviors in the 'game consensus' concept is a noteworthy contribution of the manuscript. However, the paper should provide guidance on how to determine these behaviors when utilizing the proposed framework.

Response: Thank you very much for this valuable comment! The identification of conflicting parties' heterogeneous behaviors is an important issue of our study. We added some texts to discuss how to identify these heterogeneous behaviors of conflicting parties in the revised paper:

> *Note that it is interesting to investigate how to identify heterogeneous behaviors of conflicting parties. Given that Bayesian inference can effectively predict human behaviors and has been studied in GDM [39], we consider employing Bayesian inference to predict heterogeneous behaviors in future research.*

Please refer to page 7.

Response to Reviewer 2:

Thank you very much for taking the time to provide us with feedback on our manuscript. Your comments have proven to be invaluable in enhancing the quality of our work. We have thoroughly reviewed each of your comments and have integrated them into our revised version. In instances where multiple comments addressed similar points, we have consolidated them in our response below. Additionally, we have reorganized the order of some of comments to ensure clarity and coherence in presenting our revisions.

Comment 1: As shown in Section A. Modeling, you cite the definition of "composite decision maker", the difference of this definition in this paper and previous paper should be explained.

Response: Thank you very much for this valuable comment!

In conflict issues, it often involves multiple conflicting parties. For example, in a demolition compensation conflict, there are mainly three conflicting parties: government, demolition developers and the dismantled. To the best of our knowledge, traditional graph models treat each conflicting party as a whole. However, in real-world conflict decision-making scenarios, a conflicting party may consist of multiple individuals, which is called as the composite decision-maker in existing study (Wu et al., 2021). Drawing on the concept of the composite decision-maker established in existing study paper takes a step further by considering preference divergence issue, and thus developed a consensus reaching process within each conflicting party. To distinguish our work from prior study, we introduce the notion of "composite conflicting party". For example, the demolished serves as a composite conflicting party, as it comprises multiple demolition households, each potentially holding different preferences regarding compensation strategies.

In the revised paper, we added some texts to clarify the difference between composite conflicting party and composite decision maker in Introduction section:

> *Significantly, as each individual may uphold their own distinct value system, the preferences of individuals within a given conflicting party regarding game states frequently diverge. This conflicting party is called "composite conflicting party" in this study. For instance, in the context of demolition compensation conflicts, the demolished party serves as a composite conflicting party, as it comprises multiple demolition households, each potentially holding different preferences regarding compensation strategies.*

Please refer to page 2.

Moreover, we compared composite conflicting party with decision-makers in the existing studies:

Decision-Maker vs. Composite Conflicting Party: A notable distinction between our approach and prevalent graph models lies in the treatment of conflicting parties. Compared with studies on individual decision-maker (e.g., Refs. [3, 4, 10, 13, 14, 41, 47]), this study considers multiple individuals within a composite conflicting party. Distinct from studies on composite decision-maker (e.g., Ref. [42]), our study delves into the preference divergence among individuals within each composite conflicting party, and proposes a consensus model to address this divergence, thereby augmenting the ability to navigate real-world conflict decision-making scenarios. It's important to note that if all composite conflicting party consist of only one individual, our proposed framework reverts to the conventional framework.

Please refer to page 11.

Comment 2: In Abstract, I think your definition of the dual conflict problem is problematic, as each decision-maker naturally considers personal and environmental factors when considering their own preferences. If according to your definition, all conflict issues belong to dual conflict issues.

Response: Thank you very much for this important comment!

Graph model delves into the interaction among multiple conflicting parties, seeking to derives the stability or equilibrium solution for the conflict decision-making problem. As mentioned in the response to Comment 1, our paper investigates composite conflicting parties. Within the realm of composite conflicting parties, conflict decision-making problem gradually emerge duality. This duality signifies that not only composite conflicting parties experience game conflicts stemming from different incongruent pursued objectives, but also internal individuals face preference conflicts due to varying cognitive levels and knowledge backgrounds. The conflict decision-making problem, which simultaneously involves game conflicts and preference conflicts, is termed the dual conflict decision-making problem in our study. To make the concept of dual conflict decision-making problems clear, we mainly took the following actions in the revised paper.

(1) In the revised paper, we rewrote the associated texts in Abstract to clarify dual conflict decision-making problems:

Graph model is devoted to game conflicts arising from incongruent pursued objectives among conflicting parties. Considering that each conflicting party is composed by multiple individuals, preference conflicts stemming from differing cognitive levels and knowledge backgrounds exist among internal individuals. This scenario simultaneously involving game conflicts and preference conflicts is termed the dual conflict decision-making problem.

Please refer to page 1.

(2) We added some texts to clarify dual conflict decision-making problems in Introduction section:

In a conflict decision-making problem, due to the different incongruent pursued objectives, game conflicts exist among external composite conflicting parties, and preference conflicts arise from varying cognitive levels and knowledge backgrounds among internal individuals of a composite conflicting party. This situation is called "dual conflict decision-making problem" in this study, which is commonly observed in daily life. For instance, conflict over compensation emerges between government and the dismantled, and demolition strategy selection conflict arises within demolition households.

Please refer to page 2.

Comment 3: In Abstract, why do decision-makers still need to consider external games when they have reached internal consensus? This is a contradictory issue. The resolution of conflict issues means that all decision-makers reach a consensus, and at this point, external games need to be considered? External games should be considered first, followed by internal ones.

Response: Thank you very much for this precious comment!

Within the framework of graph model, the preferences of each conflicting party regarding game states are the foundation for stability analysis. For example, the scholarly literature (Wu et al., 2021) investigated conflicting party consisting of multiple individuals in graph model. This scholarly literature predominantly involved two predominant steps: firstly, obtaining collective preferences of each conflicting party concerning game states; secondly, conducting stability analysis based on collective preferences. In the former step, preferences of all individuals within each conflicting party are directly aggregated to form collective preferences. Nevertheless, this scholarly literature overlooks the issue of preference divergence (preference conflicts) among multiple individuals within each conflicting party.

Our paper not only considered conflicting party consisting of multiple individuals, but also paid attention to the preference conflicts among individuals within a conflicting party. As the same as Wu et al. (2021), our paper also involved two predominant steps: obtaining collective preferences of each conflicting party concerning game states and stability analysis. In the step of aggregating preferences, we proposed a consensus-reaching model to manage the preference divergence among multiple individuals within each conflicting party, and further to derive the collective preferences of each conflicting party concerning game states. Subsequently, we proposed the concept of "game consensus" for stability analysis based on the collective preferences. Thus, internal consensus should be considered first, followed by external games. In the revised paper, we rewrote the following texts in Abstract to elucidate the sequence of internal consensus and external games:

> *To mitigate preference conflicts inherent to internal individuals within conflicting party concerning game states, a consensus-reaching model minimizing preference information loss is introduced. By this way, collective preferences are obtained. Based on these, the concept of "game consensus" is proposed to manage the game conflicts and the diverse behaviors exhibited by conflicting party.*

Please refer to page 1.

Comment 4: (1) In this paper, you said that the new stability definition of graph model is derived from consensus. However, consensus means that there is no conflict. I think this article needs to explain in detail the differences between consensus and stability definitions. (2) As shown in Section B Consensus reaching framework, what is the difference between consensus and stability? If consensus is reached, why does it still need to resolve the conflict? Isn't this a contradiction? (3) The abstract doesn't feel like studying graph models, but rather studying consensus decision-making. How do you express the differences and connections between the two? (4) As shown in Fig.2, What is the difference between consensus building and stability analysis? In addition, the Figure 2 is that this description is actually a process of reaching consensus in large group decision-making, rather than a conflict analysis process? Is it just moving the existing large group decision-making models over? (5) As shown in B. Consensus reaching model within composite decision-maker, this belongs to consensus research, and what is the connection between it and graph model research.

Response: Thank you very much for these precious comments! Since these comments are mainly about the differences and connections between consensus reaching process and graph model in essence, we organized and provided a unified response to them.

Graph model serves as a useful tool to manage game conflict among conflicting parties. Stability analysis constitutes a core and foundational aspect within the framework of the graph model. It pertains to the examination of conflicting parties' stability within each game state and culminates in deriving the equilibrium solution for the conflict decision-making problems. In our paper, the new stability definition of improved graph model, referred to as "game consensus", is not derived from consensus, which is achieved by extending the scope of existing stability concepts.

The consensus reaching process is a versatile tool to mitigate preference conflicts. Our paper investigates composite conflicting party that consists of multiple individuals. The preference conflicts arising from disparities in knowledge and judgment exist among internal individuals with in a composite conflicting party. In our paper, a consensus-reaching model is proposed to manage the preference conflicts among internal individuals with in a composite conflicting party.

All in all, graph model and consensus reaching process serve as versatile and adaptable tools for mitigating conflicts, while the types of conflicts they handled are different. In the revised paper, we added the following texts to explain the differences and connections between consensus reaching process and graph model in detail:

Graph model and consensus reaching process serve as versatile and adaptable tools for conflict mitigation, while the difference between them lies in the types of conflicts they handled. Stability analysis in graph model derives the equilibrium solution for managing game conflicts among conflicting parties in conflict problems. Consensus reaching process is employed to mitigate preference conflicts among individuals within each conflicting party.

Please refer to page 4.

Moreover, the graph model chiefly encompasses two primary stages: the modeling process and the stability analysis process. Fig. 2 presents the modeling process (Modeling) and stability analysis process (Game consensus analysis among external composite decision-makers). Moreover, Fig. 2 incorporate internal consensus reaching process into traditional graph model framework. Thus, Fig. 2 is a conflict analysis process, rather than a process of reaching consensus in large group decision-making. In the revised paper, we rewrote the following texts to clarify that Fig.2 is the description of conflict analysis process:

To address the dual conflict decision-making problem involving game and preference conflicts, we integrate intra-composite decision-maker consensus reaching into the framework of graph model, and propose an enhanced graph model (refer to Fig. 2).

Please refer to page 4.

Comment 5: Please restate the theoretical connotation and specific types of graph models developed in the abstract.

Response: Thank you very much for this comment! Our paper introduced consensus reaching process into graph model, and proposed an enhanced graph model to address dual conflict decision-making problems. In the revised paper, we added the following texts to clarify theoretical connotation and specific types of the proposed enhanced graph models in Abstract:

Compared to existing graph model, the proposal effectively grapples with consensus issues and heterogeneous behaviors within conflicting parties, making it more valuable in practice.

Please refer to page 1.

Comment 6: In this paper, you only provided a new game consensus of Definition 9, but cannot describe the stability definition generated by the interaction process between decision-makers.

Response: Thank you very much for this valuable comment! Because the concept of game consensus captures the diverse behaviors of decision-makers in achieving stability, the analysis of the behaviors interaction process should be grounded in the specific conflict problems under consideration. Thus, in defining game consensus, it is not imperative to outline the interaction process between decision-makers. In the revised paper, we incorporate the following texts to elucidate both the behaviors interaction process among decision-makers and the linkage between game consensus and established stability concepts:

In the framework of game consensus, conflicting parties opting to stay within the current game state can be attributed to various individual behaviors. This stands in contrast to existing global stability concepts, where conflicting parties remaining in the current game state are ascribed to uniform behavior. In this study, we posit that conflicting parties can manifest four distinct behaviors:

Nash, GMR, SMR, and SEQ behaviors. As a result, the concept of game consensus can be viewed as a more comprehensive interpretation of established global stability concepts. Specifically, when each conflicting party's choice to persist in the current game state aligns with Nash, GMR, SMR, or SEQ stability, the game consensus assumes the form of global Nash, GMR, SMR, or SEQ stability, respectively. The interrelations among game consensus, global Nash, GMR, SMR, and SEQ stabilities are illustrated in Fig. 3.

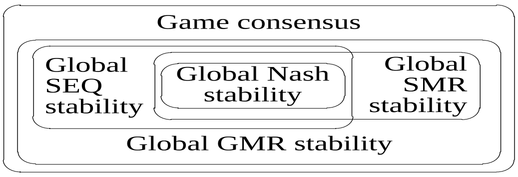

Fig. 3. The interrelationship of different solution concepts within the graph model framework

Please refer to page 7.

Response to Reviewer 3:

General comment: This study focused on the conflict mediation under a dual conflict context, in which a consensus reaching model is proposed to deal with the preference conflict within each conflicting body and the game consensus is devised to address the conflict among conflicting bodies. I think the study makes significant contributions compared to existing graph models, and I've enjoyed reading this manuscript. Overall, the paper has been well-written and well-justified. For its publication in this prestigious journal, some further revisions and improvements are needed. I provided the following comments for the authors' reference.

Response: Thank you very much for your positive comments! We have revised and improved this paper according to your valuable comments, and the specific revisions are as follows.

Comment 1: Authors proposed the concept of "composite decision-makers" in the graph model, and I think it is consistent with the practical situations. However, the explanation about this concept is not clear enough. Therefore, more information should be included for better understanding of this concept for readers. Using a small example in the introduction section may be helpful to fix this issue.

Response: Thank you very much for this precious comment! In the revised paper, to distinguish our work from prior study, we introduce the notion of composite conflicting party. We rewrote associated texts and provided an example to explain the concept of "composite conflicting party" clearly:

> *Significantly, as each individual may uphold their own distinct value system, the preferences of individuals within a given conflicting party regarding game states frequently diverge. This conflicting party is called "composite conflicting party" in this study. For instance, in the context of demolition compensation conflicts, the demolished party serves as a composite conflicting party, as it comprises multiple demolition households, each potentially holding different preferences regarding compensation strategies.*

Please refer to page 2.

