Modelling Human Vagueness and Imprecision: from Fuzzy Sets to Hesitant Fuzzy Sets

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Abstract. Hesitant fuzzy set has been able to receive an immense concern on the basis of several studies in both theoretical and practical viewpoints that have reported its adverse effects. During the last twelve years, there has been a growing interest in different extensions of hesitant fuzzy set theory which can generates and adopts to improve hesitant fuzzy capacity to address the increase in information complexity. Through this contribution, we intend to conduct a comprehensive literature review on "Hesitant Fuzzy" extensions those have been rarely reported previously. In view of this matter, this article takes the literature on "Hesitant Fuzzy" extensions in the core collection database of Web of Science as the research object, and moreover, it uses the bibliometric software package of VOSviewer to visually analyse the themes of "Hesitant Fuzzy" researches from different aspects such as general statistics, top authors, the affiliation of top authors, the country of top authors, the document types and the subject areas. Furthermore, this contribution analyses the chronological development of "Hesitant Fuzzy" extensions which have been more frequently and thoroughly reviewed, including the general-based, the linguistic term-based, the q-rung orthopair-based and the probabilistic-based extensions of hesitant fuzzy sets. Lastly, a number of directions for future research are put forward.
Keywords: Hesitant fuzzy set; Extensions; literature review; Bibliometric.

1 Introduction

The basic idea of a fuzzy set is that an element belongs to it with a membership degree from the interval $[0, 1]$ in which 0 and 1 mean respectively completely exclusive and completely inclusive to the set. However, the limitation of this concept is still more serious in case of dealing with imprecise and vague information, specially, in the case where different sources of vagueness appear simultaneously. Due to this limitation, a number of extensions of fuzzy set have been introduced in the literature. The most widely used extensions of fuzzy sets are

(i) Atanassov's intuitionistic fuzzy set [2, 97] which allows to consider simultaneously the membership degree and the non-membership degree of each element,

(ii) Type-2 fuzzy set [15] that incorporates uncertainty in the definition of membership function where a fuzzy set over $[0, 1]$ is used to model it,

(iii) Interval-valued fuzzy set [75] that assigns to each element a closed subinterval of $[0, 1]$ as the membership degree of that element such that the length of the interval may be understood as a measure of the lack of certainty for building the precise membership degree of the element,

(iv) Fuzzy multiset [51] that is based on multiset in which elements can be repeated.

The recently developed extension of fuzzy set is known as hesitant fuzzy set that was first introduced by Torra [74, 73] to cope with a number of challenging situations in which the membership degree of an element needs to be established by encountering an error margin like that arises in theory of intuitionistic fuzzy sets or some possibility distribution like that happens in type 2 fuzzy sets. Indeed, the concept of hesitant fuzzy set

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can be used to model a situation that is very usual in decision making when an expert might consider different degrees of membership of an element.

Until now, the concept of hesitant fuzzy set has attracted much more attention from scientific communities, and more than hundreds of contributions have been published to represent the development results such as hesitant fuzzy extensions, hesitant fuzzy aggregations, operations of hesitant fuzzy sets, hesitant fuzzy information measures, etc. Although there are available a number of bibliometric overviews of hesitant fuzzy set such as those by Xu [95], Rodriguez et al. [65] and Liao [45], a comprehensive literature review for "Hesitant Fuzzy" extensions is still a very fertile field. In this contribution, we are going to exhibit the expansion areas of hesitant fuzzy set theory in its 12th year and provide graphical illustrations by representing the results of most exiting literature review.

We organize this contribution into the main two portions to deal with graphical illustrations in Section 2, and the classification of "Hesitant Fuzzy" extensions in the remaining parts of article in Section 3. Describing the contents of Section 2 in a more detail, we present the graphical illustrations of the publication years, the publication sources, the authors, the affiliation of authors, the country of authors, the document types and the subject areas which are all subjected to "Hesitant Fuzzy" extensions. Section 3 presents a brief review of the most known extensions of hesitant fuzzy sets. Beside providing a picture of the latest achievements in the filed of "Hesitant Fuzzy" extensions in conclusion part, this article will provide several future research directions for scholars in this domain.

2 Extensions of hesitant fuzzy set theory with graphical illustrations

Throughout this section, we are going to illustrate all the publications which include the words "Hesitant Fuzzy" as the keywords in their body. The data source, which is adopted from Web of Science (WoS) on Jun 10, 2021, is classified according to the publication years, the publication sources, the authors, the affiliation of authors, the country of authors, the document types and the subject areas. The corresponding pictures are illustrated by Figures 1-8, and a brief description of contributions concerning "Hesitant Fuzzy" extensions are also given in Table 1.
As observed from Figure 1, we find that the frequency of documents concerning "Hesitant Fuzzy" extensions, which include

- *Hesitant fuzzy set*, *Hesitant triangular fuzzy set*, *Interval-valued hesitant fuzzy set*, *Extended hesitant fuzzy set*, *Higher order hesitant fuzzy set*, *Dual hesitant fuzzy set*, *Dual hesitant triangular fuzzy set*, *Extended dual hesitant fuzzy set*, *Interval-valued dual hesitant fuzzy set*,

- *Hesitant fuzzy linguistic term set*, *Extended hesitant fuzzy linguistic term set*, *Interval-valued hesitant fuzzy linguistic term set*, *Proportional hesitant fuzzy linguistic term set*, *Hesitant fuzzy uncertain linguistic set*, *Dual hesitant fuzzy linguistic set*, *Dual hesitant fuzzy triangular linguistic set*, *Interval-valued dual hesitant fuzzy linguistic set*,

- *q-rung orthopair hesitant fuzzy set*, *Interval-valued q-rung orthopair hesitant fuzzy set*, *Dual q-rung orthopair hesitant fuzzy set*,

- *Probabilistic hesitant fuzzy set*, *Probabilistic dual hesitant fuzzy set* , *Probabilistic linguistic hesitant fuzzy set*, *Probabilistic linguistic dual hesitant fuzzy set*, *Interval probability hesitant fuzzy linguistic variable*, *Probabilistic neutrosophic hesitant fuzzy set*, *Pythagorean probabilistic hesitant fuzzy set*, *q-rung orthopair probabilistic hesitant fuzzy set,*

indicate an increasing tendency in the annual-published number of contributions.

