A Dynamic Multi-objective Evolutionary Algorithm based on Intensity of Environmental Change

Yaru Hu[a,b], Jinhua Zheng[b,c,*], Juan Zou[b], Shengxiang Yang[d], Junwei Ou[b], Rui Wang[e]

[a] *The Department of Mathematics and Computational Science, Xiangtan University, Xiangtan 411105, China*
[b] *Key Laboratory of Intelligent Computing and Information Processing (Ministry of Education), Xiangtan University, Xiangtan, Hunan, 411105, China*
[c] *Hunan Provincial Key Laboratory of Intelligent Information Processing and Application, Hengyang Normal University, Hengyang, 421002, China*
[d] *School of Computer Science and Informatics, De Montfort University, Leicester LE1 9BH, U.K.*
[e] *College of systems Engineering, National University of Defense Technology, Changsha , Hunan, 410073, China.*

Abstract

This paper proposes a novel evolutionary algorithm based on the intensity of environmental change (IEC) to effectively track the moving Pareto-optimal front (POF) or Pareto-optimal set (POS) in dynamic optimization. The IEC divides each individual into two parts according to the evolutionary information feedback from the POS in the current and former evolutionary environment when an environmental change is detected. Two parts, the micro-changing decision and macro-changing decision, are implemented upon different situations of decision components in order to build an efficient information exchange among dynamic environments. In addition, in our algorithm, if a new evolutionary environment is similar to its historical evolutionary environment, the history information will be used for reference to guide the search toward promising decision regions. In order to verify the availability of our idea, the IEC has been extensively compared with four state-of-the-art algorithms over a range of test suites with different features and difficulties. Experimental results show that

*Corresponding author

Email addresses: huyaru1199@gmail.com (Yaru Hu), jhzheng@xtu.edu.cn (Jinhua Zheng), zoujuan@xtu.edu.cn (Juan Zou), syang@dmu.ac.uk (Shengxiang Yang), junweiou@163.com (Junwei Ou), ruiwangnudt@gmail.com (Rui Wang)

the proposed IEC is promising.

Keywords:
micro-changing decision and macro-changing decision, Evolutionary algorithms, intensity of environmental change, evolutionary information feedback

1. Introduction

There exists a large class of static multi-objective optimization problems (MOPs) [1, 2] in daily life, such as industrial scheduling [3], controller design [4], weapon selection [5] and machine learning [6]. Currently, various multi-objective evolutionary algorithms (MOEAs) are capable of attaining the optimization goals of convergence and diversity with high efficacy [7] in dealing with static MOPs. However, state-of-the-art MOEAs have not been very successful [8] in solving real-world MOPs with uncertainties and dynamics (generally, these kinds of MOPs are called DMOPs). Uncertainties and dynamics of DMOPs pose significant challenges for MOEAs in solving DMOPs, since DMOPs not only have multiple mutually conflicting objectives but also possibly changing objective functions and parameter settings (e.g., decision variables, constraints and the number of objectives).

Because DMOPs are frequently present in dynamic scheduling [9, 10, 11], path planning [12, 13, 14], resource allocation [15, 16] and other application areas in nature [17, 18], there has been a rapid increase of research in dealing with DMOPs from various application areas in recent years. Using the characteristics of various DMOPS from different fields, we can divide DMOPs into the following categories [8] : **1)** POS changes and POF does not change, **2)** Both POS and POF change, **3)** POS does not change, but POF changes. **4)** Both POS and POF do not change. Notice that the POS is the optimal decision variable in the decision space and the POF is the optimal objective value in the objective space on the time scale. Moreover, there is, of course, a possibility that while the problem is changing, several of the mentioned types of changes can occur simultaneously on the time scale. Usually, researchers mainly concentrate on the first three types of changes [8].

Recently, there has been rising interest in using strategies combined with existing MOEAs to address the most challenging DMOPs. Some of these include a steady-state and generational evolutionary algorithm [19], an algorithm based on the dynamic evolutionary environment model [20], and a hybrid of memory and prediction strategies [21]. Generally, these strategies can be roughly classified as follows: **a)** diversity introduction after a change [22]; **b)** diversity maintenance throughout the run [23]; **c)** memory approaches [24]; **d)** prediction approaches [20]; **e)** self-adaptive methods [4] and **f)** multi-population approaches [25]. These techniques have shown competitive performance in handling DMOPs, but they are usually limited [21] in solving a specific group of DMOPs. In particular, diversity-based methods are designed to handle dynamic environments that have a serious loss of diversity and memory-based techniques are not effective when the environmental change is irregular. Prediction-based methods can decrease the speed of convergence when a prediction model is not appropriate in dynamic environments [26].

To solve the aforementioned limitations, some researchers have used a series of hybrid strategies, such as a hybrid of memory and prediction strategies [21]. These hybrid tactics improve convergence and diversity of dynamic multi-objective optimization with high efficacy. Even so, DMOEAs still have plenty of room to improve the performance in dealing with DMOPs. In particular, most of the existing dynamic multi-objective optimization studies [27] assume that the change of the fitness landscape is caused by all of the decision variables in the decision space. However, in real world scenarios, the change may happen because of one or more decision variables. Few studies have focused on this issue. In addition, some researchers have used memory approaches, which reserve the obtained best solutions by DMOEAs at different evolutionary environments to speed up convergence of the population [19]. Nevertheless, this method may cause more evolutions in some generations than other algorithms, and may mislead the population's evolutionary direction because it creates a history of best solutions that do not approach the true POF.

To address these two situations, we propose an evolutionary algorithm based on the intensity of the environmental change (IEC) to efficiently solve DMOPs. The motivation of the IEC mainly considers:

(1) How can the influence of each decision variable of the decision space in the dynamic environment be detected? In this paper, we test this influence by mathematical model and then mark the micro-changing decision and macro-changing decision.

(2) What should be done to micro-changing and macro-changing decisions, respectively when the environment changes? This aim is to find the possible reasons why the current population cannot adapt to the underlying evolutionary environment and then overcome it as optimally as possible when an environmental change is detected, thereby attaining fast convergence capabilities in tracking the changing optimum solutions effectively.

(3) How can history information be preserved in order to yield the greatest returns on the performance of convergence and diversity? Rational operation not only makes the current population use reference history information to guide the search toward the changed optima, but also possesses fast convergence capabilities to track the varying optimum solutions effectively.

The rest of this paper is organized as follows. Section 2 provides background on dynamic multi-objective optimization. Section 3 presents the proposed algorithm. The underlying multi-objective evolutionary algorithm (MOEA), MOEA with Decomposition based on Differential Evolution (MOEA/D-DE), and the IEC-based prediction method are also elaborated on. Section 4 describes the experimental setting. Section 5 describes the experiment results and highlights performance comparisons. Section 6 concludes this paper and discusses potential future research directions.

2. Background

2.1. Definitions of DMOPs

In this paper, we consider that minimization problems and DMOPs [28] can be presented as follows:

$$\begin{cases} min\ \mathbf{F}(\mathbf{x},t) = \{f_1(\mathbf{x},t), f_2(\mathbf{x},t), ...f_M(\mathbf{x},t)\}, \\ s.t. g(\mathbf{x},t) \leq 0, h(\mathbf{x},t) = 0, \\ \mathbf{x} \in [L,U], \end{cases} \tag{1}$$

where t represents the time variable and $\mathbf{x} = (x_1, x_2, ..., x_n)$ is the decision vector. [L,U] = $\{\mathbf{x}=(x_1,...,x_n)|l_i \leq x_i \leq u_i, i = 1,2,..n\}$ is the decision space, where L = $(l_1, ...l_n)$ and U = $(u_1, ..., u_n)$ are the lower and upper bounds, respectively. $\mathbf{F} = (f_1, f_2, ..., f_m)$ is the m-dimensional objective vector. And g($\mathbf{x}$,t) $\leq$ 0 and h($\mathbf{x}$,t) = 0 are the inequality and equality constraints. The time step, t, is associated with the generation counter in MOEAs and can be defined as follows [8]:

$$t = \frac{1}{n_t}\lfloor\frac{\tau}{\tau_t}\rfloor, \tag{2}$$

where τ is the generation counter; n_t is change severity, and τ_t is change frequency.

Definition 1. *Pareto Dominance [29]*: Assume that p and q are any two individuals in the population; p is said to dominate q, written as $f(p) \prec f(q)$ if $f_i(p) \leq f_i(q)\ \forall i \in 1,2,...,m$ and $f_j(p) < f_j(q)\ \exists j \in 1,2,...,m$.