Comment 2: The selection of preference modeling methods is a basic issue in the area of decision making and graph model for conflict resolution, and many associated methods have been reported, such as different linguistic assessment models. The use of comparative linguistic expressions needs to be well explained. Moreover, the same linguistic assessment often denotes different meanings for different individuals. However, this issue is not considered when using comparative linguistic expressions in your proposed graph model.

Response: Thank you very much for this valuable comment! On one hand, we added the following texts to clarify the advantage of Comparative Linguistic Expressions (CLEs) in the revised paper:

Moreover, various linguistic representation models have been developed to capture the uncertain preferences of decision-makers [19, 28, 44]. Among these, Comparative Linguistic Expressions (CLEs) stand out as a preference modeling tool with enhanced flexibility, finding extensive application in GDM [39, 52].

Please refer to page 2.

On the other hand, considering the advantage of CLEs, we adopted BWM with CLEs to model individuals' preferences. In the revised paper, we rewrote associated texts to clarify the choice of CLEs:

Recognizing the challenges individuals face in expressing precise preferences, this study employs linguistic BWM. To ensure generality, the study integrates BWM with CLEs within the graph model framework. This integration aims to streamline the elicitation process of individuals' preferences regarding game states.

Please refer to page 2.

In addition, we added the following texts to discuss the personalized individual semantics in the revised paper:

In reality, personalized individual semantics (PIS) are widely present in linguistic GDM [28, 51, 52]. Furthermore, individuals may exhibit deceptive preferences to further their own interests, a phenomenon referred to as strategic behavior [32]. Therefore, we contend that integrating PIS and strategic behaviors into future research would be valuable, rendering the associated studies more practical.

Please refer to page 11.

Comment 3: In most of the existing graph models, the global stabilities are based on the homogenous behaviors of decision makers. In contrast, this paper delves into heterogeneous behaviors of decision makers in global stabilities, which makes some obvious theoretical contributions. However, the fundamental issue of identifying these heterogeneous behaviors is not addressed by this study. So, I suggest adding some discussions about this issue.

Response: Thank you very much for this useful comment! The identification of conflicting parties' heterogeneous behaviors is an important issue of our study. We added some texts to discuss how to identify these heterogeneous behaviors of conflicting parties in the revised paper:

Note that it is interesting to investigate how to identify heterogeneous behaviors of conflicting parties. Given that Bayesian inference can effectively predict human behaviors and has been studied in GDM [39], we consider employing Bayesian inference to predict heterogeneous behaviors in future research.

Please refer to page 7.

Comment 4: This paper is founded on the assumption that decision makers possess complete knowledge of their opponents' preferences. However, in practical situations, the above assumption usually does not come into existence, and individuals may only be privy to a fraction of their opponents' preferences when making decisions. It would be valuable to discuss the potential extension of your proposal to address such scenarios.

Response: Thank you very much for this valuable comment! We added the following texts to discuss this issue in the revised paper:

Within the framework of existing graph models, it is typically assumed that conflicting parties possess complete knowledge of their opponents' preferences, a scenario rarely encountered in real conflict decision-making. Hence, there is a need to broaden the current notion of stability, for example Minimax regret stability [40], to render it more applicable to real-world scenarios.

Please refer to page 11.

Comment 5: In conflict decision making problems, decision makers usually present strategic behaviors in order to obtain their own interests. I strongly recommend that the authors add relevant content to discuss this issue.

Response: Thank you very much for this precious comment! We added the following texts to discuss strategic behaviors in the revised paper:

Furthermore, individuals may exhibit deceptive preferences to further their own interests, a phenomenon referred to as strategic behavior [32]. Therefore, we contend that integrating PIS and strategic behaviors into future research would be valuable, rendering the associated studies more practical.

Please refer to page 11.

Comment 6: The background of case study over the dual-channel supply chain is insufficient. It is necessary to enhance and specify the background of this case study.

Response: Thank you for this important comment! In the revised paper, we rewrote associated texts to enhance and specify the background of case study.

The rapid advancement of E-commerce technology has ushered in a paradigm shift in the way businesses engage with consumers, further making a greater diversity in product sales channels. To tap into the vast and ever-expanding online consumer for more market share, many manufacturers are increasingly proactive in diversifying their sales strategies by establishing their online stores or participating in third-party E-commerce platforms. In this evolving landscape, manufacturers find themselves navigating between two primary sales avenues: the traditional distribution channel and the online direct channel. In the former sales avenue, manufacturers sell products to consumers through intermediary retailers, while in the latter sales avenue, manufacturers sell products directly to consumers via self-operated online stores or third-party E-commerce platforms. Consumers usually compare the prices of the two sales avenues, especially when a manufacturer's online direct channel offers the same product as a retailer's traditional distribution channel. Therefore, the two sales avenues will provide different discount strategies to attract consumers, and potential price conflicts between these two avenues may arise.

Please refer to page 8.

Response to Reviewer 4:

General comment: This article examined a highly intriguing research issue, namely network models with consensus. The key contributions of it lie in the suggested enhanced graph model, which integrates internal consensus reaching and external game processes and can successfully solve dual conflict decision-making difficulties. The article achieves tremendous achievement by merging the consensus reaching and graph model for dispute resolution. Despite its significant contributions, I observed that the following concerns need to be further explored and addressed.

Response: Thank you very much for your insightful comments! We have revised and improved this paper according to your insightful comments.

Comment 1: This article developed the notion of "game consensus", which is quite noteworthy. However, the distinction between this notion and existing solution concepts are not evident enough. More clarifications are needed.

Response: Thank you very much for this important comment! In the revised paper, we added some texts to clearly explain the notion of "game consensus" and discuss the distinction and relationship between game consensus and existing stability concepts:

In the framework of game consensus, conflicting parties opting to stay within the current game state can be attributed to various individual behaviors. This stands in contrast to existing global stability concepts, where conflicting parties remaining in the current game state are ascribed to uniform behavior. In this study, we posit that conflicting parties can manifest four distinct behaviors: Nash, GMR, SMR, and SEQ behaviors. As a result, the concept of game consensus can be viewed as a more comprehensive interpretation of established global stability concepts. Specifically, when each conflicting party's choice to persist in the current game state aligns with Nash, GMR, SMR, or SEQ stability, the game consensus assumes the form of global Nash, GMR, SMR, or SEQ stability, respectively. The interrelations among game consensus, global Nash, GMR, SMR, and SEQ stabilities are illustrated in Fig. 3.

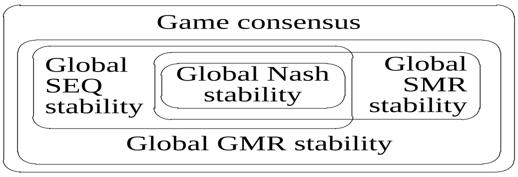

Fig. 3. The interrelationship of different solution concepts within the graph model framework

Please refer to page 7.

Comment 2: In this publication, the preference conflict issue between the persons inside each conflicting portion is handled utilizing the consensus reaching model. To my knowledge, the social trust network has been frequently regarded in the consensus reaching. I suppose the social trust also exists in your graph model. However, this problem is not examined in your proposal.

Response: Thank you for this important comment! In the revised paper, we added some texts to discuss social network:

Social network is a trust relationships structure composed of various individuals, which are widespread in groups, organizations [9, 49]. It is worth mentioning that dual conflict decision-making problems involve multiple individuals and conflicting parties. Therefore, it is interesting to extend our proposal to the social network context and investigate its impact.

Please refer to page 11.

Comment 3: The article offers some theoretical contributions in the domain of graph model. However, the description of the application value of your idea is not adequate.

Response: Thank you very much for this useful comment! In the revised paper, we mainly took the following two actions to clarify practical applicability of our proposal.

(1) We added the following texts to elucidate the practical utility of our proposal in managing conflicts within dual-channel supply chains:

The proposed enhanced graph model, which incorporates internal consensus reaching and external game processes, can effectively address practical dual conflict decision-making problems. To demonstrate the efficacy of the proposed approach, an illustrative case study pertaining to dual-channel supply chain conflicts is presented in this section.

Please refer to page 8.

(2) We have expanded upon other potential applications of our proposal:

The proposal of this study can provide the decision support to help composite conflicting party manage dual conflict decision-making problems, and this capability will be instrumental in addressing supply chain dual conflicts, trade negotiations dual conflicts and environmental dual conflicts.

Please refer to page 12.

Comment 4: The English quality should be further enhanced. Moreover, please carefully examine the material to avoid the possible errors.

Response: Thank you for your comment! We have improved the English quality of this paper and checked all material carefully.

Comment 5: The literature study over the graph model should be updated by incorporating some fresh current investigations.

Response: Thank you! We have updated the references as per your suggestion, incorporating recent works related to graph model (Ref. [40, 43]).

Comment 6: Moving the proofs of all Propositions to the appendix is a practical and effective technique to simplify the main text of the manuscript.

Response: The proofs have been moved to Appendix A. Thank you!

Response to Reviewer 5:

General comment: The paper offers a comprehensive graph model for conflict resolution, combining internal consensus reaching and external game theory. While the paper is well-structured and intriguing, several key revisions are needed to elevate its quality:

Response: Thank you very much for your positive comments! We have revised and improved the paper according to your valuable comments, and the specific revisions are as follows.

Comment 1: The authors should provide a more explicit rationale for the use of linguistic distributions to represent CLEs, as this choice is not entirely clear in the current context.

Response: Thank you very much for this valuable comment! On one hand, we added the following texts to clarify the advantage of Comparative Linguistic Expressions (CLEs) in the revised paper:

Moreover, various linguistic representation models have been developed to capture the uncertain preferences of decision-makers [19, 28, 44]. Among these, Comparative Linguistic Expressions (CLEs) stand out as a preference modeling tool with enhanced flexibility, finding extensive application in GDM [39, 52].

Please refer to page 2.

On the other hand, considering the advantage of CLEs, we adopted BWM with CLEs to model individuals' preferences. In the revised paper, we rewrote associated texts to clarify the choice of CLEs:

Recognizing the challenges individuals face in expressing precise preferences, this study employs linguistic BWM. To ensure generality, the study integrates BWM with CLEs within the graph model framework. This integration aims to streamline the elicitation process of individuals' preferences regarding game states.

Please refer to page 2.

Comment 2: In Equations 5-6, replacing $\sum_{t=0}^{g}$ with more descriptive terms such as $l_t \in TF(a_{bi})$ and $l_t \in TF(a_{iw})$ would enhance clarity and comprehension.

Response: We fixed this issue, thank you!

Comment 3: Is it possible that the optimization models do not have optimal solutions? If yes, how to deal with such situations? Eq. (8), all other constraints in model (7) may cause confusions. Please make changes.

Response: Thank you very much for this important comment!

The first optimization model (model (7)) in our paper is designed to minimize the deviation between the comparison vectors and preference vectors. For the convenience of solving, model (7) is converted into the linear programming model (8), where c_{bi}, d_{iw}, p_i, f_{bi} and f_{iw} are decision variables. We have observed that the feasible region of model (8) is consistently non-empty. For example, when setting $p_i = 1/n \ (i = 1,...,n)$, $f_{bi} = 1$, $f_{iw} = 0$, $c_{bi} = 0.5$, $b_{iw} = 0.5$, all constraints of model (8) are satisfied, which means that the feasible region of model (8) is consistently non-empty. Furthermore, both the objective function and constraints of model (8) are bounded. Thus, it is obvious that model (8) has feasible solution.

The consensus-reaching model grounded in minimizing preference information loss (model (16)) is another optimization model in our paper, with its equivalent transformed linear programming model designated as model (17). Analogous to the analysis of model (8), model (17) also guarantees the existence of feasible solution. Hence, the optimization models featured in our paper possess optimal solutions.

In addition, in the revised paper, we have modified Eq. (8) based on your comment to make it clear.

Comment 4: To enrich the literature review, consider incorporating related studies on consensus-reaching processes. Examples include "Consensus reaching for ordinal classification-based group decision making with heterogeneous preference information," "Threshold-based value-driven method to support consensus reaching in multicriteria group sorting problems: A minimum adjustment perspective," and "Consensus reaching for MAGDM with multi-granular hesitant fuzzy linguistic term sets: a minimum adjustment-based approach."

Response: We have enriched the literature review as per your suggestion (see Refs. [30, 31, 48]). Thank you!

Comment 5: Given the presence of numerous parameters in the paper, it may be beneficial to include sensitivity analyses for these parameters within the numerical results. This would provide a deeper understanding of how changes in parameters impact the model.

Response: Thank you for your important comment! The paper predominantly established two optimization models: minimum deviation model (model (7)) and consensus-reaching model grounded in minimizing preference information loss (model (16)). The only parameter involved in these models is the consensus threshold θ, which is utilized in model (16). Upon analyzing model (16), we have observed that the consensus threshold θ significantly affects its objective function. Specifically, as the θ increases, there is a clear upward trend in the objective function's value. Regrettably, due to limitation of paper's layout, we cannot provide sensitivity analyses consensus threshold θ. However, in the revised version of our paper, we have included additional text to elucidate the impact of the consensus threshold on model (16):

It is obvious that with the rising θ, the objective function's value exhibits an increasing trend.

Please refer to page 6.

Comment 6: The language should undergo further refinement to eliminate any typographical errors and enhance grammatical accuracy.

Response: Thank you very much for reading our paper carefully. We double checked the whole paper to polish the language.

Comment 7: These revisions, if implemented, will contribute to the paper's overall quality and impact.

Response: Thank you very much for your positive comments again!