Figure 2 represents the top 15 most productive journals in which "hesitant Fuzzy" extensions have been proposed and discussed so far. Furthermore, it illustrates the number of papers published yearly during 2009-2021. Confirmed by a short glance at the quantitative data of Figure 2, we find that JOURNAL OF INTELLIGENT FUZZY SYSTEMS corresponds to the source with the most publications.

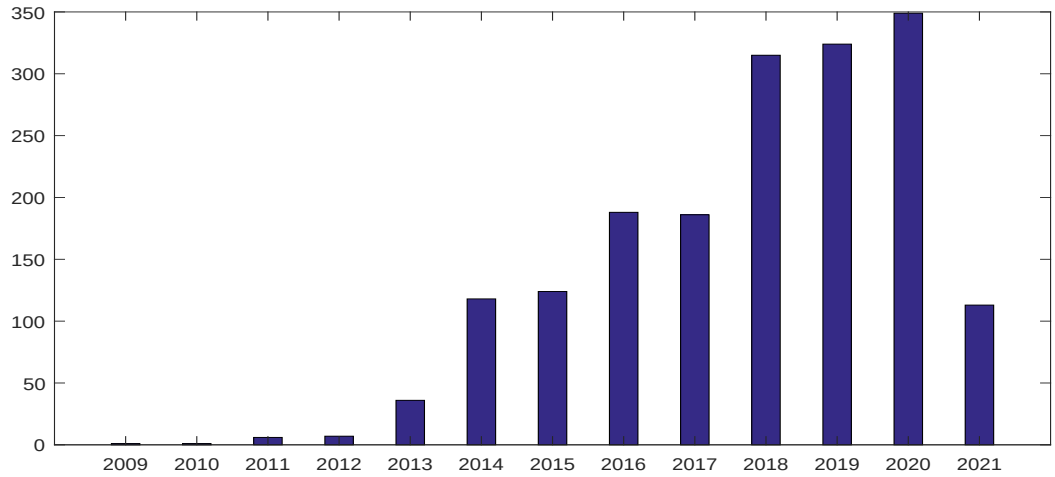

Figure 1: Frequencies of documents concerning "Hesitant Fuzzy" extensions with respect to published years.

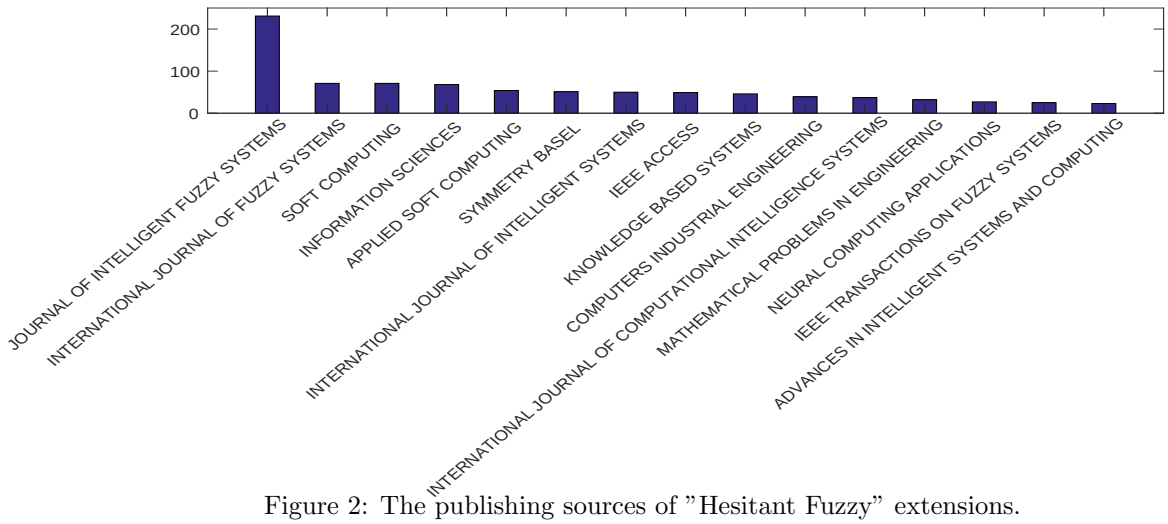

Figure 2: The publishing sources of "Hesitant Fuzzy" extensions.

Figure 3 presents the top 15 most productive authors who have contributed the published research papers in the topic of "Hesitant Fuzzy" terms cited them till 2009. XU ZS from Sichuan University is at the leading position whose number of citations is equal to 203.

Figure 4 indicates the recent affiliations with at least 27 published articles on the subject of "hesitant Fuzzy" and its extensions. SICHUAN UNIVERSITY in China occupies the first rank of affiliations list which publishes the contributions concerning "hesitant Fuzzy" and its extensions.

Figure 5 demonstrates the pioneer countries publishing articles on the subject of "hesitant Fuzzy" extensions more than seventeen times until the present. CHINA holds the top leading position among countries by the total number of 1196 articles. However, SPAIN in the second position is far behind CHINA, and the other leading countries are TURKEY, INDIA, IRAN, etc.

Figure 6 presents the document-type classification for the publications on "hesitant Fuzzy" extensions in Web of Science. Most of these documents have been prepared in the form of ARTICLEs and PROCEEDING PAPERs.