Definition 2. *Pareto-Optimal Set (POS) [28, 30]*: x is the decision vector; Ω is the decision space; F is the objective function. A solution is said to be nondominated if it is not dominated by any other solutions in Ω. Thus, the POS is the set of all nondominated solutions and can be defined mathematically as follows:

$$POS := \{x \in \Omega | \neg\exists x^* \in \Omega, F(x^*) \prec F(x)\}. \tag{3}$$

Definition 3. *Pareto Optimal-Front (POF)*: x is the decision vector; Ω is the decision space; F is the objective function. Thus, the POF is the set of all nondominated solutions with respect to the objective space and can be defined mathematically as follows:

$$POF := \{y = F(x) | x \in POS\}. \tag{4}$$

2.2. Related Work

Why have researchers chosen memory strategies [26, 31, 21]? To the best of the authors' knowledge, there are numerous DMOPs in various application areas that are both predictable and cyclic (e.g., how much electricity people use at different times of the day). For these DMOPs, memory approaches are highly effective to guide the search toward the best optimization when there is adequate memory information including different situations of the evolutionary environment in the whole optimization process. However, memory strategies still have two typical problems: **1)** how do you identify which historical information is promising for the current population without increasing the number of evolutions? **2)** how do you identify whether the current evolutionary environment has already happened?

The first question plays an important role in guiding the search toward the promising areas in the objective space or the decision space. When the memory information is immature, historical experience negatively impacts the evolution of the current population toward the POF or POS. Zhou et. al proposed a population prediction strategy (PPS) [26] which shows that the PPS is worse at the beginning stages because there is limited history information. Liu et. al proposed a memory enhanced dynamic multi-objective evolutionary algorithm [31]. There are relatively few studies that focus on the second problem. This research reduces the pressure of evaluation of the history individuals, such as in proposed hybrid of memory and prediction strategies [21].

Why do researchers research the intensity of the environmental change when the environment changes? Most research [21, 31, 26] assumes that the change of fitness landscape happens because the optimal position of each decision in the decision space changes. However, in daily life, the change of fitness landscape may happen in only part of the variables within the whole decision space. Therefore, each variable of some individual in the population is adaptively reconstructed once the environment changes, which does not accelerate the convergence process and may destroy the requirement of continuously tracking the moving POF and/or POS.

To sum up, if we can detect that the change of the evolutionary environment leads to the change of the optimal position of some variables in the decision space, it is advantageous to accelerate the goals of convergence and diversity with high efficacy [8, 32]. Building on the advantages and disadvantages of the memory method and the superiority of researching the intensity of environmental change, we put forward a novel approach combining intensity of the environmental change and a new memory method.

3. Proposed algorithm

Algorithm 1 The overall framework of IEC

Require: N (population size), t (time step), $flag$ (a boolean variable).
Ensure: P_t;

```
1: Set t = 0, Opool = θ, Dpool = θ, index = −1, flag = false;
2: Initialize a population P_t = {x_1, x_2, ··· x_N}
3: while stopping criterion is not met do
4:     if change detected then
5:         P_t is updated by Algorithm 2;
6:         P_t is updated by Algorithm 3;
7:     else
8:         Optimize the tth population using MOEA/D-DE [7] in dealing with DMOP;
9:     end if
10: end while
11: Return P_t ;
```

It is well established that the key design point for the hybridization of state-of-the-art EAs with the change response in a dynamic evolutionary environment lies in the successful promotion of convergence and diversity between the forces of evolution and the change response of individuals. When the balance between genetic evolution and the introduced change response are not appropriately made, the performance of hybrid EAs is often severely degraded. This balance naturally involves several important issues as follows:

(1) Should the prediction or memory strategy be used in an EA for a particular problem by section 2.2 ?

(2) How many of the old individuals should be replaced by the newly introduced individuals?

(3) Which individuals should be selected to perform the replaced operation?

(4) Which strategy of change response should be applied and when?

We first provide an overall framework about the MOEA/D-DE with IEC in Algorithm 1. In the following subsections, we address the aforementioned issues combining multi-objective optimization problems with complicated pareto sets, MOEA/D and NSGA-II (MOEA/D-DE) [7] with IEC. In order to understand clearly the update of population P_t by Algorithm 2 and Algorithm 3, we elaborate the process of the update as shown in Fig 1. If a similar change is detected, 30% of the individuals are updated based on historical information, 30% of the individuals are updated the storage pool and 30% of the individuals are updated based on the U-test introduced in section 3.1. This is because it can reduce the influence that the historical information may not be the best for the current evolutionary environment. Otherwise, 50% of the individuals are updated based on the U-test and 30% of the solutions are used in updating the storage pool. Therefore, in Section 3.1, we introduce the U-test [33] and its specific implementation of the IEC strategy. Finally, the population (P_t) is reconstructed according to the mechanism described in section 3.2 and 3.3 whenever the underlying environment changes.

3.1. Introducing the U-test

In this important first step, the U-test is used to distinguish some variables of the decision space that are greatly influenced by the environmental change. Then, we improve these variables in the decision space rather than improving all

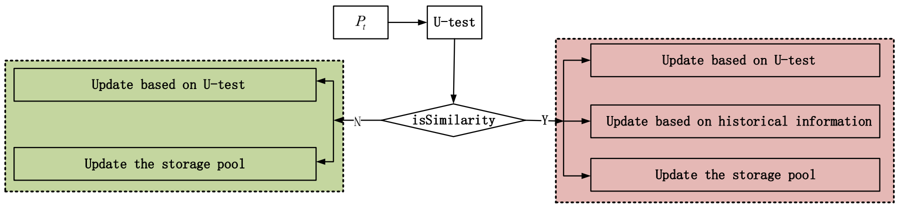

Figure 1: The process of dynamic optimization.

the variables, which like the existing change response to enhance the capacity that the current population quickly trace the moving POS and/or POF in this paper. The U-test is a violation of the parametric assumption of homogeneity of variance for equal and unequal sample sizes [33]. Although the t-test [34] can replace the U-test in many situations, we still choose the U-test when the sample is large; in other words, when the number of the sample is greater than θ [34], generally. Here, we employ the optimal population of the previous two continuous time steps as the two samples of the U-test. The U-test is defined as:

(1) If $\gamma_i > \beta$, we consider that H_0 can be accepted;

(2) If $\gamma_i \leq \beta$, we consider that H_1 can be accepted;

where β is a predefined threshold in the U-test; the H_0 that is the first assumption can be accepted if and only if the result γ_i by the U-test for ith decision variable is larger than β; in contrast, the H_1 that is the second assumption can be accepted. Notice that H_0 represents holding the large intensity of a change to the previous consecutive two historical changes on the ith decision variable in this paper, which is contrary to the second assumption H_1. The $\gamma = (\gamma_1, \gamma_2, \gamma_3, \cdots, \gamma_n)$ can be defined as:

$$\gamma_i = \frac{|E(x_i^t) - E(x_i^{t-1})|}{\sqrt{\frac{(Var(x_i^t))^2 + (Var(x_i^{t-1}))^2}{N}}}, \tag{5}$$

where N is the number of the population; n is the number of decision variables; $E(x_i^t)$ is the average value of each dimension of all the decision variables in the population at time step t; $Var(x_i^t)$ represents the variance of the sample (i.e., each dimension of the population in the paper) at time step t, and its function is calculated as:

$$Var(x_i^t) = E[(x_i^t - E(x_i^t))^2], \tag{6}$$

where x_i^t represents the ith dimension of the individual from the population at t time.

In this paper, we use the U-test to test the intensity of the environmental change on each decision variable. If the values γ are larger on decision variables than the predefined threshold, we consider that these decision variables are holding a large change due to the new environmental change (called: macro-changing decision). Macro-changing decisions possess a larger intensity of change than the rest of the decision variables (called: micro-changing decision) in the decision space. In the meantime, we consider that the optimal values of the micro-changing decisions can be obtained by the forces of evolution. Therefore, determining how to improve the variables with a large intensity of change is crucial for quickly responding to the changing environment.

3.2. Update Mechanisms based on Historical Information

As is well-known, although many DMOPs are dynamic, they still can be solved by referencing historical information when there is an existing similar historical evolution. One of the characteristics of DMOPs is their predictable properties. Additionally, the dynamic change is cyclical and/or only includes several situations in the whole process of population evolution. Hence, if DMOEAs can recognize and utilize efficiently underlying approximate historical information at each change during the whole evolutionary process, it could achieve faster convergence when solving different DMOPs. Existing memory-based approaches [31, 21] usually store several individuals by some underlying strategy, while none of them have taken the diversity of the stored individuals into consideration.

Algorithm 2 The update mechanisms based on historical information

Require: P_t; n; N; Dpool (this is a storage pool about historical changes); Opool(this is a storage pool about detective individuals); *index* (the position index of the similar environment in Dpool and Cpool).