Graph model for conflict resolution with internal consensus reaching and external game

Hengjie Zhang, Fang Wang, Rong Zhang, Yucheng Dong, Francisco Chiclana, and Enrique Herrera-Viedma

***Abstract*—Graph model is devoted to game conflicts arising from incongruent pursued objectives among conflicting parties. Considering that each conflicting party is composed by multiple individuals, preference conflicts stemming from differing cognitive levels and knowledge backgrounds exist among internal individuals. This scenario simultaneously involving game conflicts and preference conflicts is termed dual conflict decision-making problem. Tailored to effectively address this problem, this study proposes an enhanced graph model that incorporates internal consensus and external stability. The best-worst method, incorporating comparative linguistic expressions, is devised to effectively elicit individual preferences over game states. To mitigate preference conflicts inherent to internal individuals within conflicting party concerning game states, a consensus-reaching model minimizing preference information loss is introduced. By this way, collective preferences are obtained. Based on these, the concept of "game consensus" is proposed to manage the game conflicts and the diverse behaviors exhibited by conflicting party. Finally, a case study regarding price conflict within a dual-channel supply chain, accompanied by a comparative analysis, is presented to validate the effectiveness of the proposal. Compared to existing graph model, the proposal effectively grapples with consensus issues and heterogeneous behaviors within conflicting parties, making it more valuable in practice.**

***Index Terms*—Group decisions and negotiations; conflict analysis; graph model; consensus; optimization model**

I. Introduction

As society and information technology continue to advance, the interconnectedness among individuals, organizations, and nations is becoming increasingly seamless. Conflict-related matters persistently arise in various spheres of human activities, encompassing domains like economy, trade, healthcare, military operations, and the environment (e.g., [21]). Consequently, managing conflicts has emerged as a complex challenge faced by individuals, organizations, governments, and scholars alike. To facilitate conflict analysis and explore potential resolution approaches, a variety of formal methodologies have been devised, including game theory [35], metagame analysis [24], conflict analysis [11], drama theory [23], and the graph model [26].

Game theory stands out as one of the predominant methods for conflict analysis, despite its requirement for precise payoff information from conflicting parties, which is often difficult to obtain [21]. A convenient and adaptable alternative model to analyze conflict is the game theory and partial game theory based graph model [26]. In comparison with classical game theory, graph model serves as a conflict management tool that amalgamates qualitative and quantitative data, thereby reducing the demand for extensive information [20]. To put it differently, the graph model demonstrates robust applicability in management practice by relying on relative preference information, enabling optimization even with limited available data [21]. This motivates the use of the graph model in this study to address conflict decision-making issue.

Over the last few decades, substantial global endeavors have been directed towards enhancing and implementing the graph model, with particular emphasis on the following facets:

(1) *Preference modeling.* Preference modeling constitutes the central and foundational task within the framework of the graph model. Its primary objective is to analyze conflicting parties' preferences regarding game states. Research regarding to preference modeling within graph model has predominantly focused on refining various preference representation structures. In the original graph model theory, the focus was limited to two elementary preference representation structures [10]: superior ($\succ$) and equivalent ($\approx$). Hamouda et al. [14] proposed a preference representation structure enriched with intensity information, extending the conventional simple preference representation to a triad of relations that gauges the degree of preference among game states. In a similar vein, Li et al. [29] proposed uncertain preference representation structure within the graph model. Concurrently, Bashar et al. [4] and Wu et al. [41] proposed preference modeling method built upon additive preference relations.

(2) *Stability analysis*. Stability analysis constitutes another core and foundational aspect within the framework of the graph model. It pertains to the examination of conflicting parties' stability within each game state and culminates in deriving the equilibrium solution for the conflict decision-making problems. Existing research within graph model primarily predominantly focuses on four fundamental stabilities [12, 25]: Nash stability, General Metarationality (GMR) stability, Symmetric Metarationlity (SMR) stability and Sequential (SEQ) stability. Subsequently, certain scholars extended the concepts of these four fundamental stabilities. Walker et al. [6] considered conflicting parties' attitude tendencies (positive, negative and neutral) and introduced the notion of stability

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incorporating these attitudes, termed as RNash stability, RGMR stability, RSMR stability and RSEQ stability. Further, Sabino and Rêgo [40] designed minimax regret stability based on Savage rule and variable horizon.

(3) *Application of the graph model*. Some research explored the application of the graph model, and have successfully implemented them in various fields such as supply chain conflicts, trade negotiations, water resource disputes, and environmental conflicts (e.g., [20-22]). Li et al. [29] employed the graph model featuring an uncertain preference representation structure in the context of environmental conflicts. He et al. [15] developed a comprehensive hierarchical graph model framework and applied it to negotiate greenhouse gas emissions between China and the United States. Aljefri et al. [1] employed the graph model to address the challenge of conflict decision-making related to water resource management.

While the existing research on the graph model provides valuable insights, there are unresolved issues that require further attention to effectively address real-world conflict decision-making challenges:

(1) As the complexity of conflict decision-making issues continues to grow, the development of an effective methodology for modeling conflicting parties' preferences remains an open question that merits thorough exploration. Within the realm of preference modeling, significant strides have been taken in the domain of group decision making (GDM) (e.g., [44]). Thus, by adopting a more adaptable approach informed by these recent advancements, the preference modeling framework within the graph model can be enhanced to capture conflicting parties' preferences pertaining to game states more comprehensively.

(2) Conflict decision-making quandaries encompass multiple conflicting parties, some of which comprise numerous individuals, often referred to as composite decision-makers in scholarly literature (see [41]). Significantly, as each individual may uphold their own distinct value system, the preferences of individuals within a given conflicting party regarding game states frequently diverge. This conflicting party is called "composite conflicting party" in this study. For instance, in the context of demolition compensation conflicts, the demolished party serves as a composite conflicting party, as it comprises multiple demolition households, each potentially holding different preferences regarding compensation strategies. Nonetheless, the majority of prevailing graph models designed for conflict resolution treat each conflicting party or stakeholder as an indivisible entity, consequently falling short of adequately addressing the intricacies of real-world conflict decision-making predicaments.

(3) In a conflict decision-making problem, due to the different incongruent pursued objectives, game conflicts exist among external composite conflicting parties, and preference conflicts arise from varying cognitive levels and knowledge backgrounds among internal individuals of a composite conflicting party. This situation is called "dual conflict decision-making problem" in this study, which is commonly observed in daily life. For instance, conflict over compensation emerges between government and the dismantled, and demolition strategy selection conflict arises within demolition households. However, most of existing studies assumed that conflicts are singular in nature, and primarily concentrated on investigating a single method for conflict resolution, overlooking the challenges posed by fuzzy dual conflict decision-making problems.

(4) Within the prevailing graph models, global stabilities such as Nash stability, GMR stability, SMR stability, and SEQ stability have been employed to analyze potential solutions for conflict decision-making quandaries. However, the existing concepts of global stabilities are predicated on the assumption that conflicting parties possess uniform behaviors in terms of their foresight abilities and risk attitudes. In practice, conflicting parties often demonstrate heterogeneous behaviors. Therefore, there is a pressing need to broaden the concept of global stability by integrating the diversity in conflicting parties' behaviors, encompassing variations in foresight abilities and risk attitudes.

As previously discussed, advancements in preference modeling and managing preference conflicts have been realized within the realm of GDM (e.g., [18, 52]). Among the recently developed preference modeling techniques, the Best-Worst Method (BWM) [8, 34, 37] stands out. This method employs ratios derived from pairwise comparisons of the relative significance of objectives, utilizing vectors known as the Best-to-Others (BO) vector and Others-to-Worst (OW) vector. In contrast to frequently employed methods reliant on pairwise comparisons (such as multiplicative preference relations and additive preference relations), BWM not only demands fewer data inputs but also yields decision-making outcomes that are more dependable. Moreover, various linguistic representation models have been developed to capture the uncertain preferences of individuals [19, 28, 44]. Among these, Comparative Linguistic Expressions (CLEs) stand out as a preference modeling tool with enhanced flexibility, finding extensive application in GDM [38, 52]. In the context of managing preference conflicts, consensus-reaching models have been formulated and extensively employed to alleviate discord in preferences among individuals arising from variations in cognitive levels and knowledge backgrounds (e.g., [2, 16, 30 31]). In GDM, there have been reports of optimization-based consensus-reaching models that enhance the efficiency of consensus attainment, with a focus on minimizing preference information loss or consensus cost [5, 7, 27, 45, 48, 53].

Driven by the imperative to address aforementioned challenges inherent to existing graph models, and drawing inspiration from the progress made in preference modeling and managing preference conflicts within GDM, this study proposes an enhanced graph model featuring internal consensus reaching and external game processes, tailored to confront the intricacies of dual conflict decision-making. Specifically, the study undertakes following tasks:

(1) In the original framework of the BWM, a 1-9 scale is employed to formulate comparison vectors like BO and OW vectors. Recognizing the challenges individuals face in expressing precise preferences, this study employs linguistic BWM. To ensure generality, the study integrates BWM with CLEs within the graph model framework. This integration aims to streamline the elicitation process of individuals' preferences regarding game states.

(2) A consensus-reaching model, rooted in minimizing preference information loss, is formulated to tackle preference conflicts among the internal individuals within each composite conflicting party. The primary objective of this model is to curtail the information loss occurring between the individuals' comparison vectors (BO and OW vectors) and their respective individual preference vectors concerning game states. In this model, CLEs are converted into Linguistic Distribution Assessments (LDAs) within the framework of BWM, leading to the derivation of a consensual preference vector over game states for the composite conflicting party.

(3) Considering the heterogeneous behaviors of conflicting parties stemming from variations in foresight abilities and risk attitudes, a global stability concept namely "game consensus" is

introduced. Within the framework of the graph model, a game state is deemed to exhibit game consensus if it sustains stability for all conflicting parties based on their heterogeneous stability behaviors. This concept serves to delve into the game process involving external conflicting parties, achieved by broadening the applicability of existing stability concepts (namely, Nash stability, GMR stability, SMR stability, and SEQ stability).

(4) An illustrative case study pertaining to the management of conflict within a dual-channel supply chain is presented to demonstrate the efficacy of the proposed approach.

The subsequent sections of this study unfold as follows: Section II provides a foundational understanding of the graph model for conflict resolution, consensus-reaching models, and linguistic assessment models. The framework of the graph model that incorporates internal consensus reaching and external game processes is outlined in Section III. Section IV employs a case study to showcase the effectiveness of the proposed methodology. Section V conducts a comparative analysis between our proposal and pertinent graph models. Finally, section VI concludes the study by summarizing the key findings and main conclusions.

II. Preliminaries

The rest of the study relies on the basic knowledge related to graph model for conflict resolution, consensus-reaching model, and various extensively employed linguistic assessment models.

A. Graph model for conflict resolution

The graph model serves as a versatile and adaptable tool for analyzing conflicts, offering pragmatic solutions by leveraging relative preference information concerning the game state. This information is gathered from multiple conflicting parties with divergent objectives, as demonstrated in references [10, 26]. In the following, we offer a concise elucidation of the graph model, along with a delineation of its procedural analysis.

The decision-making process for conflict analysis grounded in graph model theory chiefly encompasses two primary stages [10, 20, 26, 43]: the modeling process and the stability analysis process. The central objectives within the modeling process encompass: identifying the conflicting parties, scrutinizing the strategy or option sets of diverse conflicting parties, ascertaining viable game states, sketching the diagrams for transitioning between game states, and establishing the preference information of conflicting parties concerning these game states. The main tasks within the stability analysis phase involve: conducting individual stability analysis and subsequently performing group stability analysis.

Let $E=\{E^1,...,E^m\}$ $(m\geq 2)$ be the set of conflicting parties. A graph model is often characterized through the representation of conflicting parties' options, where an option for each conflicting party represents a potential course of action they can choose to pursue or reject. A game state is defined as a configuration of options that mirrors the strategic choices of the involved conflicting parties, each striving to achieve their individual objectives. Let $S=\{s_1,...,s_n\}$ $(n\geq 2)$ denote the set of feasible game states. The collection of moves under the control of conflicting party $E^k\in E$ is denoted by T^k, wherein $(s_i, s_j)\in T^k$ signifies that decision-maker $E^k\in E$ has the capability to transition the conflict from state s_i to state s_j. Let $D^k=(S, T^k)$ be the directed graph of conflicting party E^k, and $\succsim_k$ record conflicting party E^k's crisp preference over S.

Let $R^{k,+}(s)$ denote the collection of all one-sided enhancements from state $s\in S$ linked with E^k, where $s_i\in R^{k,+}(s)$ implies that $(s, s_i)\in T^k$ and $s_i\succ_k s$. Thus, a crisp graph model can be formally represented as $<E, S, \{D^k,\succsim_k\}: E^k\in E>$ (e.g., [10, 20, 26]).

In the following, we introduce four fundamental stability concepts: Nash stability, GMR stability, SMR stability, and SEQ stability.

Definition 1: *Nash stability* [12, 25]. The game state s_t is considered Nash stability for conflicting party E^k iff $R^{k,+}(s_t)=\phi$.

Definition 2: *GMR stability* [12, 25]. The game state s_t is regarded as GMR stability for conflicting party E^k iff, for every $s_i\in R^{k,+}(s_t)$, there exists an $s_j\in R^{M/k}(s_i)$ such that $s_t\succsim_k s_j$.

Definition 3: *SMR stability* [12, 25]. The game state s_t is considered SMR stability for conflicting party E^k iff, for every $s_i\in R^{k,+}(s_t)$, there exists an $s_j\in R^{M/k}(s_i)$ such that $s_t\succsim_k s_j$, and $s_t\succsim_k s_l$ holds for all $s_l\in R^k(s_j)$.