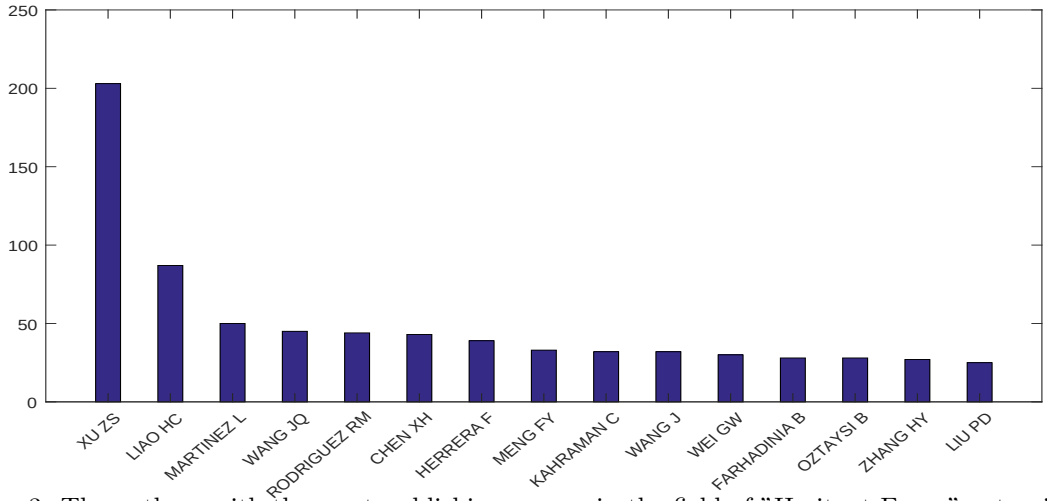

Figure 3: The authors with the most publishing papers in the field of "Hesitant Fuzzy" extensions.

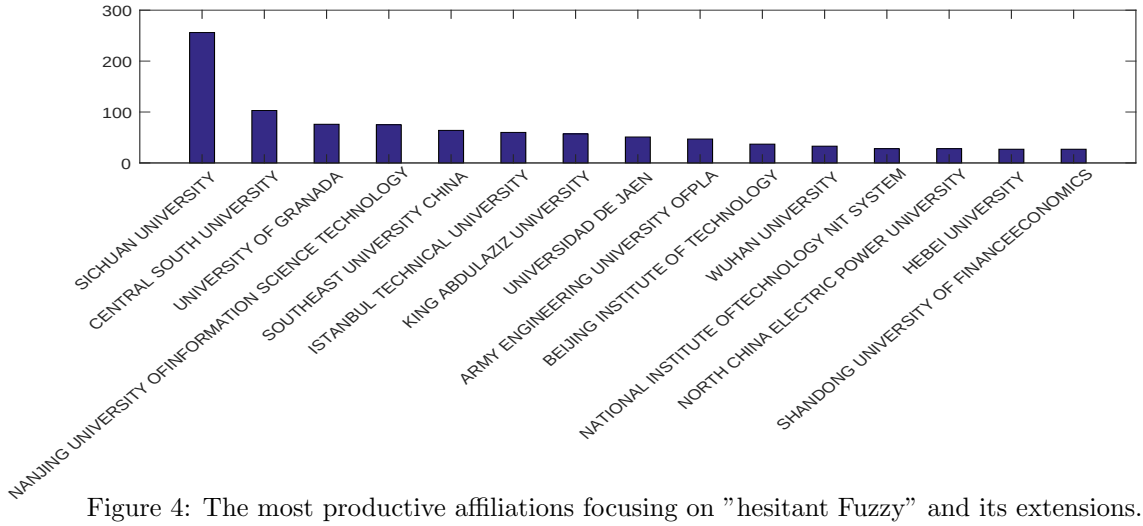

Figure 4: The most productive affiliations focusing on "hesitant Fuzzy" and its extensions.

Figure 7 covers a wide variety of journal categories ranging from COMPUTER SCIENCE ARTIFICIAL INTELLIGENCE to ENVIRONMENTAL SCIENCES. Among them which might need special attention is COMPUTER SCIENCE ARTIFICIAL INTELLIGENCE.

Figure 8 indicates the status of author collaboration in which the size of nodes is used to show the number of publications. As can be observed from Figure 8, the international coauthorship in research works on "Hesitant Fuzzy" extensions is very intuitive and almost ubiquitous in nature.

Table 1 takes a closer look at the top 20 highly cited papers in the twelve-year period from 2009 to 2021 for identifying the most crucial research topics which have drawn attention of many computer scientists and mathematicians.

3 Extensions of hesitant fuzzy set theory in a brief review

As will be demonstrated in the coming description, we briefly review the hesitant fuzzy set and it extensions which are currently available on this issue.

Table 1: Top 20 highly cited papers.

Rank	Title	Author(s)	Source	Year	Citations
1	Hesitant fuzzy sets [73]	V. Torra	International Journal of Intelligent Systems	2010	2113
2	Hesitant fuzzy linguistic term sets for decision making [61]	R.M. Rodriguez, L. Martinez, F. Herrera	IEEE Transactions on Fuzzy Systems	2012	1378
3	Hesitant fuzzy information aggregation in decision making [91]	M. Xia, Z. Xu	International Journal of Approximate Reasoning	2011	1095
4	On hesitant fuzzy sets and decision [74]	V.Torra, Y. Narukawa	2009 IEEE International Conference on Fuzzy Systems	2009	890
5	Distance and similarity measures for hesitant fuzzy sets [97]	Z. Xu, M. Xia	Information Sciences	2011	842
6	Hesitant fuzzy prioritized operators and their application to multiple attribute decision making [83]	G. Wei	Knowledge-Based Systems	2012	463
7	Distance and similarity measures for hesitant fuzzy linguistic term sets and their application in multi-criteria decision making [43]	H. Liao, Z. Xu, X.J. Zeng	Information Sciences	2012	427
8	A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets [64]	R.M. Rodriguez, L. Martinez, F. Herrera	Information Sciences	2013	414
9	Hesitant fuzzy multi-attribute decision making based on TOPSIS with incomplete weight information [99]	Z. Xu, X. Zhang	Knowledge-Based Systems	2013	399
10	On distance and correlation measures of hesitant fuzzy information [100]	Z. Xu, M. Xia	International Journal of Intelligent Systems	2011	399
11	Hesitant fuzzy geometric Bonferroni means [116]	B. Zhu, Z. Xu, M. Xia	Information Sciences	2012	388
12	Interval-valued hesitant preference relations and their applications to group decision making [11]	N. Chen, Z. Xu, M. Xia	Knowledge-Based Systems	2013	385
13	Dual hesitant fuzzy sets [119]	B. Zhu, Z. Xu, M. Xia	Journal of Applied Mathematics	2012	376
14	Correlation coefficients of hesitant fuzzy sets and their applications to clustering analysis [6]	N. Chen, Z. Xu, M. Xia	Applied Mathematical Modelling	2013	363
15	Consistency measures for hesitant fuzzy linguistic preference relations [117]	B. Zhu, Z. Xu	IEEE Transactions on Fuzzy Systems	2014	325
16	Hesitant fuzzy sets: state of the art and future directions [65]	R.M. Rodriguez, L. Martinez, V. Torra, Z.S. Xu, F. Herrera	International Journal of Intelligent Systems	2014	324
17	Qualitative decision making with correlation coefficients of hesitant fuzzy linguistic term sets [42]	H. Liao, Z. Xu, X.J. Zeng, J.M. Merigo	Knowledge-Based Systems	2015	303
18	Some hesitant fuzzy aggregation operators with their application in group decision making [92]	M. Xia, Z. Xu, N. Chen	Group Decision and Negotiation	2013	300
19	Hesitant fuzzy linguistic VIKOR method and its application in qualitative multiple criteria decision making [44]	H. Liao, Z. Xu, X.J. Zeng	IEEE Transactions on Fuzzy Systems	2015	293
20	Information measures for hesitant fuzzy sets and interval-valued hesitant fuzzy sets [21]	B. Farhadinia	Information Sciences	2013	273