Ensure: P_t, Dpool, Opool;

$flag = isSimilarity(index), i = 0$;
if flag **then**
    while $i < N/3$ **do**
        Add $individual[i * 3]$ to |Dpool|th row of Dpool.
        $individual[i * 3] = Dpool[index][i]$;
        i++;
    end while
    Carry out update operation [21] for Opool by the change identification;
    Delete *index*th row of Opool and Dpool;
else
    while $i < N/3$ **do**
        Add $individual[i * 3]$ to |Dpool|th row of Dpool;
        i++;
    end while
    Carry out update operation [21] for Opool by the change identification;
end if
Return P_t, Dpool, Opool;

Accordingly, we propose a novel memory strategy. It not only considers the convergence of the stored individuals, but also considers the diversity of the stored individuals. The maximum 30% of population members are chosen according to the relationship between the population and weight vectors. Then they are replaced by historically close individuals. One out of three individuals of the population on average is saved as a reference individual to historical information (called Dpool), whose aim is to make the historical information as diversified as possible on the whole POF. To detect the existing historical approximated evolutionary environment, we employ the change identification method [21] to produce a storage pool (called Opool) about detective individuals and return index (*index*). isSimilarity() is a distinguishable method from the hybrid of the memory and prediction strategies [21]. The similarity means that the POS and POF are similar at two different time steps. If we can distinguish similarity from historical environment, the memory strategies can be used to improve the convergence speed of the population. Thus, using the similarity can address DMOPs effectively [21]. The detailed implementation of the update mechanism based on historical information is given in Algorithm 2.

Fig. 2 gives a simple example to illustrate this update mechanism based on historical information. Assume that red points represent the individuals of the *index*th row of the Dpool. In particular, nine weight vectors divide the underlying objective space into nine subspaces, where the corresponding individual of each subspace possesses only one. The first individual is replaced by the first individual of the *index*th row of the Dpool; the fourth individual is replaced by the second individual of the *index*th row of the Dpool; in a similar way, the seventh individual is replaced by the second individual of the *index*th row of the Dpool.

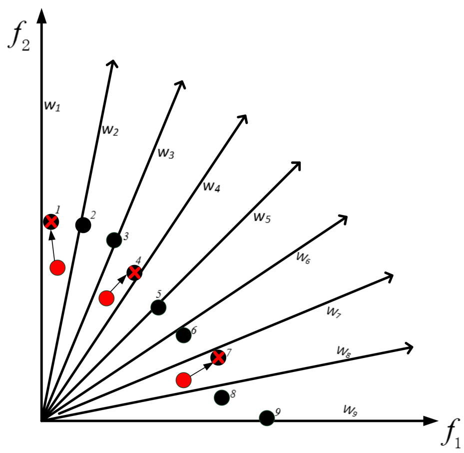

Figure 2: Update of some individuals in the current population

3.3. Update Mechanisms based on U-test

Many existing change responses act on all variables in the decision space and generate individuals by exploiting their new locations. However, with a DMOP, there often exist some variables of individuals that perform well in the decision space. Therefore, searching based on information exchange among variables of individuals in the decision space seems to be beneficial for producing better individuals. Here, we concentrate on the macro-changing decisions of all individu-

Algorithm 3 The Update Mechanisms based on U-test

Require: P_t; $D(D_1, D_2, \cdots, D_n)$ (the difference between vector C^t and C^{t-1}); n; N.

Ensure: P_t (the current population after updating mechanism)