Definition 4: *SEQ stability* [12, 25]. The game state s_t is recognized as SEQ stability for conflicting party E^k iff, for every $s_i\in R^{k,+}(s_t)$, there exists an $s_j\in R^{M/k,+}(s_i)$ such that $s_t\succsim_k s_j$.

A game state that maintains stability for all conflicting party according to a particular stability definition is referred to as a global stability within that specific definition.

B. Consensus reaching framework

The consensus reaching model has found widespread application in various GDM issues, aiming to alleviate preference conflicts arising from disparities in cognitive levels and knowledge backgrounds [9]. While a multitude of consensus-reaching models have emerged in recent times, they adhere to a common framework comprising four key stages [17, 53] (refer to Fig. 1.): (1) furnishing original or adjusted preferences using a distinct preference presentation structures (e.g., additive/multiplicative/linguistic preference relations), (2) employing an aggregation function to amalgamate individual preferences into a unified collective preference, (3) quantifying consensus level among decision-makers by applying a consensus measure technique, and (4) implementing a feedback mechanism to propose preference adjustments, guiding decision-makers to modify their preferences when consensus is deemed unacceptable.

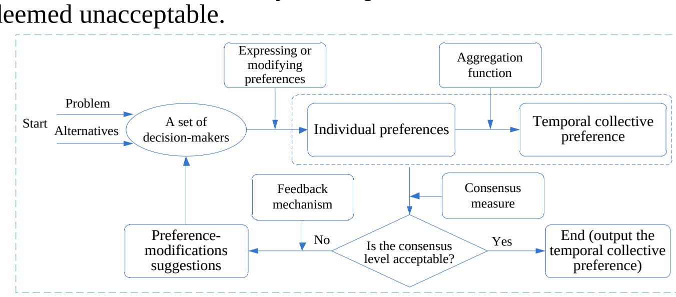

Fig. 1. The general framework for achieving consensus [17, 53]

Notably, the feedback mechanism is often propelled by the implementation of two categories of consensus rules:

(1) Identification and Direction Consensus Rules (e.g., [16, 17]). The identification rule is employed to recognize entities (such as decision-makers, alternatives and preference values) exhibiting weak consensus levels, while the direction rule provides guidance for preference modifications concerning the identified entities.

(2) Optimization-based Consensus Rules (e.g., [53]), with the goal of minimizing discrepancies between original and adjusted preferences or consensus costs. Notably, two types of consensus-reaching models with optimization-based consensus rules emerge based on the "Feedback versus No Feedback" criterion [53].

Zhang et al. [50] conducted an extensive comparative analysis of diverse consensus reaching models, revealing that consensus reaching models utilizing optimization-based consensus rules exhibit superior consensus efficiency compared to those employing identification and direction consensus rules, particularly in terms of minimizing preference information loss. Hence, in this study, the optimization-based consensus reaching model is employed to tackle preference conflicts among internal individuals within a distinct composite conflicting party.

C. Linguistic assessments

Herein, we present a range of extensively utilized linguistic assessment models, encompassing concepts such as CLEs, hesitant fuzzy linguistic term sets (HFLTSs) and LDAs.

(1) Comparative linguistic expressions

Let $L=\{l_t \mid t=0,1,...,g\}$ represent a set of $g+1$ linguistic terms, where $l_i \geq l_j$ iff $i \geq j$. Herrera et al. [19] presented a comprehensive introduction of linguistic variables. Rodríguez et al. [38] formulated a context-free grammar-based methodology to elicit CLEs.

Definition 5 (Context-free grammar) [38]. Let L be as defined previously. A context-free grammar is represented by a 4-tuple $G_H(V_N, V_T, I, PR)$, where V_N is a set of nonterminal symbols, V_T is a set of terminal symbols, I is the starting symbol, and PR is the production rules. The components of G_H are defined as follows:

$$V_N=\left\{\begin{array}{l}\langle\text{primary term}\rangle,\langle\text{composite term}\rangle,\langle\text{unary relation}\rangle,\\ \langle\text{binary relation}\rangle,\langle\text{conjunction}\rangle\end{array}\right\};$$

$$V_T=\left\{\begin{array}{l}\text{lower than, greater than, at least, at most,}\\ \text{between, and, } l_0,\ l_1,\ ...,\ l_g\end{array}\right\};$$

$$\begin{array}{l}PR=\{I ::= \langle\text{primary term}\rangle \mid \langle\text{composite term}\rangle\\ \langle\text{composite term}\rangle ::= \langle\text{unary relation}\rangle\langle\text{primary term}\rangle\\ \mid \langle\text{binary relation}\rangle\langle\text{primary term}\rangle\langle\text{conjunction}\rangle\langle\text{primary term}\rangle\\ \langle\text{primary term}\rangle ::= l_0 \mid l_1 \mid ... \mid l_g\\ \langle\text{unary relation}\rangle ::= \text{lower than} \mid \text{greater than} \mid \text{at least} \mid \text{at most}\\ \langle\text{binary relation}\rangle ::= \text{between}\\ \langle\text{conjunction}\rangle ::= \text{and}\}\end{array}$$

Let C^L encompass all CLEs across the linguistic set L.

(2) Hesitant fuzzy linguistic term sets

The concept of HFLTSs was introduced by Rodríguez et al. [38], outlined as follows.

Definition 6 [38]. Given the linguistic set $L=\{l_t \mid t=0,1,...,g\}$ as defined earlier, a HFLTS, denoted as $H=\{l_i,\ l_{i+1},\ ...,\ l_j\}$, is a finite ordered subset of consecutive linguistic terms from L. The negation function, denoted as $neg(\cdot)$, operates as follows on HFLTS: $neg(H)=\{l_{g-j},\ l_{g-j+1},\ ...,\ l_{g-i}\}$. If $H=\phi$, it is referred to as an empty HFLTS; if $H=L$, it is termed a full HFLTS.

Definition 7 (Transformation function) [38]. Let TF be a function responsible for converting the CLEs acquired through the context-free grammar G_H into HFLTSs. The linguistic expressions generated by G_H via the production rules can be converted into HFLTSs as follows:

(i) $TF(\text{greater than } l_i)=\{l_k \mid l_k \in L \text{ and } l_k > l_i\}$;

(ii) $TF(\text{lower than } l_i)=\{l_k \mid l_k \in L \text{ and } l_k < l_i\}$;

(iii) $TF(\text{at least } l_i)=\{l_k \mid l_k \in L \text{ and } l_k \geq l_i\}$;

(iv) $TF(\text{at most } l_i)=\{l_k \mid l_k \in L \text{ and } l_k \leq l_i\}$;

(v) $TF(\text{between } l_i \text{ and } l_j)=\{l_k \mid l_k \in L \text{ and } l_i \leq l_k \leq l_j\}$.

(3) Linguistic distribution assessments

The LDAs are a recently developed linguistic assessment model [28, 44]. In LDAs, symbolic proportions are allocated to all linguistic terms within the set $L=\{l_t \mid t=0,1,...,g\}$.

Definition 8 [28, 44]. Given the linguistic set $L=\{l_t \mid t=0,1,...,\ g\}$ as defined earlier, a distribution assessment of L is represented as $d=\{(l_t,\rho_t) \mid t=0,1,...,g\}$, where $l_t \in L$ and $\rho_t \in [0,1]$ represents the symbolic proportion assigned to the linguistic term l_t, satisfying the condition $\sum_{t=0}^{g}\rho_t=1$.

III. The Framework of Graph Model with Internal Consensus Reaching and External Game

Graph model and consensus reaching process serve as versatile and adaptable tools for conflict mitigation, while the difference between them lies in the types of conflicts they handled. Stability analysis in graph model derives the equilibrium solution for managing game conflicts among conflicting parties in conflict problems. Consensus reaching process is employed to mitigate preference conflicts among individuals within each conflicting party. To address the dual conflict decision-making problem involving game and preference conflicts, we integrate intra-composite decision-maker consensus reaching into the framework of graph model, and propose an enhanced graph model (refer to Fig. 2).

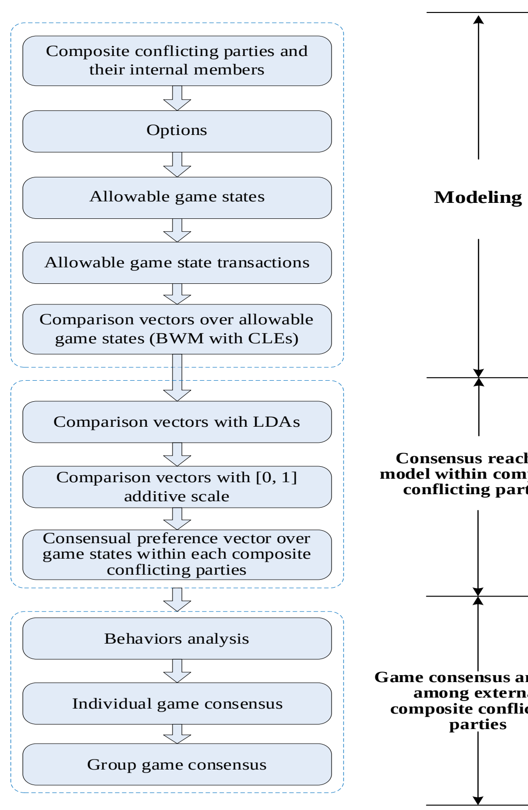

Fig. 2. The graph model framework with internal consensus reaching and external game

A. Modeling

Composite conflicting parties and their corresponding options are firstly identified in the modeling process. Subsequently, an analysis of game states and game state transition diagrams takes place. Furthermore, individual preferences concerning the game states are modeled utilizing BWM coupled with CLEs.

(1) *Identification of composite conflicting parties with multiple individuals*. Consider a set $E=\{E^1,...,E^m\}$ comprising m composite conflicting parties. Let m^k present the count of individuals within composite conflicting party E^k, such that $E^k=\{E^{k,1},...,E^{k,m^k}\}$, where $E^{k,o}$ represents the *o*th individual within E^k.

(2) *Identification of options for each composite conflicting party*. The collection of options linked to the composite conflicting party E^k is represented as $O^k=\{o_1^k,...,o_{r^k}^k\}$, where r^k signifies the count of options within O^k.

(3) *Analysis of game states*. Following the selection of choices by all composite conflicting parties from their respective options, the game states are established. The feasible game states, $S=\{s_1,...,s_n\}$, are derived by excluding certain infeasible game states.

(4) Creation of *game state transition diagrams*. In the framework of graph model, game states and their corresponding transitions for a specific composite conflicting party E^k are symbolized as $D^k=(S,\ T^k)$, wherein T^k signifies the set of directed arcs. The notation $(s_i,\ s_j)\in T^k$ represents the transition from game state s_i to s_j, which can be influenced by the composite conflicting party E^k.

Let $R^k(s_t)=\{s_i\in S\,|\,(s_t,\ s_i)\in T^k\}$ denote the collection of game states achievable from s_t through a single move by composite conflicting party E^k. The set encompassing all unilateral enhancements from $s_t\in S$ by composite conflicting party E^k is denoted as $R^{k+}(s_t)=\{s_i\,|\,s_i\in R^k(s_t)\wedge s_i\succ_k s_t\}$.

(5) *Preference modeling based on BWM with CLEs.*

Let s_b and s_w represent the best and worst game states linked to a particular individual. Subsequently, the individual's CLEs are employed to formulate the BO and OW comparison vectors.

The BO vector, utilizing the CLEs furnished by the individual, would be as follows:

$$A_b=(a_{b1},\ a_{b2},\ ...,\ a_{bn})^T \tag{1}$$

where $a_{bi}\in C^L$ signifies the CLE preference of the best s_b over s_i.

The OW vector, offered by the individual, would be:

$$A_w=(a_{1w},\ a_{2w},\ ...,\ a_{nw})^T \tag{2}$$

where $a_{iw}\in C^L$ indicates the CLE preference of s_i over the worst s_w.

Here, the most favorable and least favorable game states associated with individual $E^{k,o}$ are denoted as $s_{b^{k,o}}$ and $s_{w^{k,o}}$, respectively. The BO and OW vectors associated with individual $E^{k,o}$ are represented as $A_{b^{k,o}}^{k,o}=(a_{b^{k,o}1}^{k,o},\ ...,\ a_{b^{k,o}n}^{k,o})^T$ and $A_{w^{k,o}}^{k,o}=(a_{1w^{k,o}}^{k,o},\ ...,\ a_{nw^{k,o}}^{k,o})^T$, respectively. The subsequent notation is employed: $A_{b^{k,o}/w^{k,o}}^{k,o}=\{A_{b^{k,o}}^{k,o},\ A_{w^{k,o}}^{k,o}\}$.

Consequently, the graph model incorporating CLEs is formalized as $<E,\ S,\ \{D^k,A_{b^{k,o}/w^{k,o}}^{k,o}\}:E^k\in E,\ E^{k,o}\in E^k>$.

B. Consensus reaching model within composite conflicting party

Let $F=(f_{ij})_{n\times n}$ represent an additive preference relation, where $f_{ij}\in[0,1]$ signifies the preference degree of object x_i over x_j, with the constraint $f_{ij}+f_{ji}=1$ [36]. The additive preference relation F is deemed entirely consistent if subsequent condition is satisfied [46]:

$$f_{ij}=\frac{n-1}{2}(p_i-p_j)+0.5\quad \forall i,\ j\in\{1,\ 2,\ ...,\ n\} \tag{3}$$

where $P=(p_1,\ p_2,\ ...,\ p_n)^T$, constrained by $p_i\in[0,1]$ and $\sum_{i=1}^n p_i=1$, represents the priority vector of the compared objects.