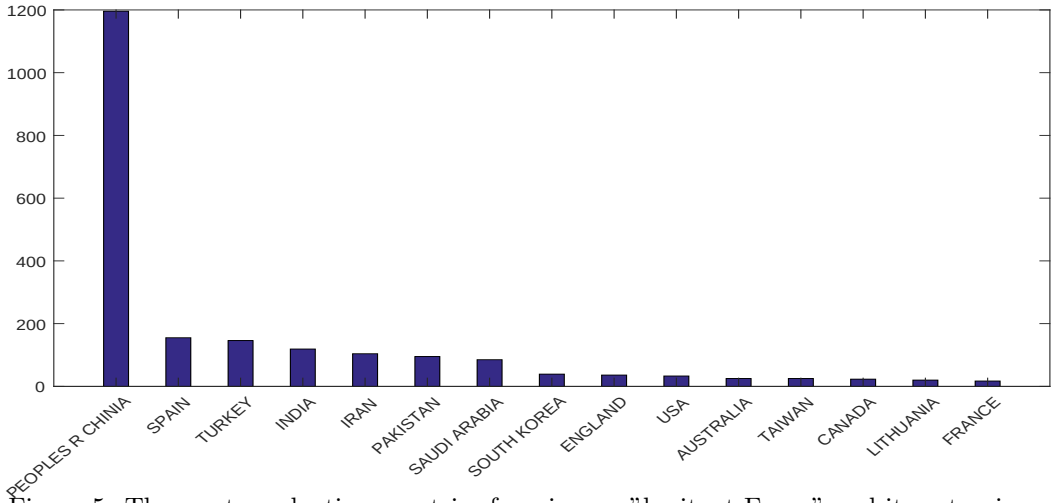

Figure 5: The most productive countries focusing on "hesitant Fuzzy" and its extensions.

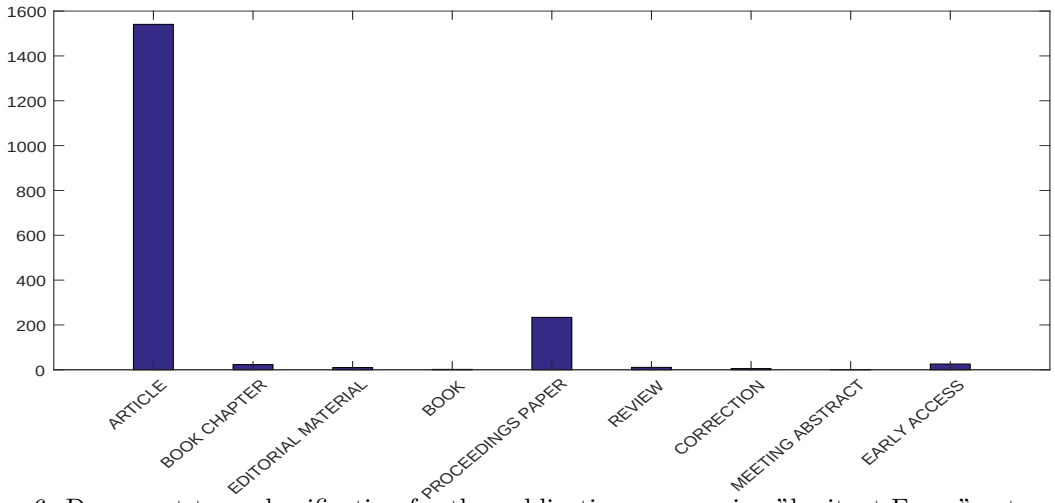

Figure 6: Document-type classification for the publications concerning "hesitant Fuzzy" extensions.

Hesitant fuzzy set has moved beyond its infancy, and is now entering a maturing phase with increased numbers and types of extensions. Up to now, the existing extensions of hesitant fuzzy set have been encountering an increasing interest, and attracting more and more attentions. It is not an exaggeration if we say that the recent decade has seen the blossoming of a larger set of techniques and theoretical outcomes for hesitant fuzzy sets and their extensions as well as applications.

By the way, hesitant fuzzy set is introduced [73] for modelling a situation of decision making in which an expert(s) might consider different degrees of membership of an element in a set. A hesitant fuzzy set is semantically described by a function

$$A : X \to \wp([0,1]) \tag{1}$$

that returns a set of membership degrees for any element in the referential set X from the non-empty subset $\wp([0,1])$ of values in $[0,1]$.