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ϒ is obtained by Formula 5;
if flag ==false then
    Set i = 0; j = 0;
    while i < n do
        while j < N do
            if (γ_i > β) ⋃ (j%2 ≡ 0) then
                individual[j][i] is updated by Formula 7;
            end if
            j++;
        end while
        i++;
    end while
else
    Set i = 0; j = 0;
    while i < n do
        while j < N/3 do
            if γ_i > β then
                individual[j * 3 + 1][i] is updated by Formula 7;
            end if
            j + +;
        end while
        i + +;
    end while
end if
Return P_t ;
```

als in the current population. The macro-changing decisions should be urgently addressed since these decision space variables will have a negative influence on individuals of the population in regards to adapting to the new evolutionary environment. This, to some extent, indicates that the gained optimal values of the macro-changing decisions are more difficult by general genetic operators in a limited time. Therefore, we concentrate on all of the macro-changing decisions of the current population and implement a strategy to ameliorate them.

Specifically, we first calculate the population centroid [21] of the population C^t and C^{t-1} at the t and $t-1$ time steps; then, the difference between vector $D(D_1, D_2, \cdots, D_n)$ between centers C^t and C^{t-1} is computed. Finally, the macro-changing decisions in the $\delta\%$ population are updated by the following equation:

$$x_i = x_i + D_i, \tag{7}$$

where x_i is the ith dimension of the individual x in the decision space; n represents the size of the decision space. These steps only apply to the macro-changing decisions of the individuals. The detailed implementation of this update mechanism based on the U-test is given in Algorithm 3.

4. Experimental Setting

This section introduces the benchmark problems, performance metrics, parameter settings, and state-of-the-art dynamic multi-objective algorithms used in the experimental studies.

4.1. Test problems and compared algorithms

Test problems play important roles in assessing and analyzing the performance of an algorithm, thereby guiding its further development. The proposed algorithm is tested on problems from three test benchmark suites FDA [28], dMOP [35], and JY [17]. The FDA benchmark suite is commonly used in the performance evaluation of DMOEAs. The dMOP benchmark problems are an extension of the FDA benchmark suite to test further performance characteristics of dynamic multi-objective optimization algorithms, such as learning that the POS/POF do not change. The problem suite proposed in [17] is very recent and considers the nonlinear correlation of the underlying decision space. In particular, the JY test suite introduces novel characteristics, such as mixed POFs and nonmonotonic and time-varying relationships between variables, which is beneficial for testing the performance of algorithms.

In this empirical study, four popular DMOEAs with different operating principles are selected to verify the advantages of our approach. They are the multi-objective optimization evolution algorithm based on decomposition (MOEA/D) [36], population prediction strategy (PPS) [26], hybrid of memory and prediction strategies (HMPS) [21] and steady-state and generational evolution algorithm (SGEA) [19]. PPS and MOEA/D are representative algorithms of metaheuristics, which are frequently used in comparative studies; SGEA [19] and HMPS [21] are proposed in recent by Jiang et al. and Liang et al., respectively. Each compared algorithm gives a particular description in the corresponding literature.

Table 1: Parameter Settings

Number of decision variables, n	10 for all test problems
Neighborhood; n_r	Size: 20.
Population size	100 for all test problems
Probability that parents are selected from the archive	0.5
Decomposition method	Tchebycheff [36]
Differential evolution	CR=1.0 and F=0.2
Polynomial mutation	$P_m = 0.05$
Crossover probability	$P_c = 0.8$
Severity of change	$n_t = 10$;
Frequency of change	$\tau_t = 30$
The significance level of [37]	0.05 significance level
Number of changes, num	50
Number of generations	$\tau_t * num$

4.2. Performance metrics and parameter settings

To assess the optimal performance of the proposed algorithm in regards to convergence and diversity in dynamic environments, a number of performance metrics are used, including the inverted generational distance (IGD) metric [26], hypervolume difference (HVD) metric [19], generational distance (GD) metric [38] and Schott's spacing (SP) metric [19]. The IGD and HVD synthetical metrics mainly depend on the closeness, distribution, and coverage of an approximation to the true POF. We can use IGD and HVD together with SP and GD to deeply and extensively reveal the algorithms' performance on the test instances. Hence, the convergence GD metric and distribution SP metric also are selected.

1)GD: The GD [38] indicator usually measures the convergence of the population and can be defined as follows:

$$GD(POF_t, P_t) = \frac{\sum_{v \in P_t} d(POF_t, v)}{|P_t|}, \tag{8}$$

where $d(POF_t, v) = min_{u \in POF_t} \sqrt{\sum_{j=1}^{m} (f_j^v - f_j^u)^2}$ is the minimum Euclidian distance between v and the point in POF_t. POF_t is a set of uniformly distributed Pareto optimal points in the POF at time t; P_t is the solution obtained by the algorithms.

2)SP: The SP [19] performance metric is adopted to measure the distribution of the discovered Pareto front. The SP can be expressed by the following formula:

$$SP = \sqrt{\frac{1}{|P_t| - 1} \sum_{i=1}^{|P_t|} (D_i - \overline{D})^2}, \tag{9}$$

where D_i is the Euclidean distance between the ith member in P_t and its nearest member in P_t, and $\overline{D}$ is the average value of D_i. SP measures how evenly the solutions in $|P_t|$ are distributed.

3)IGD: The IGD [26] is a metric which assesses the convergence and diversity of the obtained solution set. The IGD is calculated as follows:

$$IGD(POF_t, P_t) = \frac{\sum_{v \in POF_t} d(v, P_t)}{|POF_t|}, \tag{10}$$

where $d(v, P_t) = min_{u \in P_t} \sqrt{\sum_{j=1}^{m} (f_j^v - f_j^u)^2}$ is the minimum Euclidian distance between v and the point in P_t. POF_t is a set of uniformly distributed Pareto optimal points in the POF at time t; P_t is the solution obtained by the algorithms.

4) HVD: The HVD [19] measures the gap between the hypervolume of the obtained POF and that of the true POF.

$$HVD(POF_t, P_t) = HV(POF_t) - HV(P_t), \tag{11}$$

where P_t is the solution obtained by the algorithm at time t and POF_t is the solution of the true POF at t time. $HV(S)$ is the hypervolume of a set S. The reference point for the computation of hypervolume is $(z_1^t + 0.5, z_2^t + 0.5, ..., z_M^t + 0.5)$, where z_j^t is the maximum value of the jth objective of the true POF at t time and M is the number of objectives.

Fundamental parameter settings of the experiments were introduced by Table 1. Additional parameters of the selected DMOEAs in the experiment are implemented as guided by their original papers. To show the fairness of the results, the Wilcoxon rank-sum test [37] was used to point out significance between different results at the 0.05 significance level.