Drawing inspiration from the notion of additive preference relation, the BWM with an [0, 1] additive scale is presented. Here, $F_b=(f_{b1},\ f_{b2},\ ...,\ f_{bn})^T$ and $F_w=(f_{1w},\ f_{2w},\ ...,\ f_{nw})^T$ denote the BO and OW vectors with the [0, 1] additive scale, respectively, where $f_{bi}\in[0,\ 1]$ and $f_{iw}\in[0,\ 1]$. For convenience, let $F_{b/w}=\{F_b,\ F_w\}$. The deviation between P and $F_{b/w}$ is quantified by:

$$d(F_{b/w},P)=\sum_{i=1}^n(|\frac{n-1}{2}(p_b-p_i)+0.5-f_{bi}|+|\frac{n-1}{2}(p_i-p_w)+0.5-f_{iw}|) \tag{4}$$

Evidently, $d(F_{b/w},P)\in[0,\ 1]$. The smaller the $d(F_{b/w},P)$ value, the higher the consistency degree in the preference information $F_{b/w}=\{F_b,\ F_w\}$. Notably, $d(F_{b/w},P)=0$ means that $F_{b/w}=\{F_b,F_w\}$ is impeccably consistent.

The BWM with LDAs are further presented. Let $D_b=(d_{b1},\ d_{b2},\ ...,\ d_{bn})^T$ denote the BO vector with LDAs, where $d_{bi}=\{(l_t,\rho_{bi,t})\,|\,t=0,1,...,g\}$. Let $D_w=(d_{1w},\ d_{2w},\ ...,\ d_{nw})^T$ represent the OW vector with LDAs, where $d_{iw}=\{(l_t,\rho_{iw,t})\,|\,t=0,1,...,g\}$. The BO and OW vectors with LDAs can be transformed into BO and OW with the [0, 1] additive scale, respectively, using the following expressions:

$$f_{bi}=\sum_{l_t\in TF(a_{bi})}NS(l_t)\cdot\rho_{bi,t} \tag{5}$$

and

$$f_{iw}=\sum_{l_t\in TF(a_{iw})}NS(l_t)\cdot\rho_{iw,t} \tag{6}$$

where $NS(l_t)=t/g$, representing numerical scale, corresponding to the linguistic term l_t. For a comprehensive understanding of the numerical scale function, please consult Ref. [51].

In the proposed graph model framework, internal individuals present their preferences based on CLEs. Therefore, effectively managing CLEs becomes a pivotal concern. In traditional research regarding to linguistic computational models, CLEs are frequently translated into HFLTSs following Definition 7. Nevertheless, it's worth highlighting that the assumption of equal occurrence probabilities for linguistic terms within HFLTSs might not always accurately reflect the genuine preferences of individuals. LDAs serves as an effective approach to address this concern. When transforming CLEs into LDAs, the objective is to minimize the deviation between comparison vectors with [0, 1] additive scale and their corresponding preference vectors. Guided by this fundamental concept, subsequent optimization model (7) is presented.

$$\min\sum_{i=1}^n(|\frac{n-1}{2}(p_b-p_i)+0.5-f_{bi}|+|\frac{n-1}{2}(p_i-p_w)+0.5-f_{iw}|)$$

$$s.t.\begin{cases}\sum_{i=1}^n p_i=1 & (a)\\ f_{bi}=\sum_{l_t\in TF(a_{bi})}NS(l_t)\cdot\rho_{bi,t},\ i=1,...,n & (b)\\ f_{iw}=\sum_{l_t\in TF(a_{iw})}NS(l_t)\cdot\rho_{iw,t},\ i=1,...,n & (c)\\ \sum_{l_t\in TF(a_{bi})}\rho_{bi,t}=1,\ i=1,...,n & (d)\\ \sum_{l_t\in TF(a_{iw})}\rho_{iw,t}=1,\ i=1,...,n & (e)\\ \rho_{bi,t}\geq 0,\ l_t\in TF(a_{bi});\ \ \rho_{iw,t}\geq 0,\ l_t\in TF(a_{iw}) & (f)\\ p_i\geq 0,\ i=1,...,n & (g)\end{cases} \tag{7}$$

where $\rho_{bi,t}$, $\rho_{iw,t}$, f_{bi}, f_{iw} are decision variables.

Proposition 1: Through the introduction of a new set of variables $c_{bi},\ d_{iw}\geq 0\quad(i=1,2,...,n)$, the formulation represented by model (7) can be converted into the subsequent linear programming model (8):

$$\min \sum_{i=1}^{n}(c_{bi}+d_{iw})$$
$$s.t.\begin{cases} \frac{n-1}{2}(p_b-p_i)+0.5-f_{bi}\le c_{bi},\ i=1,...,n & (a)\\ -\frac{n-1}{2}(p_b-p_i)-0.5+f_{bi}\le c_{bi},\ i=1,...,n & (b)\\ \frac{n-1}{2}(p_i-p_w)+0.5-f_{iw}\le d_{iw},\ i=1,...,n & (c)\\ -\frac{n-1}{2}(p_i-p_w)-0.5+f_{iw}\le d_{iw},\ i=1,...,n & (d)\\ c_{bi},\ d_{iw}\ge 0,\ i=1,...,n & (e)\\ \text{Other constraints as the same as that in model (7)} \end{cases} \quad (8)$$

Model (7) (or (8)) presents an effective approach to handle CLEs. In this model, CLEs are translated into LDAs representations and further converted into numerical values within the [0, 1] range. Given that each composite conflicting party encompasses multiple individuals, a method to handle CLEs and preference conflicts within a group context is proposed next.

For a specific composite conflicting party, their individual members are represented as $\{e_1, e_2, ..., e_h\}$. Let s_{b^k} and s_{w^k} symbolize the optimal and poorest game states associated with individual e_k, respectvely. The BO and OW vectors related to CLEs for individual e_k are denoted as $A^k_{b^k}=(a^k_{b^k1}, a^k_{b^k2}, ..., a^k_{b^kn})^T$ and $A^k_{w^k}=(a^k_{1w^k}, a^k_{2w^k}, ..., a^k_{nw^k})^T$, respectively.

Let $P^k=(p^k_1, p^k_2, ..., p^k_n)^T$ be the priority vector obtained from $A^k_{b^k}$ and $A^k_{w^k}$, wherein $p^k_i\in[0,1]$ signifies the preference value of the game state s_i and $\sum_{i=1}^{n}p^k_i=1$. Let $A^k_{b^k/w^k}=\{A^k_{b^k}, A^k_{w^k}\}$. The deviation between P^k and $A^k_{b^k/w^k}$ across all individuals $\{e_1, e_2, ..., e_h\}$ is quantified as follows:

$$\sum_{k=1}^{h}d(P^k, A^k_{b^k/w^k})=\sum_{k=1}^{h}\sum_{i=1}^{n}(|\frac{n-1}{2}(p^k_{b^k}-p^k_i)+0.5-f^k_{b^ki}|+|\frac{n-1}{2}(p^k_i-p^k_{w^k})+0.5-f^k_{iw^k}|) \quad (9)$$

where

$$f^k_{b^ki}=\sum_{l_t\in TF(a^k_{b^ki})}NS(l_t)\rho^k_{b^ki,t} \quad (10)$$

and

$$f^k_{iw^k}=\sum_{l_t\in TF(a^k_{iw^k})}NS(l_t)\rho^k_{iw^k,t} \quad (11)$$

The consensus level among $\{e_1, e_2, ..., e_h\}$ is given by:

$$CL(e_1, ..., e_h)=1-\frac{1}{2h}\sum_{k=1}^{h}\sum_{i=1}^{n}|p^k_i-p^c_i| \quad (12)$$

where $P^c=(p^c_1, p^c_2, ..., p^c_n)^T$, representing group priority vector, is:

$$p^c_i=\sum_{k=1}^{h}\lambda_k\cdot p^k_i \quad (13)$$

and $\lambda=(\lambda_1,..., \lambda_h)^T$ is the weight vector of individuals $\{e_1,...,e_h\}$.

The objective is to minimize the deviation between P^k and $A^k_{b^k/w^k}$ across all individuals $\{e_1,...,e_h\}$, i.e.:

$$\min \sum_{k=1}^{h}d(P^k, A^k_{b^k/w^k}) \quad (14)$$

Meanwhile, the consensus level among the individuals $\{e_1,...,e_h\}$ is deemed satisfactory, i.e.:

$$CL(e_1, ..., e_h)\ge\theta \quad (15)$$

where $\theta\in[0, 1]$ represents the consensus threshold.

Building upon the preceding analysis, the subsequent model introduces a consensus-reaching approach grounded in minimizing preference information loss:

$$\min \sum_{k=1}^{h}\sum_{i=1}^{n}(|\frac{n-1}{2}(p^k_{b^k}-p^k_i)+0.5-f^k_{b^ki}|+|\frac{n-1}{2}(p^k_i-p^k_{w^k})+0.5-f^k_{iw^k}|)$$
$$s.t.\begin{cases} \sum_{i=1}^{n}p^k_i=1,\ k=1,...,h & (a)\\ p^c_i=\sum_{k=1}^{h}\lambda_k\cdot p^k_i,\ i=1,...,n & (b)\\ CL(e_1, ..., e_h)=1-\frac{1}{2h}\sum_{k=1}^{h}\sum_{i=1}^{n}|p^k_i-p^c_i|\ge\theta & (c)\\ f^k_{b^ki}=\sum_{l_t\in TF(a^k_{b^ki})}NS(l_t)\cdot\rho^k_{b^ki,t},\ k=1,...,h;\ i=1,...,n & (d)\\ f^k_{iw^k}=\sum_{l_t\in TF(a^k_{iw^k})}NS(l_t)\cdot\rho^k_{iw^k,t},\ k=1,...,h;\ i=1,...,n & (e)\\ \sum_{l_t\in TF(a^k_{b^ki})}\rho^k_{b^ki,t}=1,\ k=1,...,h;\ i=1,...,n & (f)\\ \sum_{l_t\in TF(a^k_{iw^k})}\rho^k_{iw^k,t}=1,\ k=1,...,h;\ i=1,...,n & (g)\\ \rho^k_{b^ki,t}\ge 0,\ l_t\in TF(a^k_{b^ki});\ \rho^k_{iw^k,t}\ge 0,\ l_t\in TF(a^k_{iw^k}) & (h)\\ p^k_i,\ p^c_i\ge 0,\ k=1,...,h;\ i=1,...,n & (h) \end{cases} \quad (16)$$

In model (16), p^k_i, p^c_i, $\rho^k_{b^ki,t}$, $\rho^k_{iw^k,t}$, $f^k_{b^ki}$ and $f^k_{iw^k}$ are decision variables, and their optimal values are represented as $p^{k,*}_i$, $p^{c,*}_i$, $\rho^{k,*}_{b^ki,t}$, $\rho^{k,*}_{iw^k,t}$, $f^{k,*}_{b^ki}$ and $f^{k,*}_{iw^k}$, respectively. It is obvious that with the rising θ, the objective function's value exhibits an increasing trend. Moreover, the setting of consensus threshold is an open problem in consensus reaching process. Notably, preference learning method is a useful tool to estimate parameters in the field of decision making [33]. Thus, it would be a promising direction to apply this method to tackle consensus threshold configuration challenge.

$$\min \sum_{k=1}^{m}\sum_{i=1}^{n}(c^k_{b^ki}+d^k_{iw^k})$$
$$s.t.\begin{cases} \frac{n-1}{2}(p^k_{b^k}-p^k_i)+0.5-f^k_{b^ki}\le c^k_{b^ki},\ k=1,...,h;\ i=1,...,n & (a)\\ -\frac{n-1}{2}(p^k_{b^k}-p^k_i)-0.5+f^k_{b^ki}\le c^k_{b^ki},\ k=1,...,h;\ i=1,...,n & (b)\\ \frac{n-1}{2}(p^k_i-p^k_{w^k})+0.5-f^k_{iw^k}\le d^k_{iw^k},\ k=1,...,h;\ i=1,...,n & (c)\\ -\frac{n-1}{2}(p^k_i-p^k_{w^k})-0.5+f^k_{iw^k}\le d^k_{iw^k},\ k=1,...,h;\ i=1,...,n & (d)\\ \sum_{i=1}^{n}p^k_i=1,\ k=1,...,h;\ i=1,...,n & (e)\\ p^c_i=\sum_{k=1}^{m}\lambda_k\cdot p^k_i,\ i=1,...,n & (f)\\ CL(e_1, ..., e_h)=1-\frac{1}{2m}\sum_{k=1}^{m}\sum_{i=1}^{n}e^k_i\ge\theta & (g)\\ p^k_i-p^c_i\le e^k_i,\ k=1,...,h;\ i=1,...,n & (h)\\ -p^k_i+p^c_i\le e^k_i,\ k=1,...,h;\ i=1,...,n & (i)\\ f^k_{b^ki}=\sum_{l_t\in TF(a^k_{b^ki})}NS(l_t)\cdot\rho^k_{b^ki,t},\ k=1,...,h;\ i=1,...,n & (j)\\ f^k_{iw^k}=\sum_{l_t\in TF(a^k_{iw^k})}NS(l_t)\cdot\rho^k_{iw^k,t},\ k=1,...,h;\ i=1,...,n & (k)\\ \sum_{l_t\in TF(a^k_{b^ki})}\rho^k_{b^ki,t}=1,\ k=1,...,h;\ i=1,...,n & (l)\\ \sum_{l_t\in TF(a^k_{iw^k})}\rho^k_{iw^k,t}=1,\ k=1,...,h;\ i=1,...,n & (m)\\ \rho^k_{b^ki,t}\ge 0,\ l_t\in TF(a^k_{b^ki});\ \rho^k_{iw^k,t}\ge 0,\ l_t\in TF(a^k_{iw^k}) & (n)\\ p^k_i, p^c_i, c^k_{b^ki}, d^k_{iw^k}, e^k_i\ge 0,\ k=1,...,h;\ i=1,...,n & (o) \end{cases} \quad (17)$$

Proposition 2: By introducing a new set of variables, $c_{b^k i}^k$, $d_{iw^k}^k$, e_i^k $(k=1,2,...,h;\ i=1,2,...,n)$, model (16) can be reformulated into the subsequent linear programming model (17).