Torra [73], Xu [91], Herrera [62], Wei [83], Liao [41], and Farhadinia [21] are among the pioneer researchers who provided a broad conceptualization and application of hesitant fuzzy set and their works were followed by different researchers on the area later.

Hesitant triangular fuzzy set [103] is an extension of hesitant fuzzy set in which the membership degree of each element is stated by the use of triangular fuzzy numbers. By taking on the concept of triangular fuzzy

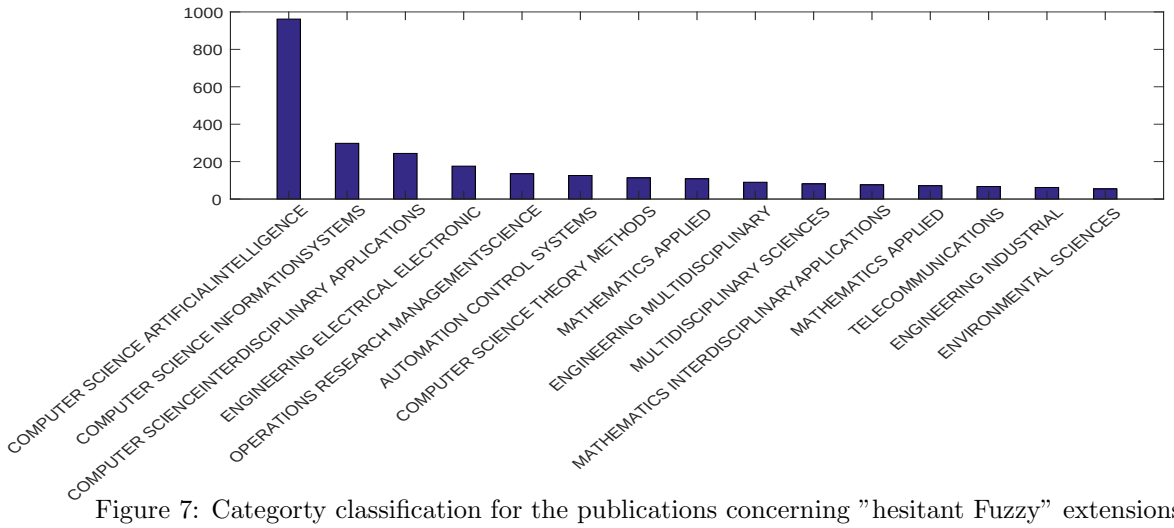

Figure 7: Categorty classification for the publications concerning "hesitant Fuzzy" extensions.

number into account, Yu [103] represented the concept of triangular fuzzy hesitant number, and further, defined the corresponding arithmetic operators. Shi et al. [72] defined Einstein correlated averaging operator to evaluate the wireless network security whose information is in the form of hesitant triangular fuzzy set. A development of hesitant triangular fuzzy set was given by Rashid and Husnine [58] who proposed trapezoidal valued hesitant fuzzy set. Peng [53] and Zhang et al. [107] developed some different multiple criteria decision making techniques whose criteria values are expressed by hesitant trapezoidal fuzzy elements. In a different manner of construction, Deli and Karaaslan [14] introduced the concept of generalized trapezoidal hesitant fuzzy set which is defined as a generalization of hesitant fuzzy sets and generalized fuzzy numbers.

Interval-valued hesitant fuzzy set as a generalization of hesitant fuzzy set is similar to that encountered in intuitionistic fuzzy environments, where the concept of intuitionistic fuzzy set has been extended to that of interval-valued intuitionistic fuzzy set. In many practical applications, the uncertainty is usually specified between some different values, and determination of crisp membership degrees is not easy task. This impulsed Chen et al. [7] to define a generalization of hesitant fuzzy set as the concept of interval-valued hesitant fuzzy set which dominates the barrier, and helps a decision maker to assign some membership degrees for an object under a set to have several interval-values [56]. The theory of interval-valued hesitant fuzzy set has attracted more attention in decision making fields [37, 88], and in group decision-making processing [7]. Li and Peng [37] introduced the interval-valued hesitant fuzzy Hamacher synergetic weighted aggregation operators; Wei et al. [88] proposed a number of hesitant interval-valued fuzzy aggregation operators; and Chen et al. [7] presented some interval-valued hesitant preference relations.

As an extension of the hesitant fuzzy set concept, Zhu and Xu [118] proposed the concept of *extended hesitant fuzzy set* as a tool to avoid loss of information. In fact, such a consideration enables us to present information represented by the decision makers using possible value-groups. However, this definition of extended hesitant fuzzy set suffers from some drawbacks which were mentioned by a study conducted by Farhadinia and Herrera-Viedma [23]. They revisited the concept of extended hesitant fuzzy set proposed by Zhu and Xu using the Cartesian product of hesitant fuzzy sets. In the sequel, Farhadinia and Xu [22] dealt with an aspect of emergency event that is used instead of usual aggregation procedure, and they developed the fusion technique on the basis of modified version of extended hesitant fuzzy set.

Generalization of hesitant fuzzy set, known as the generalized hesitant fuzzy set [55], has its inherent drawbacks, because it expresses the membership degrees of an element to a given set only by crisp numbers or intuitionistic fuzzy sets. In many practical decision making problems, the information provided by a decision maker might often be described by fuzzy sets (instead of crisp numbers) or other fuzzy set extensions (instead of intuitionistic fuzzy sets). Therefore, it is difficult for the decision makers to provide exact crisp values or just intuitionistic fuzzy sets for the membership degrees. This difficulty can be avoided using a *higher order of*