5. Experimental Analysis

In this section, we first observe the GD, SP, IGD and HVD performance of the five popular DMOEAs under all change severity levels and frequencies $(n_t, \tau_t) = (10, 30)$. Tables 2, 3, 4 and 5 give the mean and standard deviation values of the corresponding metric values obtained by different algorithms under various circumstances. In particular, the best metric values are in bold face. In addition to the metric values, we also keep the Wilcoxon rank-sum test of the GD, SP, IGD and HVD values obtained by different algorithms at each time step. And followed, if its value is smaller than 5%, then one concludes that a significant difference between the two exists; this observation is labeled as † after the value. Otherwise, there is a ‡ after the result value. ‡ and † indicate IEC performs significantly better than and equivalently to the corresponding algorithm, respectively. We give a detailed analysis of the results on the selected

Table 2: Mean and standard deviation values of GD obtained by five algorithms.

Problems	(n_t, τ_t)	IEC	HMPS	SGEA	MOEA/D	PPS
FDA1	$(10, 30)$	**2.24e-3(3.59e-4)**	2.29e-3(2.67e-4)†	3.22e-3(5.56e-5)‡	3.89e-3(3.36e-5)‡	6.34e-3(8.78e-4)‡
FDA2	$(10, 30)$	2.77e-1(4.52e-5)	2.77e-1(5.31e-5)†	**4.38e-2(2.23e-4)**	5.04e-2(1.73e-4)	5.64e-2(1.15e-3)
FDA3	$(10, 30)$	7.28e-2(6.91e-4)	8.70e-2(6.31e-4)‡	**4.25e-2(1.27e-3)**	7.60e-2(2.28e-3)‡	7.64e-2(1.01e-2)‡
dMOP1	$(10, 30)$	**1.94e-3(2.98e-4)**	3.86e-3(1.16e-3)‡	2.11e-3(3.62e-5)†	2.44e-3(2.80e-4)‡	3.23e-3(1.40e-4)‡
dMOP2	$(10, 30)$	**2.42e-3(1.14e-4)**	2.65e-3(9.60e-5)‡	3.32e-3(7.45e-5)‡	4.64e-3(2.03e-4)‡	1.67e-2(2.37e-2)‡
dMOP3	$(10, 30)$	**2.17e-3(7.46e-5)**	2.40e-3(2.33e-4)‡	3.22e-3(5.56e-5)‡	4.07e-3(2.84e-4)‡	1.65e-2(5.66e-3)‡
JY1	$(10, 30)$	**1.59e-3(5.80e-5)**	1.64e-3(6.00e-5)‡	2.85e-3(1.09e-4)‡	1.82e-3(4.50e-5)‡	6.82e-3(3.22e-3)‡
JY2	$(10, 30)$	**4.89e-2(1.72e-4)**	4.89e-2(1.79e-4)†	4.99e-2(5.60e-4)†	4.94e-2(5.00e-5)†	5.91e-2(9.80e-3)‡
JY3	$(10, 30)$	**1.48e-1(4.22e-2)**	1.83e-1(8.13e-4)‡	1.62e-1(5.44e-2)‡	1.66e-1(2.84e-2)‡	2.40e-1(1.23e-2)‡
JY4	$(10, 30)$	**2.26e-3(6.94e-5)**	2.99e-3(3.72e-5)‡	2.60e-3(1.87e-5)‡	2.44e-3(8.69e-5)‡	6.67e-2(5.06e-2)‡
JY5	$(10, 30)$	**2.05e-3(8.59e-4)**	2.12e-3(1.94e-4)‡	2.14e-3(5.30e-5)‡	8.56e-2(5.19e-6)‡	2.36e-3(2.76e-4)‡
JY6	$(10, 30)$	**1.00e-2(1.08e-1)**	8.48e-1(8.18e-2)‡	7.39e-2(9.35e-3)‡	7.26e-2(2.26e-3)‡	5.11e+0(2.26e-1)‡
JY7	$(10, 30)$	1.82e+0(7.16e-1)	1.78e+0(1.24e+0)‡	**4.77e-1(3.52e-1)**	1.33e+0(2.07e-1)	1.11e+1(1.18e+1)‡
JY8	$(10, 30)$	1.19e-2(7.25e-4)	1.43e-2(1.09e-3)‡	3.01e+0(0.00e+0)‡	**6.24e-3(1.42e-4)**	4.61e-3(1.76e-4)
JY9	$(10, 30)$	**2.02e-3(4.32e-4)**	2.46e-3(5.01e-4)‡	2.69e-2(8.16e-5)‡	2.37e-3(2.07e-4)‡	1.12e-2(2.41e-3)‡

DMOPs in Sections 5.1 and 5.2. Finally, we also discuss the effects of being under different change severity levels and frequencies in Section 5.3. Note: if the metrics values of some problems are analogous on the GD, SP, IGD and HVD, we analyze them together.

5.1. Results on FDA1-FDA3 and dMOP

FDA1: From tables 2 and 3, we can see that the convergence of the IEC is the best but its distribution is worse than the SGEA. For all this, tables 4 and 5 still show that the overall performance (which includes the convergence, distribution and extensive coverage of the population from approaching the POF) of the IEC is better than the other algorithms because it obtained the best IGD and HVD values under the underlying dynamic environment. For the results of the IEC, a conclusion can be drawn which is that the SP metric obtained by the different algorithms is only one part of the overall performance of an algorithm (i.e., the distribution of the obtained solutions by the IEC is

Table 3: Mean and standard deviation values of SP obtained by five algorithms.

Problems	(n_t, τ_t)	IEC	HMPS	SGEA	MOEA/D	PPS
FDA1	(10, 30)	4.99e-3(3.29e-5)	5.07e-3(4.30e-5)‡	**3.57e-3(8.64e-5)**	6.50e-3(1.23e-4)‡	3.61e-3(1.13e-4)
FDA2	(10, 30)	**3.05e-3(1.42e-5)**	3.11e-3(2.57e-5)‡	6.61e-3(2.04e-3)‡	1.81e-2(4.27e-4)‡	4.68e-3(7.32e-5)‡
FDA3	(10, 30)	**1.00e-2(1.29e-4)**	1.05e-2(1.02e-4)†	3.24e-2(1.01e-3)‡	7.01e-1(2.65e-4)‡	4.77e-3(3.56e-4)†
dMOP1	(10, 30)	**3.08e-3(3.53e-5)**	3.39e-3(4.93e-5)‡	3.19e-3(1.45e-4)†	4.31e-3(2.98e-5)‡	3.22e-3(8.57e-5)†
dMOP2	(10, 30)	**3.46e-3(4.65e-5)**	3.55e-3(4.36e-5)‡	3.79e-3(1.07e-4)‡	6.05e-3(8.96e-5)‡	3.84e-3(9.82e-4)‡
dMOP3	(10, 30)	**2.95e-3(3.55e-5)**	5.12e-3(3.20e-5)‡	3.57e-3(8.64e-5)‡	6.67e-3(1.37e-4)‡	5.06e-3(7.13e-4)‡
JY1	(10, 30)	**1.04e-2(2.45e-4)**	1.04e-2(1.95e-4)†	6.39e-2(2.14e-4)‡	1.18e-2(9.57e-5)	5.55e-2(6.48e-4)‡
JY2	(10, 30)	**4.38e-3(1.50e-4)**	7.12e-3(1.03e-3)‡	4.98e-3(1.57e-4)‡	7.69e-3(2.31e-4)‡	5.99e-3(2.10e-3)‡
JY3	(10, 30)	**1.04e-2(1.56e-3)**	1.80e-2(4.91e-3)‡	1.06e-2(1.41e-3)†	1.11e-2(5.50e-4)‡	1.18e-2(1.06e-3)‡
JY4	(10, 30)	**1.80e-3(5.67e-5)**	1.86e-3(3.13e-5)†	1.91e-2(2.34e-4)‡	2.47e-3(2.60e-4)‡	7.56e-3(1.89e-3)‡
JY5	(10, 30)	1.24e-2(4.28e-3)	1.01e-2(1.64e-3)	**3.48e-3(3.37e-4)**	8.52e-3(3.40e-4)	3.53e-3(1.84e-4)
JY6	(10, 30)	**3.05e-2(1.20e-2)**	1.19e-1(1.74e-2)‡	3.67e-2(2.90e-3)‡	5.93e-2(2.03e-2)‡	2.18e-1(1.20e-2)‡
JY7	(10, 30)	5.37e-2(2.39e-2)	1.60e-1(6.18e-2)‡	5.47e-2(1.27e-2)‡	**4.63e-2(8.58e-3)**	9.05e-1(7.72e-1)‡
JY8	(10, 30)	1.97e-2(1.24e-3)	1.90e-2(8.09e-4)	**3.06e-3(3.11e-3)**	1.15e-2(2.06e-4)‡	4.57e-3(5.83e-5)
JY9	(10, 30)	**1.10e-2(6.13e-4)**	1.10e-2(3.97e-4)†	4.90e-2(2.71e-4)‡	1.16e-2(1.50e-4)†	6.18e-3(1.22e-4)†

not better than SGEA and PPS, but its overall metric is better than the other algorithms.).

FDA2: It can be observed from Table 3 that although the distribution of the IEC is better than the other algorithms for FDA2 (i.e., the obtained small value of the SP by IEC), its overall performance is significantly inferior to the other four algorithms across the underlying environmental changes. This may be because its convergence is worse than the other algorithms (i.e., the biggish GD values of Table 2 on FDA2). Considering as a whole, the convergence of IEC may need to be enhanced to effectively deal with FDA2. The Wilcoxon rank-sum test for the corresponding metric values obtained by different algorithms under various circumstances shows that IEC is better than HMPS, but not significantly better than HMPS on FDA2 (i.e., as shown in tables 2, 3, 4 and 5 on FDA2).

FDA3: Tables 3, 4 and 5 show that IEC with the use of random solutions is very capable of tracking the changing POF on FDA3; in other words, the

Table 4: Mean and standard deviation values of IGD obtained by five algorithms.

Problems	(n_t, τ_t)	IEC	HMPS	SGEA	MOEA/D	PPS
FDA1	(10, 30)	**4.57e-3(3.91e-5)**	4.61e-3(1.80e-5)‡	5.25e-3(1.46e-4)‡	8.25e-3(1.55e-4)‡	7.71e-3(7.05e-4)‡
FDA2	(10, 30)	3.53e-1(4.26e-5)	3.64e-1(3.67e-5)†	**2.86e-2(5.48e-5)**	4.08e-2(1.56e-3)‡	4.10e-1(3.00e-4)‡
FDA3	(10, 30)	**1.08e-2(1.84e-3)**	1.24e-2(8.25e-4)‡	2.13e-2(1.366e-3)‡	4.36e-2(1.69e-3)‡	7.33e-2(1.00e-3)‡
dMOP1	(10, 30)	**4.57e-3(3.46e-4)**	4.83e-3(2.59e-4)‡	8.17e-3(2.17e-3)‡	5.51e-3(6.18e-4)‡	5.58e-3(5.41e-4)‡
dMOP2	(10, 30)	**4.62e-3(4.62e-5)**	4.72e-3(1.88e-5)‡	6.07e-3(8.38e-4)‡	8.24e-3(5.68e-4)‡	1.40e-2(1.47e-2)‡