The proof of Proposition 2 follows a similar logic as that of Proposition 1 and is omitted for brevity.

Employing the above model, we derive the priority vector corresponding to each composite conflicting party. Let $P^{c,k}=(p_1^{c,k},...,p_n^{c,k})^T$ symbolize the priority vector linked to the composite conflicting party E^k.

As mentioned earlier, current research on CLEs primarily revolves around their conversion into HFLTSs. Subsequently, the HFLTSs are translated into LDAs by assigning equal possibility values to the linguistic terms within the HFLTSs. Proposition 3 contrasts our proposed approach with existing methodologies.

Proposition 3: Let $\bar{D}_{b^k}^k=(\bar{d}_{b^k 1}^k,\ \bar{d}_{b^k 2}^k,\ ...,\ \bar{d}_{b^k n}^k)$ denote the BO vector incorporating LDAs through the HFLTSs transformation-based approach, where $\bar{d}_{b^k i}^k=\{(l_t,\bar{\rho}_{b^k i,t}^k)\,|\,t=0,1,...,g\}$:

$$\begin{cases}\bar{\rho}_{b^k i,t}^k=\dfrac{1}{\#TF(a_{b^k i}^k)}, & l_t\in TF(a_{b^k i}^k)\\ \bar{\rho}_{b^k i,t}^k=0, & l_t\notin TF(a_{b^k i}^k)\end{cases} \tag{18}$$

Let $\bar{D}_{w^k}^k=(\bar{d}_{1w^k}^k,\ \bar{d}_{2w^k}^k,\ ...,\ \bar{d}_{nw^k}^k)^T$ represent be the OW vector with LDAs through the HFLTSs transformation-based approach, where $\bar{d}_{iw^k}^k=\{(l_t,\bar{\rho}_{iw^k,t}^k)\,|\,t=0,1,...,g\}$:

$$\begin{cases}\bar{\rho}_{iw^k,t}^k=\dfrac{1}{\#TF(a_{iw^k}^k)}, & l_t\in TF(a_{iw^k}^k)\\ \bar{\rho}_{iw^k,t}^k=0, & l_t\notin TF(a_{iw^k}^k)\end{cases} \tag{19}$$

In model (16), we set $\rho_{b^k i,t}^k=\bar{\rho}_{b^k i,t}^k$ and $\rho_{iw^k,t}^k=\bar{\rho}_{iw^k,t}^k$. Subsequently, we determine the optimal values for p_i^k, $f_{b^k i}^k$, and $f_{iw^k}^k$, represented as $\bar{p}_i^k$, $\bar{f}_{b^k i}^k$, and $\bar{f}_{iw^k}^k$, respectively.

Then, it can be deduced that

$$\begin{aligned}&\sum_{k=1}^{h}\sum_{i=1}^{n}\left(\left|\frac{n-1}{2}(p_{b^k}^{k,*}-p_i^{k,*})+0.5-f_{b^k i}^{k,*}\right|+\left|\frac{n-1}{2}(p_i^{k,*}-p_{w^k}^{k,*})+0.5-f_{iw^k}^{k,*}\right|\right)\le\\&\sum_{k=1}^{h}\sum_{i=1}^{n}\left(\left|\frac{n-1}{2}(\bar{p}_{b^k}^{k}-\bar{p}_i^{k})+0.5-\bar{f}_{b^k i}^{k}\right|+\left|\frac{n-1}{2}(\bar{p}_i^{k}-\bar{p}_{w^k}^{k})+0.5-\bar{f}_{iw^k}^{k}\right|\right)\end{aligned} \tag{20}$$

Proposition 3 underscores that our method leads to a reduced loss of preference information compared to existing approach.

C. Game consensus analysis among external composite conflicting parties

In this section, we introduce the concept of game consensus to address game conflicts among external composite conflicting parties. Prior to that, we thoroughly analyze the four fundamental stability concepts within graph model from the perspectives of behavior concerning foresight ability and risk attitude.

In the context of global Nash stability, conflicting parties are assumed to be solely concerned with whether the current game state could be improved by altering their options. If such an improvement is feasible for a particular conflicting party, they will shift to the corresponding improved game state. If not, they will remain in the current game state. In simpler terms, conflicting parties in this scenario do not consider the potential counterattacks from opponents, lacking foresight and neglecting any risk factors.

In the context of global GMR stability, conflicting parties hold the belief that their actions will trigger counterattacks from their opponent(s). They take into account all potential counterattack scenarios from their opponent(s), even if those counterattacks could lead the opponent(s) to a less favorable game state. Consequently, global GMR stability indicates that conflicting parties possess a two-step foresight mechanism and tend to be cautious when confronted with risks.

For global SMR stability, conflicting parties exhibit heightened foresight capabilities, as they delve deeply into assessing whether they can evade the retribution of their opponent(s). Therefore, in the realm of global SMR stability, conflicting parties demonstrate a three-step foresight mechanism and adopt a cautious stance when dealing with risks.

In the realm of global SEQ stability, conflicting parties hold the belief that their opponents will prioritize their own interests when responding, rather than engaging in retaliatory actions at any cost. Nevertheless, opponents might still opt for retaliatory actions, even if it would result in harm to their interests. In such cases, conflicting parties must be willing to take on certain risks. Consequently, global SEQ stability implies that conflicting parties possess a two-step foresight mechanism and harbor a spirit of adventure.

The four global stability concepts assume uniform behaviors among all conflicting parties in terms of foresight ability and risk attitude. However, in reality, conflicting parties can exhibit heterogeneous behaviors. For instance, some conflicting parties may lack foresight, while others possess strong foresight. Similarly, some conflicting parties might lean towards risk-averse conservatism, while others lean towards risk-taking eagerness. Note that it is interesting to investigate how to identify heterogeneous behaviors of conflicting parties. Given that Bayesian inference can effectively predict human behaviors and has been studied in GDM [39], we consider employing Bayesian inference to predict heterogeneous behaviors in future research. Consequently, a more nuanced stability concept is required within the graph model framework to accommodate this diversity in behaviors. In the following, we introduce the concept of game consensus, which takes into account the heterogeneous behaviors of conflicting parties.

Definition 9: Game consensus. For game state s_t, if all conflicting party opt not deviate from it, then s_t is considered a state exhibiting game consensus within the graph model framework. If S_t represents the set of game states demonstrating game consensus, then $S_t=S^1\cap S^2\cap\cdots\cap S^m$ where S^k represents the set of game states that conflicting party E^k does not deviate from contingent on their own behavior.

In the framework of game consensus, conflicting parties opting to stay within the current game state can be attributed to various individual behaviors. This stands in contrast to existing global stability concepts, where conflicting parties remaining in the current game state are ascribed to uniform behavior. In this study, we posit that conflicting parties can manifest four distinct behaviors: Nash, GMR, SMR, and SEQ behaviors. As a result, the concept of game consensus can be viewed as a more comprehensive interpretation of established global stability concepts. Specifically, when each conflicting party's choice to persist in the current game state aligns with Nash, GMR, SMR, or SEQ stability, the game consensus assumes the form of global Nash, GMR, SMR, or SEQ stability, respectively. The interrelations among game consensus, global Nash, GMR, SMR, and SEQ stabilities are illustrated in Fig. 3.

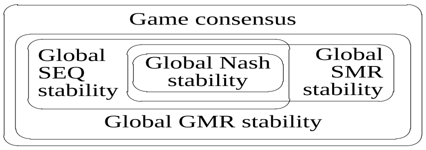

Fig. 3. The interrelationship of different solution concepts within the graph model framework

The subsequent example, drawn from Ref. [54], concerning the conflict within the Paraxylene (PX) project, is utilized to demonstrate the application of the game consensus concept.

Example 1: Consider the PX project conflict involving three conflicting parties:

(1) Local government (LG). LG has two options:

(i) **continue** with the construction of the PX project to pursue economic benefits;

(ii) LG has to decide whether or not to **compensate** the local residents as the local residents have to bear the risk of environmental pollution and associated health hazards.

(2) Local residents (LR). LR has one option:

(i) whether or not to **oppose** the PX project.

(3) Non-Governmental Organizations (NGO), consisting of environmental, health and other kinds of organizations which may communicate using social media.

(i) NGO can decide whether or not to **positively** influence public opinion about construction of PX project.

The PX project conflict encompasses a total of 16 game states, among which 10 feasible game states (represented as $\{s_1, s_2, ..., s_{10}\}$) are delineated in Table 1 after excluding the infeasible ones. In the table, "Y" denotes the selection of the corresponding option, whereas "N" indicates the option not being chosen.

The game state transition diagrams for the LG, LR, and NGO strategies in the PX project conflict are illustrated in Fig. 4. Each vertex corresponds to a feasible game state, while arcs with one or two arrowheads denote the game state transitions influenced by the particular conflicting party.

Table 1: Options and feasible game states for the PX project conflict

Conflicting parties	**Options**	**Feasible game states**									
		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
LG	Continue	N	N	Y	Y	Y	Y	Y	Y	Y	Y
	Compensate	N	N	N	N	N	N	Y	Y	Y	Y
LR	Oppose	N	N	N	N	Y	Y	N	N	Y	Y
NGO	Positive	N	Y	N	Y	N	Y	N	Y	N	Y

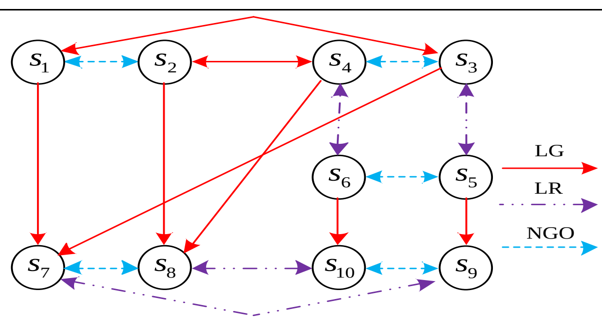

Fig. 4. Game state transition diagrams of LG, LR and NGO

The preferences of LG, LR and NGO are given below:

LG: $s_8 \succ s_7 \succ s_4 \succ s_3 \succ s_2 \succ s_1 \succ s_{10} \succ s_9 \succ s_6 \succ s_5$;

LR: $s_8 \succ s_7 \succ s_{10} \succ s_9 \succ s_4 \succ s_3 \succ s_6 \succ s_5 \succ s_2 \succ s_1$;

NGO: $s_8 \succ s_7 \succ s_4 \succ s_2 \succ s_3 \succ s_1 \succ s_{10} \succ s_9 \succ s_6 \succ s_5$.

In the subsequent analysis, we consider two distinct cases:

(1) LG has SEQ behavior, LR has Nash behavior, and NGO has SEQ behavior. According to Definition 4, it is $S^1 = \{s_7, s_8, s_9, s_{10}\}$ and $S^3 = \{s_2, s_4, s_6, s_8, s_{10}\}$, while according to Definition 1 it is $S^2 = \{s_1, s_2, s_3, s_4, s_7, s_8\}$. As a result, $S_t = S^1 \cap S^2 \cap S^3 = \{s_8\}$.

(2) LG has Nash behavior, LR has SEQ behavior, and NGO has GMR behavior. In this case, $S^1 = \{s_7, s_8, s_9, s_{10}\}$ (according to Definition 1), $S^2 = \{s_1, s_2, s_3, s_4, s_7, s_8\}$ (according to Definition 4) and $S^3 = \{s_1, s_2, s_3, s_4, s_6, s_7, s_8, s_{10}\}$ (according Definition 2). Thus, $S_t = S^1 \cap S^2 \cap S^3 = \{s_7, s_8\}$.

IV. Case Study

The proposed enhanced graph model, which incorporates internal consensus reaching and external game processes, can effectively address practical dual conflict decision-making problems. To demonstrate the efficacy of the proposed approach, an illustrative case study pertaining to dual-channel supply chain conflicts is presented in this section.

The rapid advancement of E-commerce technology has ushered in a paradigm shift in the way businesses engage with consumers, further making a greater diversity in product sales channels. To tap into the vast and ever-expanding online consumer for more market share, many manufacturers are increasingly proactive in diversifying their sales strategies by establishing their online stores or participating in third-party E-commerce platforms. In this evolving landscape, manufacturers find themselves navigating between two primary sales avenues: the traditional distribution channel and the online direct channel. In the former sales avenue, manufacturers sell products to consumers through intermediary retailers, while in the latter sales avenue, manufacturers sell products directly to consumers via self-operated online stores or third-party E-commerce platforms. Consumers usually compare the prices of the two sales avenues, especially when a manufacturer's online direct channel offers the same product as a retailer's traditional distribution channel. Therefore, the two sales avenues will provide different discount strategies to attract consumers, and potential price conflicts between these two avenues may arise.

Certainly, let's delve into the provided case study which pertains to a price conflict within a dual-channel supply chain. This case study effectively demonstrates the efficacy of the proposed approach. In the dual-channel supply chain price conflict, the primary composite conflicting parties comprise the manufacturer (referred to as E^1) and the retailer (denoted as E^2). Additionally, the manufacturer's conflicting party encompasses three individual members denoted as $E^1 = \{E_1^1, E_2^1, E_3^1\}$, while the retailer's conflicting party also encompasses three individual members denoted as $E^2 = \{E_1^2, E_2^2, E_3^2\}$.