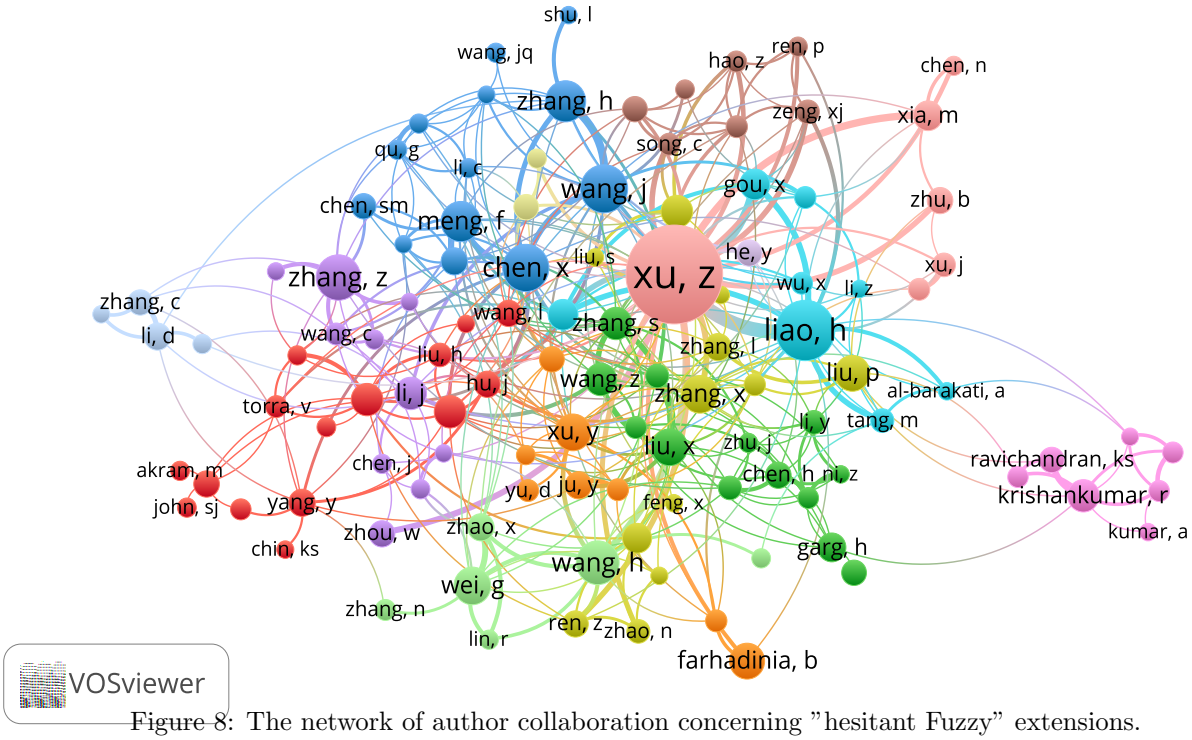

Figure 8: The network of author collaboration concerning "hesitant Fuzzy" extensions.

hesitant fuzzy set introduced by Farhadinia [18] for the membership degrees. The higher order hesitant fuzzy set is more fit for the case when the decision makers have a hesitation among several possible memberships with uncertainties.

Dual hesitant fuzzy set is a representation which seeks to combine intuitionist fuzzy concept and hesitant fuzzy concept by uniting the advantages of these concepts. Indeed, Zhu et al. [119] introduced the concept of dual hesitant fuzzy set which can encompass fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets and fuzzy multisets as special cases. It is seen that dual hesitant fuzzy sets can better deal with the situations that permit the membership and the non-membership of an element to a given set having a few different values. Such situations often arise in group decision making problems, such as, the study deals with a number of dual hesitant fuzzy information aggregation operators including mean and geometric forms of dual hesitant fuzzy Heronian concept [104], the dual hesitant fuzzy-based group decision making technique which assists the assessment of network information system security [105], a variety of distance measures for dual hesitant fuzzy sets [82], and

the investigation of multiple criteria decision making problems based on Heronian mean, in which the attribute values are assumed in the form of interval-valued dual hesitant fuzzy information [106].

Dual hesitant triangular fuzzy set, which is also known as hesitant triangular intuitionistic fuzzy [8], extends the concept of generalized trapezoidal hesitant fuzzy set introduced by Deli and Karaaslan [14]. Zhao et al. [108] gave the hesitant triangular fuzzy information together with a number of aggregating operators for aggregating all the related information of multiple attribute decision making problems. Chen and Huang [8] standardized hesitant triangular intuitionistic fuzzy aggregation operators, and they proposed several distance measures to explore their application in multiple criteria decision making.

Extended dual hesitant fuzzy set was introduced by Alcantud et al. [1] based on the combination of extended hesitant fuzzy set with dual hesitant fuzzy set. Indeed, the extended dual hesitant fuzzy set of degree $\aleph$ is nothing else except the set of elements in the form of $\aleph$-tuple of dual hesitant fuzzy sets. In view of this vein, the set and algebraic operations on extended dual hesitant fuzzy sets are the $\aleph$-tuple forms of those presented for dual hesitant fuzzy sets.

Interval-valued hesitant fuzzy set was extended by Ju et al. [35] to the interval-valued dual hesitant fuzzy set using the assignment of same importance to the possible non-membership interval values in the interval-valued hesitant fuzzy set. In the sequel, Peng et al. [54] employed the concepts of Archimedean t-norm and Archimedean t-conorm to propose a number of interval-valued dual-hesitant fuzzy aggregation operators. A divers type of interval-valued dual-hesitant fuzzy operators were developed by Zang et al. [106], Sarkar and Biswas [66] and Jiang et al. [34] for solving a variety of group decision-making problems.

Rodriguez et al. [63, 64] represented the concept of *hesitant fuzzy linguistic term set* which permits a linguistic variable to own various linguistic terms. Under hesitant fuzzy linguistic situation, Liao et al. [43] developed a satisfactory based decision making technique. Wei et al. [89] extended the comparison rule for hesitant fuzzy linguistic term sets, and studied the theory of hesitant fuzzy linguistic term set aggregation. By the help of pessimistic and optimistic attitudes of decision makers, Chen and Hong [10] represented a multiple criteria linguistic decision technique. Zhu and Xu [117] introduced the concept of hesitant fuzzy linguistic preference relation, and moreover, they investigated its consistency. By taking comparative linguistic expressions into account, Liu et al. [48] improved the additive consistency of the hesitant fuzzy linguistic preference relations.