dMOP3	(10, 30)	**4.56e-3(3.67e-5)**	4.60e-3(4.26e-5)†	5.25e-3(1.46e-4)‡	8.23e-3(7.33e-4)‡	1.26e-2(2.37e-3)‡
JY1	(10, 30)	**6.11e-3(4.03e-5)**	6.22e-3(4.39e-5)‡	7.89e-3(4.01e-4)‡	7.95e-3(1.21e-4)‡	8.36e-3(2.13e-3)‡
JY2	(10, 30)	**4.98e-2(3.92e-5)**	5.00e-2(2.45e-5)†	5.00e-2(1.17e-4)†	5.09e-2(0.00e+0)†	5.60e-2(7.10e-3)‡
JY3	(10, 30)	**3.10e-1(1.64e-3)**	3.26e-1(4.94e-4)‡	3.38e-1(9.23e-3)‡	3.18e-1(7.89e-3)†	3.14e-1(5.00e-4)†
JY4	(10, 30)	**1.97e-2(6.75e-5)**	2.00e-2(3.54e-5)†	2.08e-2(3.31e-4)†	2.31e-2(2.87e-4)‡	5.96e-2(3.38e-2)‡
JY5	(10, 30)	6.52e-3(6.41e-5)	6.63e-3(5.70e-5)‡	**4.17e-3(1.47e-5)**	6.53e-3(3.26e-5)†	1.06e-2(7.53e-3)‡
JY6	(10, 30)	**4.03e-1(6.38e-2)**	4.18e-1(5.23e-2)‡	9.26e-1(7.75e-3)‡	8.43e-1(1.90e-2)‡	1.87e+0(7.22e-2)‡
JY7	(10, 30)	2.02e+0(4.86e-1)	1.09e+0(7.72e-1)	**3.69e-1(2.49e-1)**	1.00e+0(5.36e-2)	1.72e+0(2.36e+0)
JY8	(10, 30)	1.51e-2(2.67e-4)	1.58e-2(5.53e-4)†	3.69e+0(0.00e+0)‡	1.66e-2(7.22e-4)‡	**8.44e-3(3.77e-5)**
JY9	(10, 30)	**6.66e-3(1.19e-4)**	6.99e-3(1.06e-4)‡	3.44e-2(6.84e-4)‡	9.63e-3(7.05e-4)‡	9.97e-3(8.05e-4)‡

obtained performance significantly outperforms the other algorithms by a clear margin in terms of the IGD and HVD metrics. On the other hand, the GD metric shown in Table 2 is negatively affected by random solutions. In short, the convergence metric GD is only one part of the whole performance of a certain algorithm, but it may impact the overall performance result (i.e., IGD and HVD).

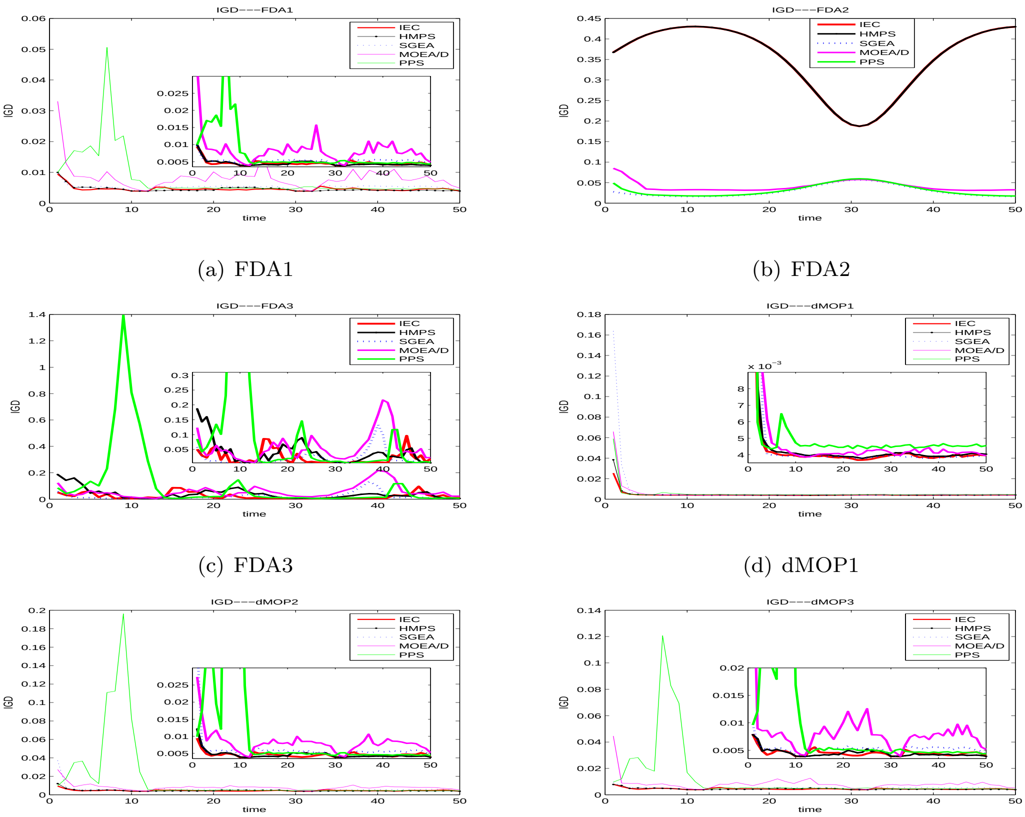
dMOP1: As shown in tables 4 and 5, the performance of IEC is robust as it obtained the best GD and SP values under all dynamic change, implying that it maintains better distribution and convergence of its approximations over changes than the other algorithms on dMOP1. The better distribution and convergence can effectively promote the overall performance of the IEC (i.e., relatively small IGD and HVD metrics in tables 4 and 5). For all the compared algorithms, PPS failed to show encouraging performance on the IGD and HVD metrics; this may be because PPS had slightly poor distribution and convergence

Table 5: Mean and standard deviation values of HVD obtained by five algorithms.

Problems	(n_t, τ_t)	IEC	HMPS	SGEA	MOEA/D	PPS
FDA1	(10, 30)	**7.81e-3(2.82e-4)**	8.37e-3(1.50e-4)‡	1.10e-2(4.32e-4)‡	1.80e-2(5.19e-4)‡	1.60e-2(1.43e-3)‡
FDA2	(10, 30)	5.21e-1(7.05e-5)	5.21e-1(9.47e-5)†	**3.30e-2(5.48e-5)**	3.47e-2(9.57e-5)	5.57e-1(4.69e-4)‡
FDA3	(10, 30)	**4.81e-1(5.92e-4)**	5.18e-1(1.33e-3)‡	5.92e-1(6.35e-3)‡	5.27e-1(2.75e-2)‡	4.96e-1(1.60e-2)‡
dMOP1	(10, 30)	**1.00e-2(1.18e-3)**	1.31e-2(1.37e-3)‡	1.14e-2(1.00e-3)‡	1.07e-2(5.83e-4)†	1.26e-2(9.43e-4)‡
dMOP2	(10, 30)	**1.97e-3(2.19e-4)**	9.79e-3(1.72e-4)‡	1.50e-2(1.27e-3)‡	2.31e-2(1.66e-3)‡	3.44e-2(3.53e-2)‡
dMOP3	(10, 30)	**8.35e-3(1.52e-4)**	8.41e-3(1.60e-4)†	1.10e-2(4.32e-4)‡	1.87e-2(2.06e-4)‡	2.68e-2(5.45e-3)‡
JY1	(10, 30)	**4.79e-3(6.76e-5)**	4.98e-3(7.06e-5)‡	7.82e-3(6.12e-4)‡	6.95e-3(6.60e-4)‡	1.12e-2(3.94e-3)‡
JY2	(10, 30)	**2.15e-3(8.45e-5)**	7.46e-3(1.31e-4)‡	9.33e-3(2.06e-4)‡	8.78e-3(3.77e-5)‡	2.45e-2(1.55e-2)‡
JY3	(10, 30)	**3.31e-3(1.00e-1)**	3.67e-1(9.03e-4)‡	3.27e-1(1.22e-1)‡	3.32e-1(6.79e-2)‡	3.73e-1(7.39e-3)‡
JY4	(10, 30)	**2.15e-2(8.55e-5)**	2.24e-2(4.45e-5)‡	2.40e-2(7.31e-4)‡	2.57e-2(8.18e-4)‡	1.39e-1(9.54e-2)‡
JY5	(10, 30)	**3.92e-3(4.66e-4)**	6.45e-3(9.73e-5)‡	4.72e-3(5.31e-5) ‡	5.28e-3(2.06e-5)‡	7.04e-3(3.32e-4)‡
JY6	(10, 30)	**4.10e+0(6.07e-1)**	5.76e+0(2.30e-1)‡	1.35e+1(1.67e-2)‡	1.28e+1(9.60e-3)‡	1.46e+1(8.96e-1)‡
JY7	(10, 30)	2.20e+1(1.49e+1)	1.34e+1(1.22e+1)	**3.51e+0(3.94e+0)**	1.26e+1(5.67e+0)†	6.16e+1(8.23e+1)‡
JY8	(10, 30)	2.02e-1(1.11e-2)	2.04e-1(1.88e-3)†	1.63e+0(4.59e-2)‡	**1.16e-1(2.87e-3)**	1.20e-1(5.00e-4)
JY9	(10, 30)	**5.41e-3(2.38e-4)**	5.55e-3(2.97e-4)‡	1.40e-2(1.15e-3)‡	9.10e-3(1.76e-3)‡	1.50e-2(1.63e-3)‡

(i.e., large SP and GD metrics on dMOP1 shown in tables 2 and 3). It is interesting to note that the convergence and distribution of SGEA are better than HMPS, MOEA/D and PPS (i.e., slightly small SP and GD values) but SGEA's overall performance is worse than these three algorithms. This may be because the solutions obtained by SGEA have a poor spread coverage of the POF for dMOP1.

dMOP2 and dMOP3: For problems dMOP2 and dMOP3, IEC significantly outperformed the other algorithms by a clear margin in terms of the IGD and HVD metrics of tables 4 and 5, since the solutions obtained by the IEC maintain a uniform distribution and good convergence on the POF, in other words, the values obtained by the IEC algorithm on the corresponding metrics are smaller than the other algorithms whenever the underlying dynamic environment across the whole evolution process when handling the underlying problems dMOP2 and dMOP3. It is worth noting that both PPS and MOEA/D failed to show encouraging performance on all selected metrics, and MOEA/D struggled to maintain a uniform distribution of its obtained POF for dynamic optimization, as indicated by the large SP values in Table 3.



(a) FDA1 (b) FDA2

(c) FDA3 (d) dMOP1

(e) dMOP2 (f) dMOP3

Figure 3: Evolution curves of average IGD values for problems with n_t =10 and τ_t=30.

5.2. Results on JY1-JY9

JY1: The results obtained by different algorithms on different metrics all are quite good in dealing with JY1 across the whole evolution process. Particularly, the IEC and HMPS algorithms possess the same distribution, which is shown by the metric SP (i.e., the SP values of the IEC and HMPS all are 1.04e-2 as shown in table 3) and their whole performance shows better performance than SGEA, MOEA/D and PPS in dealing with JY1 (i.e., the values of the IGD and HVD metrics are significantly less than the obtained values by the other corresponding algorithms during the whole evolution process). Even so, we can still see that the performance gained by IEC slightly overmatch HMPS, which is verified by the Wilcoxon rank sum test at the 5% significance level in the comparisons.

JY2: All of the algorithms are considered on JY2 of tables 2, 3, 4 and 5; IEC obtained the best results on the GD, SP, IGD and HVD metrics in addressing the tested JY2, implying that it maintains better diversity and convergence of its approximations over changes than the other algorithms. It is interesting to note that the IGD is not significantly different (i.e., this is shown in table 4) from IEC, HMPS, SGAEA and MOEA/D, signifying that the population convergence speed of HMPS, SGAEA and MOEA/D are almost like IEC. The HVD values in table 5 on JY2 are roughly consistent with those of IGD illustrated in table 4, but the HVD values acquired by IEC, HMPS, SGAEA and MOEA/D are significantly different. For the situation mentioned, probably because there are the owning boundary solutions in the different extent in HMPS, SGEA, MOEA/D and PPS.

JY3, JY4, JY6 and JY9: During the whole evolution process, the IEC obtained an evolutionary population with better distribution, convergence and coverage of the POF in various dynamic environments. This conclusion can be drawn because the values on all selected metrics in tables 2, 3, 4 and 5 are relatively smaller than the other algorithms. In particular, although the IGD values obtained by IEC slightly precede the HMPS, the HVD values are significantly superior in dealing with JY4. This may be because the several solutions obtained by HMPS are dominance resistant solutions (DRS) which affects the calculation of the HVD metric for solving JY4. In addition, for JY9, the distributions gained by IEC, HMPS and MOEA/D are not markedly different from each other, which may be because they all use the decomposition-based framework.