Three options for manufacturer are denoted as $O^1 = \{o_1^1, o_2^1, o_3^1\}$.

(i) *Low price competition* (o_1^1).Given that the manufacturer's direct channel has the capability to eliminate numerous intermediate retail steps, thereby reducing sales-related costs, it enjoys a noticeable price advantage over the retailer. Consequently, the manufacturer can capitalize on this price advantage and opt for a low-price competition strategy to swiftly capture the market.

(ii) *Bundling* (o_2^1). To alleviate pricing conflicts with traditional retail channels, the manufacturer can employ a bundling strategy.

This entails bundling the primary product with complementary items or value-added services.

(iii) *Stop online direct selling* (o_3^1). As the retailer's dominant position becomes increasingly pronounced, the retailer may express strong dissatisfaction upon the manufacturer's launch of online direct marketing channels. As a result, the retailer may threaten the manufacturer to stop direct online sales by reducing orders or terminating cooperation.

In addition, the retailer has two options available to consider, denoted as $O^2 = \{o_1^2, o_2^2\}$.

(i) *Low price competition* (o_1^2). The retailer may stabilize the market position through price reduction, that is, compete on price with manufacturer' online direct channels.

(ii) *Binding sales* (o_2^2). The main product and its related ancillary products, value-added services are bundled for selling.

In the dual-channel supply chain price conflict, the manufacturer and the retailer independently select their strategies, and the combination of the strategy choices will form a game state of the price conflict. This dual-channel supply chain price conflict consists of ten feasible game states, as shown in the Table 2.

Table 2: Feasible game states in the price conflict of the dual-channel supply chain

Conflicting party	Options	Feasible game states									
		s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}
Manufacturer	Low price competition	N	Y	N	N	N	Y	N	Y	N	-
	Bundling sales	N	N	Y	N	N	N	Y	N	Y	-
	Stop online direct selling	N	N	N	N	N	N	N	N	N	Y
Retailer	Low price competition	N	N	N	Y	N	Y	Y	N	N	-
	Binding sales	N	N	N	N	Y	N	N	Y	Y	-

From feasible game state table, the state transition diagrams for the manufacturer and the retailer can be plotted as shown in Fig. 5.

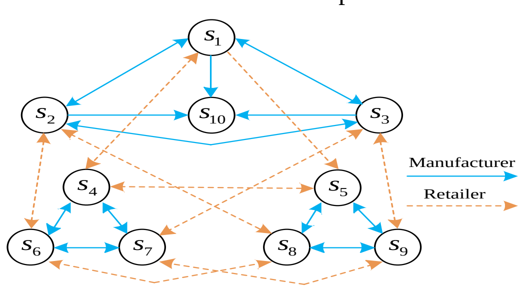

Fig. 5. State transition diagrams of the manufacturer and the retailer

The conflicting parties employ the BWM with CLEs to articulate their preferences concerning the feasible game states. Furthermore, a nine-grade linguistic term set is utilized as outlined below:

$$L = \{l_0 = \text{absolutely worse},\ l_1 = \text{much worse},\ l_2 = \text{worse},\ l_3 = \text{slightly worse},\ l_4 = \text{indifferent},\ l_5 = \text{slightly better},\ l_6 = \text{better},\ l_7 = \text{much better},\ l_8 = \text{absolutely better}\}.$$

The numerical scales are set as:

$NS(l_0) = 0$, $NS(l_1) = 1/8$, $NS(l_2) = 2/8$, $NS(l_3) = 3/8$, $NS(l_4) = 4/8$, $NS(l_5) = 5/8$, $NS(l_6) = 6/8$, $NS(l_7) = 7/8$, and $NS(l_8) = 1$.

For the manufacturer, it is assumed that

$s_{b^{1,1}} = s_9$, $s_{b^{1,2}} = s_9$, $s_{b^{1,3}} = s_9$, $s_{w^{1,1}} = s_{10}$, $s_{w^{1,2}} = s_{10}$, and $s_{w^{1,3}} = s_{10}$.

For the retailer, it is assumed that

$s_{b^{2,1}} = s_{10}$, $s_{b^{2,2}} = s_{10}$, $s_{b^{2,3}} = s_{10}$, $s_{w^{2,1}} = s_2$, $s_{w^{2,2}} = s_2$, and $s_{w^{2,3}} = s_2$.

Tables 3 and 4 present the BO and OW vectors, respectively, for both the manufacturer and the retailer.

Table 3: BO and OW vectors with CLEs of the manufacturer

	E_1^1		E_2^1		E_3^1	
	$A_9^{1,1}$	$A_{10}^{1,1}$	$A_8^{1,2}$	$A_{10}^{1,2}$	$A_8^{1,3}$	$A_{10}^{1,3}$
s_1	at least l_6	between l_3 and l_6	at least l_7	between l_5 and l_6	between l_6 and l_7	between l_5 and l_6
s_2	greater than l_4	between l_4 and l_6	between l_3 and l_5	at least l_6	between l_5 and l_7	l_6
s_3	between l_5 and l_6	between l_5 and l_7	between l_5 and l_7	between l_5 and l_7	between l_5 and l_6	between l_6 and l_7
s_4	greater than l_6	between l_3 and l_5	between l_6 and l_7	between l_4 and l_5	at least l_6	between l_4 and l_5
s_5	between l_4 and l_6	between l_6 and l_7	between l_4 and l_6	between l_5 and l_7	between l_4 and l_6	between l_6 and l_7
s_6	greater than l_5	between l_4 and l_5	between l_4 and l_5	between l_6 and l_7	between l_6 and l_7	l_5
s_7	between l_5 and l_7	between l_4 and l_7	between l_6 and l_7	between l_4 and l_6	between l_5 and l_7	between l_5 and l_7
s_8	between l_4 and l_5	greater than l_5	l_4	at least l_7	l_4	at least l_7
s_9	l_4	at least l_7	between l_4 and l_6	between l_6 and l_7	between l_4 and l_5	at least l_6
s_{10}	l_8	l_4	l_8	l_4	at least l_7	l_4

Table 4: BO and OW vectors with CLEs of the retailer

	E_1^2		E_2^2		E_3^2	
	$A_{10}^{2,1}$	$A_2^{2,1}$	$A_{10}^{2,2}$	$A_2^{2,2}$	$A_{10}^{2,3}$	$A_2^{2,3}$
s_1	greater than l_5	between l_4 and l_5	l_7	between l_4 and l_5	greater than l_5	between l_4 and l_5
s_2	l_8	l_4	at least l_7	l_4	l_8	l_4
s_3	between l_6 and l_7	between l_4 and l_6	between l_5 and l_7	between l_5 and l_6	between l_5 and l_6	between l_6 and l_7
s_4	between l_5 and l_6	between l_6 and l_7	between l_4 and l_6	l_7	between l_6 and l_7	between l_4 and l_6
s_5	l_7	between l_4 and l_6	between l_6 and l_7	l_5	l_6	l_6
s_6	l_6	between l_5 and l_7	between l_5 and l_6	between l_6 and l_7	at least l_7	l_5
s_7	between l_4 and l_6	greater than l_5	l_5	at least l_7	l_5	between l_5 and l_7
s_8	at least l_7	between l_4 and l_5	between l_6 and l_7	between l_3 and l_5	between l_6 and l_7	between l_5 and l_6
s_9	between l_5 and l_7	between l_5 and l_7	l_6	between l_5 and l_7	between l_4 and l_6	between l_6 and l_8
s_{10}	l_4	at least l_7	l_4	l_8	l_4	at least l_7

Table 5: BO and OW vectors with LDAs of the manufacturer

	E_1^1		E_2^1		E_3^1	
	$D_9^{1,1}$	$D_{10}^{1,1}$	$D_8^{1,2}$	$D_{10}^{1,2}$	$D_8^{1,3}$	$D_{10}^{1,3}$
s_1	$\{(l_8,1)\}$	$\{(l_3,0.25), (l_4,0.5), (l_5,0.25)\}$	$\{(l_7, 1)\}$	$\{(l_5, 1)\}$	$\{(l_7,1)\}$	$\{(l_5, 1)\}$
s_2	$\{(l_5,0.05), (l_6,0.038), (l_7,0.862), (l_8,0.05)\}$	$\{(l_4,0.2), (l_5,0.512), (l_6,0.288)\}$	$\{(l_5,1)\}$	$\{(l_6,0.3), (l_7,0.4), (l_8,0.3)\}$	$\{(l_5,0.3), (l_6,0.4), (l_7,0.3)\}$	$\{(l_6, 1)\}$
s_3	$\{(l_6,1)\}$	$\{(l_5, 0.3), (l_6, 0.4), (l_7,0.3)\}$	$\{(l_5,0.188), (l_6,0.712), (l_7,0.1)\}$	$\{(l_5,0.15), (l_6,0.612), (l_7,0.238)\}$	$\{(l_6,1)\}$	$\{(l_6, 1)\}$
s_4	$\{(l_8,1)\}$	$\{(l_3, 0.35), (l_4,0.3), (l_5,0.35)\}$	$\{(l_7,1)\}$	$\{(l_5,1)\}$	$\{(l_8,1)\}$	$\{(l_4, 1)\}$
s_5	$\{(l_5,1)\}$	$\{(l_7,1)\}$	$\{(l_4,0.2), (l_5,0.6), (l_6,0.2)\}$	$\{(l_7,1)\}$	$\{(l_4,0.2), (l_5,0.556), (l_6,0.244)\}$	$\{(l_6,0.044), (l_7,0.956)\}$
s_6	$\{(l_8,1)\}$	$\{(l_4, 1)\}$	$\{(l_5,1)\}$	$\{(l_7,1)\}$	$\{(l_7,1)\}$	$\{(l_5, 1)\}$
s_7	$\{(l_7,1)\}$	$\{(l_4,0.3), (l_5,0.4), (l_6,0.3)\}$	$\{(l_7,1)\}$	$\{(l_4,0.35), (l_5,0.3), (l_6,0.35)\}$	$\{(l_7,1)\}$	$\{(l_5, 1)\}$
s_8	$\{(l_4,0.6184), (l_5,0.3816)\}$	$\{(l_6,0.05), (l_7,0.2816), (l_8,0.6684)\}$	$\{(l_4,1)\}$	$\{(l_8,1)\}$	$\{(l_4,1)\}$	$\{(l_8,1)\}$
s_9	$\{(l_4, 1)\}$	$\{(l_8,1)\}$	$\{(l_6,1)\}$	$\{(l_6,1)\}$	$\{(l_5,1)\}$	$\{(l_6,0.2), (l_7,0.6), (l_8,0.2)\}$
s_{10}	$\{(l_8, 1)\}$	$\{(l_4,1)\}$	$\{(l_8,1)\}$	$\{(l_4,1)\}$	$\{(l_8,1)\}$	$\{(l_4,1)\}$

In this example, the following parameter values are set: $\theta = 0.95$, $\lambda^1 = (1/3,\ 1/3,\ 1/3)^T$, and $\lambda^2 = (1/3,\ 1/3,\ 1/3)^T$. Based on (17), the following specific optimization model is presented to the manufacturer.

Tables 5 and 6 show the manufacturer's BO and OW vectors with LDAs and [0, 1] additive scale, respectively, that are derived after solving the above optimization model.

Table 6: BO and OW vectors with [0, 1] additive scale for the manufacturer

	E_1^1		E_2^1		E_3^1	
	$F_9^{1,1}$	$F_{10}^{1,1}$	$F_8^{1,2}$	$F_{10}^{1,2}$	$F_8^{1,3}$	$F_{10}^{1,3}$
s_1	1	0.5	0.875	0.625	0.875	0.625
s_2	0.8640	0.6360	0.625	0.875	0.75	0.75
s_3	0.75	0.75	0.739	0.7610	0.75	0.75
s_4	1	0.5	0.875	0.625	1	0.5
s_5	0.625	0.875	0.625	0.875	0.6305	0.8695
s_6	1	0.5	0.625	0.875	0.875	0.625
s_7	0.875	0.625	0.875	0.625	0.875	0.625
s_8	0.5477	0.9523	0.5	1	0.5	1
s_9	0.5	1	0.75	0.75	0.625	0.875
s_{10}	1	0.5	1	0.5	1	0.5