By considering this fact that a group of experts may think of several possible linguistic values or richer expressions than a single term for evaluating an alternative in group decision making, Wang [76] discussed linguistic terms involved in an expression which is derived by the multiple decision makers being not always consecutive. Therefore, Wang generalized the concept of hesitant fuzzy linguistic term set to *extended hesitant fuzzy linguistic term set* in which the family of linguistic term sets is to be taken an ordered subset.

Wang et al. [80] proposed the concept of *interval-valued hesitant fuzzy linguistic term set* to combine the advantages of both linguistic term sets and interval-valued hesitant fuzzy sets. Indeed, the main advantage of interval-valued hesitant fuzzy linguistic term set is that it can describe two fuzzy criteria of an object, namely a linguistic term and an interval-valued hesitant fuzzy element. The former provides an evaluation value, such as "good" or "excellent", and the latter describes the hesitancy for the given evaluation value and denotes the interval-valued membership degrees associated with the specific linguistic term. Meng et al. [50] in a re-phrased concept, which was called linguistic interval hesitant fuzzy set, implemented interval-valued hesitant fuzzy linguistic term set concept to cope with the situations where the membership degrees of linguistic terms are intervals rather than real numbers.

Wang and Hao [81] introduced the proportional two-tuple model that enables experts to assign symbolic proportions to two successive linguistic terms. Then, Zhang et al. [109] included the proportional information into the model of linguistic representation. Motivated by the idea of Zhang et al. [109], Wu and Xu [90] represented a possibility distribution-based technique for addressing multiple criteria group decision making with hesitant fuzzy linguistic information. In order to propose a linguistic representation model which considers simultaneously the hesitant linguistic assessment of experts and the proportional information of each linguistic

term under a group decision making environment, Chen et al. [9] introduced the notion of *proportional hesitant fuzzy linguistic fuzzy term set*.

Lin et al. [46] introduced the concept of *hesitant fuzzy uncertain linguistic set* to overcome the limitation that arises from the case where the decision information about alternatives is usually uncertain due to the vagueness of inherent subjective nature of human think. In the sequel, Li et al. [38] presented a number of aggregation operators to deal with hesitant fuzzy uncertain linguistic multiple criteria decision making problems. Hou and Zhuo [31] investigated the multiple criteria decision making problems involving hesitant fuzzy uncertain linguistic information, and furthermore, they developed a number of aggregation operators of hesitant fuzzy uncertain linguistic information.
Some other relevant concepts including interval valued hesitant fuzzy uncertain linguistic set [85], linguistic interval hesitant fuzzy set [50], interval-valued 2-tuple hesitant fuzzy linguistic term set [71] and multi-hesitant fuzzy linguistic term set [79] can be defined similarly, and therefore, we do not show them here to save space.

Inspired by the idea of hesitant fuzzy linguistic variable, Yang and Ju [101] combined dual hesitant fuzzy set with linguistic term set to construct the concept of *dual hesitant fuzzy linguistic set* containing a linguistic term and a set of membership and non-membership degrees. Wei et al. [86] developed a number of aggregation operators with dual hesitant fuzzy linguistic information, and utilized them in developing some approaches for dealing with hesitant fuzzy linguistic multiple criteria decision making problems. Zhang et al. [110] extended Archimedean t-norm and t-conorm in order to aggregate the dual hesitant fuzzy linguistic information. Li et al. [40] defined an extended TODIM method under dual hesitant fuzzy linguistic information, and then they applied that technique for dealing with a stock selection problem.

On the basis of hesitant fuzzy linguistic set, triangular linguistic term set and dual hesitant fuzzy set, Ju et al. [36] introduced the concept of *dual hesitant fuzzy triangular linguistic set* which is composed of a triangular linguistic term. Dual hesitant fuzzy triangular linguistic set is a set of membership and non-membership degrees for overcoming the shortcomings of hesitant fuzzy linguistic set.

Interval-valued dual hesitant fuzzy linguistic set concept has been introduced for facilitating the calculation whenever we are required to consider linguistic evaluation sets by considering the interval-valued dual hesitant fuzzy sets [87]. Following this line of thought, Qi et al. [57] presented the concept of interval-valued dual hesitant fuzzy unbalanced linguistic set together with its power aggregation operators. Xian et al. [93] utilized the concept of generalized interval-valued intuitionistic fuzzy linguistic variable to define an induced hybrid operator for dealing with TOPSIS-based linguistic group decision making problems.
It should be mentioned that the concept of interval-valued dual hesitant fuzzy linguistic set is also called in the literature [93] as the interval-valued intuitionistic hesitant fuzzy linguistic set, and in [84] as the interval-valued dual hesitant fuzzy uncertain linguistic set.

Liu et al. [47] proposed a number of operations, score and accurate functions for *q-rung orthopair hesitant fuzzy sets* together with a ranking technique of q-rung orthopair hesitant fuzzy sets. Furthermore, they established a distance measure for q-rung orthopair hesitant fuzzy sets in order to handle the uncertainty involved in a related TOPSIS approach. Moreover, Wang et al. [78] developed some distance and similarity measures of q-rung orthopair hesitant fuzzy sets, and investigated their properties. In addition to that, they defined the axiomatic form of entropy measure for q-rung orthopair hesitant fuzzy sets. Hussain et al. [30] developed a series of operations, score and accurate functions together with comparison rule for q-rung orthopair hesitant fuzzy sets, and then implemented them in a q-rung orthopair hesitant fuzzy-based decision making process.