JY5: On balance by all selected performance metrics in tables 2, 3, 4 and 5, IEC may need some improvement. In particular, the convergence of IEC is the best. However, the set of solutions obtained by the IEC algorithm for approximating the POF has the worst distribution in the comparative experiments. This may be because IEC cannot deal with the loss of diversity well when each environmental change is detected in the whole evolutionary process. So IEC must be improved to enhance the distribution of the gained solutions covering the POF. From tables 4 and 5, we find an interesting situation in that the best IGD values and the best HVD values do not correspond to the same algorithm. This may be because the solutions obtained by the SGEA do not cover the whole POF when dealing with JY5 during the evolutionary process.

JY7 and JY8: JY7 takes into account the shift of the POS, multimodality, and the overall shape of the POF. The number of local optima in JY7 remains fixed, and the overall POF shape can be concave or convex due to environmental changes. Tables 4 and 5 clearly show that IEC mainly loses on JY7 and JY8, whereas SGEA is the best performer in terms of the IGD metric. Clearly, the uncompetitive distribution (i.e., slightly large SP metric) and poor convergence (i.e., relatively large GD metric) of the obtained approximations are the main reasons for the low performance of IEC on JY7 and JY8. For MOEA/D, the IGD and HVD performance were not competitive for the test instances JY7, and this may be caused by their poor solution distribution, as indicated by their large SP values. However, not having the best SP and GD values necessarily result in satisfying the IGD metric, and this can be particularly observed from the case of PPS on JY8, suggesting that PPS converges worse than MOEA/D although it does not provide the best SP and MS metrics on JY8.

Overall, it can be observed from tables 4 and 5 that IEC obtained the best results on the majority of the tested instances, implying that it maintains better distribution, convergence and coverage of an approximation to the true POF than the other algorithms in most cases. Apart from tabular presentation, we provide evolution curves of the average IGD values on the test instances in Fig. 3 on FDA and dMOP problems. It can be clearly seen that, compared with the other algorithms, IEC responds to changes more stably and recovers faster for most of the test problems, thereby obtaining higher convergence performance. The only exception is FDA2, where SGEA performs the best, and due to the lack of population convergence (indicated by poor GD values) when a change occurs.

5.3. Further Discussion

Table 6: Mean and standard deviation values of IGD and HVD metric obtained by five algorithms with different value of τ_t.

Problems	(n_t, τ_t)	metric	IEC	HMPS	SGEA	MOEA/D	PPS
FDA1	(10,10)	IGD	**8.53e-3(5.02e-4)**	3.33e-2(4.73e-3)‡	1.29e-2(9.92e-4)‡	3.19e-2(3.03e-3)‡	8.24e-2(6.49e-2)‡
		HVD	**2.03e-2(1.50e-3)**	8.26e-2(1.26e-2)‡	2.89e-2(4.68e-4)‡	6.34e-2(2.84e-3)‡	1.60e-1(1.17e-1)‡
	(10,20)	IGD	**5.27e-3(9.28e-5)**	1.17e-2(7.28e-4)‡	6.63e-3(3.33e-4)‡	1.19e-2(4.89e-4)‡	3.91e-2(1.70e-2)‡
		HVD	**1.11e-2(1.64e-4)**	2.93e-2(3.17e-3)‡	1.51e-2(5.72e-4)‡	2.86e-2(6.92e-4)‡	9.26e-2(4.27e-2)‡
	(10,30)	IGD	**4.57e-3(3.91e-5)**	4.61e-3(1.80e-5)‡	5.25e-3(1.46e-4)‡	8.25e-3(1.55e-4)‡	7.71e-3(7.05e-4)‡
		HVD	**7.81e-3(2.82e-4)**	8.37e-3(1.50e-4)‡	1.10e-2(4.32e-4)‡	1.80e-2(5.19e-4)‡	1.60e-2(1.43e-3)‡
dMOP2	(10,10)	IGD	**1.02e-2(7.22e-4)**	5.21e-2(7.98e-3)‡	1.78e-2(4.03e-3)‡	3.35e-2(9.67e-3)‡	2.00e-1(5.77e-2)‡
		HVD	**3.41e-2(1.67e-3)**	1.27e-1(2.11e-2)‡	4.07e-2(3.00e-3)‡	8.52e-2(4.07e-3)‡	3.16e-1(6.07e-3)‡
	(10,20)	IGD	**5.56e-3(7.35e-5)**	1.55e-2(2.36e-3)‡	8.33e-3(1.40e-3)‡	1.18e-2(3.83e-4)‡	3.62e-2(3.72e-2)‡
		HVD	**1.23e-2(4.69e-4)**	3.94e-2(5.70e-3)‡	2.06e-2(1.22e-3)‡	3.51e-2(4.71e-4)‡	7.70e-2(6.09e-2)‡
	(10,30)	IGD	**4.62e-3(4.62e-5)**	4.72e-3(1.88e-5)‡	6.07e-3(8.38e-4)‡	8.24e-3(5.68e-4)‡	1.40e-2(1.47e-2)‡
		HVD	**1.97e-3(2.19e-4)**	9.79e-3(1.72e-4)‡	1.50e-2(1.27e-3)‡	2.31e-2(1.66e-3)‡	3.44e-2(3.53e-2)‡
JY7	(10,10)	IGD	2.08e+0(9.57e-1)	1.47e+1(1.27e+0)‡	**1.02e+0(3.27e-1)**	1.84e+0(1.64e-1)	9.36e+0(1.22e+0)
		HVD	6.44e+1(4.14e+1)	1.25e+3(2.10e+2)‡	**7.74e+0(2.75e+0)**	1.34e+1(4.84e+0)	5.02e+2(8.07e+1)
	(10,20)	IGD	1.09e+0(1.00e+0)	5.87e+0(2.26e+0)‡	**4.78e-1(2.04e-1)**	8.67e-1(4.55e-1)	5.88e+0(1.28e+0)‡
		HVD	2.58e+1(2.81e+1)	3.21e+2(1.42e+2)‡	**2.89e+0(2.03e+0)**	1.48e+1(1.36e+1)	2.41e+2(6.96e+1)
	(10,30)	IGD	2.02e+0(4.86e-1)	1.09e+0(7.72e-1)	**3.69e-1(2.49e-1)**	1.00e+0(5.36e-2)	1.72e+0(2.36e+0)
		HVD	2.20e+1(1.49e+1)	1.34e+1(1.22e+1)	**3.51e+0(3.94e+0)**	1.26e+1(5.67e+0)	6.16e+1(8.23e+1)‡
JY9	(10,10)	IGD	**1.76e-2(2.53e-3)**	2.47e-1(8.38e-2)‡	3.94e-1(5.02e-2)‡	3.18e-2(1.02e-2)‡	2.61e-1(2.26e-1)‡
		HVD	**2.82e-2(1.84e-2)**	2.54e+0(1.06e+0)‡	2.82e+0(6.75e-1)‡	2.98e-2(4.76e-3)‡	1.60e+0(1.67e+0)‡
	(10,20)	IGD	**8.39e-3(4.97e-4)**	7.31e-2(2.57e-2)‡	4.87e-2(3.19e-3)‡	1.31e-2(4.50e-4)‡	2.12e-2(2.77e-3)‡
		HVD	**1.03e-2(1.29e-3)**	4.37e-1(1.80e-1)‡	4.06e-2(7.42e-3)‡	1.16e-2(1.30e-3)‡	3.92e-2(8.74e-3)‡
	(10,30)	IGD	**6.66e-3(1.19e-4)**	6.99e-3(1.06e-4)‡	3.44e-2(6.84e-4)‡	9.63e-3(7.05e-4)‡	9.97e-3(8.05e-4)‡
		HVD	**5.41e-3(2.38e-4)**	5.55e-3(2.97e-4)‡	1.40e-2(1.15e-3)‡	9.10e-3(1.76e-3)‡	1.50e-2(1.63e-3)‡

To examine the effect of frequency of change on the algorithms' performance, experiments were carried out on FDA1, dMOP2, JY7 and JY9 problems with n_t fixed to 10, and τ_t set to 10, 20 and 30, which represent severe, moderate, and slight frequencies of change, respectively. Experimental results of five algorithms on the HVD and IGD metrics are given in Table 6. Additionally, to examine the effect of severity of change on the algorithms' performance, experiments

Table 7: Mean and standard deviation values of IGD and HVD metric obtained by five algorithms with different value of n_t.

Problems	(n_t, τ_t)	metric	SSE	DNSGA-II	SGEA	MOEA/D	PPS
FDA1	(5, 10)	HV	**4.57e-2(5.92e-3)**	1.50e-1(4.55e-2)‡	5.31e-2(2.99e-3)‡	9.64e-2(3.90e-3)‡	1.32e-1(4.49e-2)‡
		IGD	**2.00e-2(2.38e-3)**	5.87e-2(1.58e-2)‡	2.22e-2(6.49e-3)‡	7.59e-2(8.44e-3)‡	6.21e-2(2.30e-2)‡
	(10, 10)	HV	**2.03e-2(1.50e-3)**	8.26e-2(1.26e-2)‡	2.51e-2(5.72e-4)‡	2.86e-2(6.92e-4)‡	9.26e-2(4.27e-2)‡
		IGD	**8.53e-3(5.02e-4)**	3.33e-2(4.73e-3)‡	1.29e-2(9.92e-4)‡	3.19e-2(3.03e-3)‡	8.24e-2(6.49e-2)‡
dMOP2	(5, 10)	HV	**5.88e-2(6.46e-3)**	1.97e-1(4.62e-2)‡	6.89e-2(6.58e-3)‡	1.27e-1(3.20e-3)‡	1.55e-1(7.07e-2)‡
		IGD	**2.94e-2(2.43e-3)**	9.01e-2(1.77e-2)‡	3.28e-2(1.16e-2)‡	7.66e-2(4.17e-2)‡	6.79e-2(3.16e-2)‡
	(10, 10)	HV	**2.81e-2(1.67e-3)**	1.27e-1(2.11e-2)‡	3.06e-2(1.22e-3)‡	3.51e-2(4.71e-4)‡	7.70e-2(6.09e-2)‡
		IGD	**1.02e-2(7.22e-4)**	5.21e-2(7.98e-3)‡	1.78e-2(4.03e-3)‡	3.35e-2(9.67e-3)‡	2.00e-1(5.77e-2)‡