Table 7: BO and OW vectors with LDAs for the retailer

	E_1^2		E_2^2		E_3^2	
	$D_{10}^{2,1}$	$D_2^{2,1}$	$D_{10}^{2,2}$	$D_2^{2,2}$	$D_{10}^{2,3}$	$D_2^{2,3}$
s_1	$\{(l_8,1)\}$	$\{(l_4,1)\}$	$\{(l_7,1)\}$	$\{(l_5,1)\}$	$\{(l_6,0.1),(l_7,0.14),(l_8,0.76)\}$	$\{(l_4,0.66),(l_5,0.34)\}$
s_2	$\{(l_8,1)\}$	$\{(l_4,1)\}$	$\{(l_8,1)\}$	$\{(l_4,1)\}$	$\{(l_8,1)\}$	$\{(l_4,1)\}$
s_3	$\{(l_6,0.4216),(l_7,0.5784)\}$	$\{(l_4,0.2),(l_5,0.1784),(l_6,0.6216)\}$	$\{(l_7,1)\}$	$\{(l_5,1)\}$	$\{(l_6,1)\}$	$\{(l_6,1)\}$
s_4	$\{(l_5,1)\}$	$\{(l_7,1)\}$	$\{(l_4,0.2),(l_5,0.6),(l_6,0.2)\}$	$\{(l_7,1)\}$	$\{(l_7,1)\}$	$\{(l_4,0.4),(l_5,0.2),(l_6,0.4)\}$
s_5	$\{(l_7,1)\}$	$\{(l_4,0.35),(l_5,0.3),(l_6,0.35)\}$	$\{(l_7,1)\}$	$\{(l_5,1)\}$	$\{(l_6,1)\}$	$\{(l_6,1)\}$
s_6	$\{(l_6,1)\}$	$\{(l_5,0.4),(l_6,0.2),(l_7,0.4)\}$	$\{(l_5,1)\}$	$\{(l_7,1)\}$	$\{(l_7,1)\}$	$\{(l_5,1)\}$
s_7	$\{(l_4,0.5924),(l_5,0.3576),(l_6,0.05)\}$	$\{(l_6,0.1),(l_7,0.2576),(l_8,0.6424)\}$	$\{(l_5,1)\}$	$\{(l_7,1)\}$	$\{(l_5,1)\}$	$\{(l_7,1)\}$
s_8	$\{(l_7,0.9216),(l_8,0.0784)\}$	$\{(l_4,0.0784),(l_5,0.9216)\}$	$\{(l_7,1)\}$	$\{(l_5,1)\}$	$\{(l_7,1)\}$	$\{(l_5,1)\}$
s_9	$\{(l_7,1)\}$	$\{(l_5,1)\}$	$\{(l_6,1)\}$	$\{(l_5,0.25),(l_6,0.5),(l_7,0.25)\}$	$\{(l_6,1)\}$	$\{(l_6,1)\}$
s_{10}	$\{(l_4,1)\}$	$\{(l_8,1)\}$	$\{(l_4,1)\}$	$\{(l_8,1)\}$	$\{(l_4,1)\}$	$\{(l_8,1)\}$

Table 8: BO and OW vectors with [0, 1] additive scale of the retailer

	E_1^2		E_2^2		E_3^2	
	$F_{10}^{2,1}$	$F_2^{2,1}$	$F_{10}^{2,2}$	$F_2^{2,2}$	$F_{10}^{2,3}$	$F_2^{2,3}$
s_1	1	0.5	0.875	0.625	0.9575	0.5425
s_2	1	0.5	1	0.5	1	0.5
s_3	0.8223	0.6777	0.875	0.625	0.75	0.75
s_4	0.625	0.875	0.625	0.875	0.875	0.625
s_5	0.875	0.625	0.875	0.625	0.75	0.75
s_6	0.75	0.75	0.625	0.875	0.875	0.625
s_7	0.5572	0.9428	0.625	0.875	0.625	0.875
s_8	0.8848	0.6152	0.875	0.625	0.875	0.625
s_9	0.875	0.625	0.75	0.75	0.75	0.75
s_{10}	0.5	1	0.5	1	0.5	1

Meanwhile, the following preference vectors associated with the three members in manufacturer are achieved:

$P^{1,1} = (0.0591, 0.0894, 0.1147, 0.0591, 0.1425, 0.0591, 0.0869, 0.1597, 0.1703, 0.0592)^T$,

$P^{1,2} = (0.0720, 0.1275, 0.1022, 0.0720, 0.1275, 0.1275, 0.0720, 0.1553, 0.0998, 0.0442)^T$,

$P^{1,3} = (0.0806, 0.1085, 0.1085, 0.0529, 0.1350, 0.0807, 0.0807, 0.1640, 0.1362, 0.0529)^T$.

The consensual preference vector associated with the manufacture is:

$P^{1,C} = (0.0706, 0.1085, 0.1085, 0.0613, 0.1350, 0.0891, 0.0799, 0.1597, 0.1354, 0.0520)^T$.

Therefore, it is: $s_8 \succ s_9 \succ s_5 \succ s_2 \sim s_3 \succ s_6 \succ s_7 \succ s_1 \succ s_4 \succ s_{10}$.

Based on model (17), the retailer's BO and OW vectors with LDAs and [0, 1] additive scale, are obtained and offered in Tables 7 and 8, respectively.

Meanwhile, the following preference vectors associated with the three members in the retailer are obtained:

$P^{2,1} = (0.0531, 0.0531, 0.0925, 0.1364, 0.0809, 0.1087, 0.1515, 0.0787, 0.0809, 0.1642)^T$,

$P^{2,2} = (0.075, 0.0472, 0.075, 0.1306, 0.075, 0.1306, 0.1306, 0.075, 0.1027, 0.1583)^T$,

$P^{2,3} = (0.0640, 0.0546, 0.1102, 0.0824, 0.1102, 0.0824, 0.1380, 0.0824, 0.1101, 0.1657)^T$.

The consensual preference vector associated with the retailer is:

$P^{2,c} = (0.0640, 0.0516, 0.0926, 0.1165, 0.0887, 0.1072, 0.14, 0.0787, 0.0979, 0.1627)^T$.

Thus, it is: $s_{10} \succ s_7 \succ s_4 \succ s_6 \succ s_9 \succ s_3 \succ s_5 \succ s_8 \succ s_1 \succ s_2$.

It is assumed that E^1 has SEQ behavior while E^2 has GMR behavior, which implies that $S^1 = \{s_2, s_3, s_5, s_6, s_8, s_9, s_{10}\}$ and $S^2 = \{s_4, s_6, s_7, s_{10}\}$. Finally, the set of states with consensus is $S_t = S^1 \cap S^2 = \{s_6, s_{10}\}$.

V. Comparison Analysis

In this section, we undertake a comprehensive comparison between our enhanced graph model and existing graph models [3, 4, 10, 13, 14, 41, 42, 47]. Notably, we highlight the salient attributes of our proposed graph model by juxtaposing them with the principal characteristics of these ten related graph models. This analysis aims to underscore the unique features and advantages that our model brings to conflict resolution and decision-making frameworks.

Table 9: Comparative analysis of our enhanced graph model with select relevant graph models

Graph models	Preference modeling	Decision-maker VS Composite conflicting party	Consensus issue	Stability concept
Fang et al. [10]	Crisp preference	Individual decision-maker	Not existed	Homogenous behaviors
Wu et al. [42]	Hesitant fuzzy preferences	Composite decision-maker	Not considered	Homogenous behaviors
Bashar et al. [3]	Interval fuzzy preference	Individual decision-maker	Not existed	Homogenous behaviors
Bashar et al. [4]	Fuzzy and interval fuzzy preferences	Individual decision-maker	Not existed	Homogenous behaviors
Wu et al. [41]	Incomplete fuzzy reciprocal preference relations	Individual decision-maker	Not existed	Homogenous behaviors
Garcia et al. [13]	Crisp preference	Individual decision-maker	Not existed	Homogenous behaviors
Yu et al. [47]	Unknown and fuzzy preferences	Individual decision-maker	Not existed	Homogenous behaviors
Hamouda et al. [14]	Crisp preference with intensity information	Individual decision-maker	Not existed	Homogenous behaviors
Graph model with internal consensus reaching and external game	**BWM with CLEs**	**Composite conflicting party**	**Preference information loss based consensus reaching model**	**Heterogeneous behaviors**

(1) *Preference modeling:* Within this study, an innovative preference modeling methodology called "BWM with CLEs" is introduced to capture conflicting parties' preferences regarding game states within the graph model framework. The BWM, being a

pairwise comparison-based technique, not only demands less data but also yields more coherent outcomes as compared to existing methods of pairwise comparison-based preference determination. Furthermore, CLEs offer a versatile approach to expressing preferences, making them particularly adept at addressing uncertainty in intricate decision scenarios. Furthermore, CLEs, compared to existing linguistic assessment models, is more adept at tackling uncertainty in intricate decision scenarios.

(2) Decision-Maker vs. Composite Conflicting Party: A notable distinction between our approach and prevalent graph models lies in the treatment of conflicting parties. Compared with studies on individual decision-maker (e.g., Refs. [3, 4, 10, 13, 14, 41, 47]), this study considers multiple individuals within a composite conflicting party. Distinct from studies on composite decision-maker (e.g., Ref. [42]), our study delves into the preference divergence among individuals within each composite conflicting party, and proposes a consensus model to address this divergence, thereby augmenting the ability to navigate real-world conflict decision-making scenarios. It's important to note that if all composite conflicting party consist of only one individual, our proposed framework reverts to the conventional framework.

(3) Addressing Consensus: Acknowledging that each composite conflicting party might comprise multiple individuals, the potential for diverse value systems or preference values over game states within the group emerges. To effectively navigate preference conflicts among individuals within a given composite conflicting party, we've devised a novel consensus reaching model rooted in minimizing preference information loss. This pioneering model addresses the challenge of achieving consensus within composite conflicting parties during conflict resolution, culminating in collective decision outcomes harmonized within each composite conflicting party entity. Importantly, this stands as the inaugural graph model to effectively grapple with consensus issues among individuals within composite conflicting parties, demonstrating its ability to foster harmonious collective decisions amidst conflicts complexities.

(4) *Stability concept:* It's worth highlighting that prevailing stability concepts in extant graph models uniformly presuppose uniform behaviors among conflicting parties, encompassing foresight ability and risk attitude. However, this study pioneers a groundbreaking perspective by introducing the concept of game consensus. This paradigm shift considers the inherent diversity in behaviors pertaining to foresight ability and risk attitude. Unlike existing stability concepts, the proposed game consensus acknowledges and accommodates the heterogeneous behaviors exhibited by conflicting parties, heralding a remarkable stride towards a more nuanced and realistic understanding of stability in complex decision scenarios.

A detailed comparison between the existing graph models and the graph model with internal consensus reaching and external game is summarized in Table 9.

The aforementioned comparison underscores how the outcomes of this study contribute to the enhancement and enrichment of the existing landscape of decision-making research. These findings significantly bolster decision-making support for intricate and multifaceted conflict resolution challenges.

VI. Conclusion

This study introduces an enhanced graph model featuring an internal consensus reaching process and an external game process, specifically tailored to address dual conflict decision-making scenarios. These scenarios involve both preference conflicts among internal individuals within a particular composite conflicting party, and game conflicts among different external composite conflicting parties. The primary contributions of this study include:

(1) Enrichment of Preference Modeling: This research enhances the scope of preference modeling within the graph model framework by introducing the BWM combined with CLEs. This preference modeling approach effectively captures individuals' preferences concerning game states. Furthermore, the study introduces variations such as BWM with LDAs and the application of [0, 1] additive scale to enhance the handling of preference information obtained through BWM with CLEs.

(2) Resolution of Internal Preference Conflict: The study effectively tackles preference conflicts among internal individuals within a composite conflicting party through a consensus reaching model. This is achieved using an optimization approach that minimizes the discrepancy between the preference information held by individuals and their corresponding priority vectors related to game states. Furthermore, the research presents a method to transform CLEs into LDAs, which are then further translated into numerical values within the [0, 1] range using the BWM methodology.

(3) Introduction of Game Consensus: The study introduces the concept of game consensus to capture the external game dynamics between composite conflicting parties. This novel concept extends the familiar stability principles within the graph model framework by accounting for heterogeneous behaviors in terms of foresight ability and risk attitude. Compared to existing stability concepts, game consensus provides a more accurate representation of real-world scenarios in conflict problems, thus enhancing the comprehensiveness of the model.

(4) The effectiveness of the graph model with internal consensus reaching and external game is validated with a case study on the price conflict in dual-channel supply chain. Moreover, comparisons show that this study provides a new and more effective method for conflict analysis, as well as novel theoretical insights and practical implications on how to improve conflict resolution.

The proposal of this study can provide the decision support to help composite conflicting party manage dual conflict decision-making problems, and this capability will be instrumental in addressing supply chain dual conflicts, trade negotiations dual conflicts and environmental dual conflicts. Moreover, three promising directions for future research warrant attention:

(1) Social network is a trust relationships structure composed of various individuals, which are widespread in groups, organizations [9, 49]. It is worth mentioning that dual conflict decision-making problems involve multiple individuals and conflicting parties. Therefore, it is interesting to extend our proposal to the social network context and investigate its impact.

(2) In reality, personalized individual semantics (PIS) are widely present in linguistic GDM [28, 51, 52]. Furthermore, individuals may exhibit deceptive preferences to further their own interests, a phenomenon referred to as strategic behavior [32]. Therefore, we contend that integrating PIS and strategic behaviors into future research would be valuable, rendering the associated studies more practical.

(3) Within the framework of existing graph models, it is typically assumed that conflicting parties possess complete knowledge of their opponents' preferences, a scenario rarely encountered in real conflict decision-making. Hence, there is a need to broaden the current notion of stability, for example Minimax regret stability [40], to render it more applicable to real-world scenarios.

Appendix

Appendix. Proofs of Propositions.

Proof of Proposition 1: In model (8), constraints (a) and (b) guarantee that $|(n-1)(p_b-p_i)/2+0.5-f_{bi}|\le c_{bi}$, and constraints (c) and (d) ensure that $|(n-1)(p_i-p_w)/2+0.5-f_{iw}|\le d_{iw}$. The objective function attains its optimal value solely when $|(n-1)(p_b-p_i)/2+0.5-f_{bi}|=c_{bi}$ and $|(n-1)(p_i-p_w)/2+0.5-f_{iw}|=d_{iw}$. Consequently, model (7) can be equitably transformed into model (8). This concludes the proof of Proposition 1.

Proof of Proposition 2: Consider Ω as the feasible region of model (16). Let Ω_1 be feasible region of model (16) when setting $\rho^{k}_{b^{k}i,t} = \bar{\rho}^{k}_{b^{k}i,t}$ and $\rho^{k}_{iw^{k},t} = \bar{\rho}^{k}_{iw^{k},t}$. It is clear that $\Omega_1 \subseteq \Omega$. So, (20) holds.

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