In [96], Xu et al. presented some operations, comparison technique and aggregation operators of *interval-valued q-rung dual hesitant fuzzy sets*. Then, Feng et al. [24] extended the operator of power Hamy mean to operators for interval-valued q-rung dual hesitant fuzzy sets, called the interval-valued q-rung dual hesitant fuzzy power Hamy mean and the interval-valued q-rung dual hesitant fuzzy power weighted Hamy mean. Xu et al. [96] presented a number of operations and a comparison technique for interval-valued q-rung dual hesitant fuzzy sets, and then they developed some interval-valued q-rung dual hesitant fuzzy aggregation operators.
In some references (for instance, [77]), the authors use the concept "dual" in extending the notion of q-rung orthopair fuzzy set to the *dual hesitant q-rung orthopair fuzzy set*. This is while, the notion of "dual" is hidden

in the definition of q-rung orthopair fuzzy set, and so need not to be mentioned again. Therefore, it is enough to say that only the hesitant q-rung orthopair fuzzy set, the interval-valued hesitant q-rung orthopair fuzzy set, and so on.

If some experts assign the same value for specified alternative in a hesitant fuzzy-based decision making, then we cannot determine what their preferences are. To avoid the loss of hesitant fuzzy information in such a decision-making process, Zhu [115] brought probability to the concept of hesitant fuzzy set and introduced the *probabilistic hesitant fuzzy set*. Zhang et al. [112] developed the probabilistic hesitant fuzzy operations and integrations to apply in decision making. Song et al. [70] represented a comparison technique for making the multiple criteria decision making more efficiently. Zhou and Xu [114] defined a probabilistic hesitant fuzzy preference relation, and then, they tested the consistency of the same for group decision-making. Jiang and Ma [32] developed a number of aggregation operators under arithmetic and geometric context for probabilistic hesitant fuzzy sets.

As a pioneer work concerning on the probabilistic dual hesitant fuzzy set, Hao et al. [28] proposed a class of operational laws and aggregation operators for *probabilistic dual hesitant fuzzy sets*. Then, they developed an entropy measure of probabilistic dual hesitant fuzzy sets for enhancing a visual analysis technique. In the sequel, Ren et al. [59] extended TODIM technique for probabilistic dual hesitant fuzzy sets in the application of enterprise strategic assessment. Further, Garg and Kaur [25] proposed a robust correlation coefficient for probabilistic dual hesitant fuzzy sets. Ren et al. [60] investigated a strategy selection process with an integrated AHP and VIKOR technique under probabilistic dual hesitant fuzzy set information.
It should be mentioned that the concept of probabilistic dual hesitant fuzzy set is also referred to as the dual hesitant fuzzy probability set [5], and the probabilistic intuitionistic hesitant fuzzy set [111] in some references.

Gong et al. [26] presented the probabilistic linguistic hesitant fuzzy preference relation on the basis of *probabilistic linguistic hesitant fuzzy set* which not only provides flexible linguistic expression for decision makers, but also gives the occurrence probability of each element in the probabilistic linguistic hesitant fuzzy preference relation. Zhao and Huang [113] developed the notions of score and variance functions for probabilistic linguistic hesitant fuzzy sets together with a set of basic operations. Then, they implemented a probabilistic linguistic hesitant fuzzy-based distance measure in multiple criteria decision making.

Based on the probabilistic linguistic term set proposed by Pang et al. [52], Gong and Chen [27] gave the definition of *probabilistic linguistic dual hesitant fuzzy set* by combining the concept of probabilistic dual hesitant fuzzy set with the concept of dual hesitant fuzzy linguistic term set.

Xian et al. [94] introduced the *interval probability hesitant fuzzy linguistic variable* by taking the concept of hesitant fuzzy linguistic term set as the evaluation part together with a novel element-reliability of evaluation. They also gave the definition of operation rules and comparison techniques for interval probability hesitant fuzzy linguistic variables.

Shao et al. [67] presented *probabilistic neutrosophic hesitant fuzzy set*, and they investigated the operation laws together with the averaging and geometric operators for probabilistic neutrosophic hesitant fuzzy sets. Furthermore, Shao et al. [68] developed Shao et al.'s [67] Choquet averaging and Choquet geometric operators for probabilistic neutrosophic hesitant fuzzy sets.

Batool et al. [3] introduced the concept of *Pythagorean probabilistic hesitant fuzzy set* with the constraint that the square sum of positive and negative hesitant membership degrees is less than or equal to one. Batool et al. [4] established a multiple criteria decision making technique on the basis of EDAS approach under Pythagorean probabilistic hesitant fuzzy information, and further, they developed an algorithm for addressing the uncertainty in the selection of drugs in EmDM issues in correspondence with the clinical analysis.

Li et al. [39] introduced the concept of *q-rung probabilistic dual hesitant fuzzy set* by the use of capturing the probability of each element in q-rung dual hesitant fuzzy sets. Q-rung probabilistic dual hesitant fuzzy set is indeed q-rung probabilistic hesitant fuzzy set. The most prominent character of q-rung probabilistic dual hesitant fuzzy set is that it allows experts to have a flexible manner for evaluating criterion values in a

complicated and realistic multiple criteria decision making situation. In order to more effectively exploit the opportunities for applying q-rung probabilistic dual hesitant fuzzy sets, Li et al. [39] proposed some basic operational rules, comparison technique and distance measure of q-rung probabilistic dual hesitant fuzzy sets.

4 Conclusions

Throughout this study, we tried to present a literature review on the main research results in the field of "Hesitant Fuzzy" extensions which has been not yet reported in the literature.

By presenting the results of a comparative study of Web of Science, VOSviewer and Scopus databases, we found that

- JOURNAL OF INTELLIGENT FUZZY SYSTEMS is the journal with the highest number of articles citing "hesitant Fuzzy" extensions;
- XU ZS and SICHUAN UNIVERSITY occupy both the most cited author and the first rank of affiliations list;
- CHINA holds the top leading position among countries;
- COMPUTER SCIENCE ARTIFICIAL INTELLIGENCE is the productive journal in the field of "hesitant Fuzzy" extensions.

In addition to the above, we analysed the chronological development of most known "Hesitant Fuzzy" extensions including the general-based, the linguistic term-based, the q-rung orthopair-based and the probabilistic-based extensions of hesitant fuzzy sets.
A number of directions for future research may be expected to show the structural and the dynamic aspects of scientific research by the help of science mapping analysis which was proposed in [12, 13, 29].

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