JY1	(5, 10)	HVD	**1.37e-2(1.44e-3)**	1.66e-1(3.90e-2)‡	3.45e-2(1.35e-3)‡	2.93e-2(1.37e-3)‡	1.63e-1(1.21e-1)‡
		IGD	**1.00e-2(5.81e-4)**	6.00e-2(1.10e-2)‡	3.32e-2(5.28e-4)‡	3.16e-2(6.53e-4)‡	6.33e-2(3.69e-2)‡
	(10, 10)	HVD	**3.67e-3(6.94e-5)**	7.71e-2(8.89e-3)‡	1.08e-2(2.14e-4)‡	9.54e-3(1.37e-3)‡	9.16e-2(7.59e-2)‡
		IGD	**4.62e-3(7.50e-5)**	3.42e-2(4.00e-3)‡	2.41e-2(1.56e-3)‡	2.27e-2(8.16e-4)‡	3.50e-2(6.00e-3)‡
JY8	(5, 10)	HVD	2.36e-1(1.04e-2)	3.20e-1(8.42e-2)‡	1.76e+0(1.58e-1)‡	**1.16e-1(7.32e-3)**	1.36e-1(7.89e-3)
		IGD	1.72e-2(5.20e-4)	2.89e-2(2.98e-3)‡	3.75e+0(0.00e+0)‡	3.53e-2(5.05e-3)‡	**1.52e-2(3.51e-3)**
	(10, 10)	HVD	2.37e-1(1.20e-2)	3.25e-1(7.22e-2)‡	1.59e+0(2.66e-2)‡	**1.14e-1(3.00e-3)**	1.13e-1(1.00e-3)
		IGD	1.76e-2(7.09e-4)	2.44e-2(2.82e-3)‡	3.69e+0(0.00e+0)‡	3.04e-2(7.16e-3)‡	**9.41e-3(3.65e-4)**

were carried out on FDA1, dMOP2, JY1 and JY8 problems with tau_t fixed to 10, and n_t set to 10, 20 and 30, which represent severe, moderate, and slight severities of change, respectively. Experimental results of five algorithms on the HVD and IGD metrics are given in Table 7.

It can be observed from tables 6 and 7 that all the algorithms are very sensitive to the frequency and severity of change. This can be seen from the improvement of the metrics when increasing the value of τ_t or n_t. For different frequency levels, IEC is able to produce impressive performance and wins in the majority of the instances. For the test problems JY7 of table 6 and JY8 7, our proposed algorithm is enhanced, with results similar to the previous conclusion from tables 4 and 5. IEC is robust as it obtains the IGD and HVD values under all frequencies of change and severity of change according to the appearance of tables 6 and 7.

5.4. Study of Different Components of IEC

In order to study the different components of IEC, IEC algorithms have been transformed into two critical versions. The first version (IEC-v1) only uses Algorithm 2 to update P_t, the other (IEC-v2) reacts to the environmental

Table 8: Mean and standard deviation values of IGD and HVD metric obtained by five algorithms.

Problems	metric	IEC	IEC-v1	IEC-v2	p-Value
FDA1	IGD	4.57e-3(3.91e-5)	5.56e-3(5.25e-5)‡	7.76e-3(5.44e-5)‡	0.002
	HVD	7.81e-3(2.81e-4)	8.75e-3(1.94e-4)‡	1.28e-2(2.76e-4)‡	0.009
dMOP1	IGD	4.57e-3(3.46e-4)	5.34e-3(9.55e-5)‡	4.77e-3(1.09e-3)†	0.010
	HVD	1.00e-2(1.18e-3)	3.94e-2(4.57e-4)‡	2.01e-2(1.11e-3)‡	0.000
JY1	IGD	6.11e-3(4.03e-5)	7.15e-3(3.78e-5)‡	8.40e-3(9.22e-5)‡	0.002
	HVD	4.79e-3(6.76e-5)	5.17e-3(7.01e-5)‡	7.60e-3(1.66e-4)‡	0.011
JY9	IGD	6.66e-3(1.19e-4)	7.59e-3(8.17e-5)‡	8.81e-3(1.03e-4)‡	0.007
	HVD	5.41e-3(2.38e-4)	6.24e-3(2.25e-4)‡	8.54e-3(2.73e-4)‡	0.032

change by Algorithm 3. To show the significant difference between the three algorithms, the Friedman test [39, 40] was carried out to indicate significance between different results at the 0.05 significance level. p-Value represents the test value of all the algorithms on the Friedman test with setting of $(n_t,\, n_t) =$ (10, 30) and the best values are shown with a deep gray background and the second best with a light gray background. Moreover, FDA1, dMOP1, JY1 and JY9 are selected to compare algorithms by IGD and HVD values. As shown in table 8, IEC significantly performed better than the other two algorithms. It can be concluded that IEC has an advantage in dealing with DMOPs. The results clearly indicate the significant and indispensable role of each component in coping with a dynamic environment.

5.5. Computational Complexity Analysis

The overall framework of IEC is based on MOEA/D-DE, which consumes the most computational resources. Its computational complexity is O(NMT), where M is the number of objectives; N is the population size, and T is the number of subproblems. Here, we mainly analyze the computational complexity of Algorithms 2 and 3. Algorithm 2 takes O(Nn) and Algorithm 3 also spends O(Nn), where n is size of decision variable. Therefore, the overall computational complexity of the IEC strategy is O(Nn).

6. Conclusions and future work

A novel DMOEA based on intensity of environmental change (i.e., IEC) in the decision space is proposed for handling DMOPs with time-varying characteristics. Different from existing dynamism handling approaches, IEC reacts to changes in the intensity of the environmental change. In the first situation, if the environmental change is detected but this change is not similar to the existing historical environment, half of the solutions of the current population are updated according to the intensity of the environmental change of each decision where the U-test is used in the decision space; the rest of the solutions are maintained. Furthermore, if the environmental change is detected and this change is similar to the existing historical environment, nearly 2/3 of the solutions are updated according to the two different methods, respectively. Otherwise, a generational cycle of the static optimization algorithm is carried out. The main contributions of our work can be summarized as follows:

(1) The update method introduces the intensity of environmental change via the U-test in the decision space to roughly test the effected biggish decision in the current change. Then the update method is carried out for this decision with the effected biggish decision to make the current population adapt to the new evolutionary environment more quickly. However, each decision of some solutions are updated in the decision space in most research when the environmental change is detected, but in nature, not all decision variables in the decision space are affected markedly by the new evolutionary environment. Therefore, IEC has and advantage in dealing with DMOPs.

(2) We have exhibited a novel memory approach which equally keeps individuals of the current optimization population according to the distribution of the vector. The aim of this method is to direct the current population in the new environment toward **each promising search region** when there exists a historical evolutionary environment. Nevertheless, most of the research

about the memory tactics only takes the convergence of the solutions into consideration, but these convergent solutions may be located in a small regions on the whole POF in real life. So, this novel memory strategy is very advantageous in dealing with DMOPs.

Although IEC performed a better performance compared with the other algorithms, it had some weakness in handling DMOPs with more complicated characteristics, such as multimodal problems (JY7). Moreover, if there are some changes that do not vary in an irregular way or have little similarity in dynamic environments, it might be desirable to combine IEC with other dynamic handling techniques in the future. Thus, we will explore the use of the different strategies as well as consider the development of other novel prediction models for solving DMOPs using evolution algorithms. In addition, we also plan to design a new operator in dynamic adaption under the dynamic environment, then make the population adapt to the new evolutionary environment by operator.

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