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Designing optimal proactive replacement strategies for degraded systems subject to two types of external shocks

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ABSTRACT
This paper mainly investigates a proactive replacement policy for a stochastically deteriorating system concurrently subject to two types of shocks. Firstly, the closed-form representation of system reliability function suffering from both a degradation process and environmental shocks is derived based on the degradation-threshold-failure (DTS) modelling framework. An age- and state-dependent competing risks model with mutual dependence between the two failure processes is embedded into system reliability modelling, where two types of shocks are taken into consideration upon arrival of an external shock including a minor one and a major one. Based on which, a bivariate maintenance policy is put forward for the deteriorating system, where the system is proactively replaced before failure at a planned time, or at an appropriate number of minimal repairs, whichever takes place first. The expected long-run cost rate (ELRCR) is formulated, and optimal solutions are evaluated analytically for two special cases. Finally, an illustrative example is redesigned to validate the theoretical results, exploring the significance of two types of shocks and mutual dependence in system reliability modelling, and illustrating the potential applications in maintenance decisions in various manufacturing systems.

KEYWORDS
Degradation modelling; Two types of shocks; Mutual dependence; Proactive replacement; Optimization

1. Introduction

It is acknowledged that the importance of production systems' reliability has been increasingly realized with the purpose of manufacturing satisfying products with high quality and designing highly resilient systems (Lee and Pan, 2017; Ouyang et al., 2020). Amongst the multiple factors that contribute to system failure, either a catastrophic failure that results in an immediate stoppage of operation or a very severe malfunction, continuous performance deterioration such as wear, fatigue, fracture, corrosion and erosion occurred internally and random shocks stemming from the external environment are viewed as two main causes. In such cases, gradual failure due to the

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internal deterioration is usually partial and moderate, whilst catastrophic failure resulted from environmental shocks is mostly sudden and complete (Cherkaoui et al., 2020).

Degradation models such as general degradation paths and stochastic processes, for example, the Wiener process, the Gamma process, and the Inverse Gaussian (IG) process are promising probability approaches in describing production system deterioration behavior over dynamic time. The rationale of degradation modelling is to make full use of degradation characteristics that reflect the health conditions of a product or a system, seeking the perceived link between a degradation model and system's hidden lifetime distribution (Huynh, 2020; Lu et al., 2020). Therefore, the premise of reliability modelling based on the so called degradation-failure mechanism is to choose an appropriate degradation model for the product in light of the physics of failure and collected degradation data. An excellent review on the degradation models for highly reliable products is presented, expounding the pros and cons of these two classes, i.e., general degradation paths and stochastic processes, as well as their corresponding applications in accelerated degradation test (ADT) planning and degradation-based burn-in modelling (Ye and Xie, 2015).

External shocks from the outside environment often arrive at discrete time epochs randomly. Another stochastic process, such as the Poisson process (PP) and the Markov point process (MPP), is progressively introduced to describe the stochastic behavior of random shocks. More specifically, a homogeneous Poisson process (HPP) and a non-homogeneous Poisson process (NHPP) are commonly considered as effective approaches to model the arrival numbers of external shocks for a system operating in a stable environment and a turbulent environment, respectively (Dong et al., 2019; Kim and Makis, 2013). Shocks are vividly classified into different categories according to their load magnitudes and effectiveness (Dong et al., 2020c; Dong et al., 2021).

Reliability analysis for systems that experience only degradation or random shocks has always been an important issue in literature (Liu et al., 2017). However, in a large majority of most industrial production systems are subject to both stochastic degradation and random shocks concurrently, and thus, the interaction or dependence between the degradation process and the shock process is being more and more important to be taken into consideration in performability evaluation. In total, there are mainly two directions to research system failure phenomenon in the case of interacting the degradation process and the shock process. On one side, it is assumed that damage loads due to random shocks cause an abrupt jump onto the current degradation level of the system, and at the same time, accelerate the degradation rate as well. The influence of the shock process on the degradation process is repeatedly viewed as shock-degradation dependence (Cao et al., 2020). On the other side, random shocks can be influenced by the degradation process, and this is commonly seen in structures such as buildings and bridges, where a system is more vulnerable to external shocks at a severe deteriorating condition. Similarly, degradation-shock dependence is steadily formed considering the effect of the degradation process on the shock process (Huynh et al., 2012).

More recently, a few reliability evaluation models have been proposed for stochastically deteriorating systems suffering from random shocks in regard of describing the mutually dependent relationship incorporating the shock-degradation dependence and the degradation-shock dependence simultaneously. Che et al. (2018) introduced a Facilitation model, which is a special type of MPP (Cha and Finkelstein, 2018) to model the shock process, and based on which, a novel analytical reliability model considering mutual dependence is developed. In contrast to PP, the Facilitation model has a good property that the arrival points can facilitate the points arriving in the future, and is further investigated in Yousefi et al. (2020). Fan et al. (2017) developed a new reliability model for systems subject to dependent and competing failure processes (DCFPs), where the degradation-shock dependence is modelled by assuming that the intensity function of the NHPP describing the random shock process is dependent on the degradation process. Along with that, the dependence effect is modelled with reference to a classification of the random shocks in three zones according to their magnitudes, that is, a damage zone, a fatal zone, and a safety zone. Later in Wang et al. (2020b), an interdependent structure is proposed in a two-fold way: (a) the impact of random shocks on the degradation process is expressed by a compound Poisson distribution; (b) the influence of natural degradation process toward random shocks is reflected by the natural-degradation-state varying thresholds and the varying cumulative damage of shocks.

The results reported in the above mentioned works support the idea that both current degradation state and system age have an impact on the subsequence of shocks, but none of them takes the two factors into consideration in unison. With a system degrading, it becomes more sensitive to the influence that a shock exerts upon it. Consequently, an age- and state- dependent competing failure model embedded with random shocks is proposed by Wang et al. (2020c), in which age-dependence means that the system's degradation is only due to the effect of its age, and state-dependence means that the degradation is partly or totally related to the current state. The closed-form reliability function is derived for the continuous age- and state-dependent model, as well as for the discrete age- and state-dependent model (Wang et al., 2020a).

As failures of a system are usually costly, and sometimes disastrous, the importance of maintenance, especially preventive maintenance (PM) is mightily enhanced during past decades (Dong et al., 2020a; Khatab et al., 2019). The most important issue of maintenance theory in the field of production research is to determine when and how to maintain a product or a system preventively before failure, and therefore, arranging the frequency and timing of PM according to either maintenance cost criterion or benefit criterion is becoming the kernel objective in the framework of maintenance optimization (Panagiotidou and Tagaras, 2010). Normally, PM policies on operating systems are always assumed to be adopted at some continuous measures (e.g., age, running distance, damage thresholds, condition level, etc.), or at some discrete quantities (e.g., number of usage, repairs, failures, shocks, etc.), or at some bivariate maintenance decision variables (Celen and Djurdjanovic, 2020; Dong et al., 2022; Kim, 2016; Wang et al., 2019; Wang and Ye, 2020).

From actual engineering point of view, not each shock on a system has a catastrophic production disruption, and thus, different maintenance activities are suggested based on the damage degree of random shocks. A minimal repair, which does not change system degradation rate, is always adopted in the face of minor failure while a corrective replacement should be arranged for a failed system when confronted with major failure. The concept of two types of failures was initially put forward by Brown and Proschan (1983), in which a unit is returned to be the "good-as-new" state with a constant probability and returned to be the "bad-as-old" state with another constant probability after its repair. More researches on two types of failures are developed (Sheu et al., 2014; Sheu et al., 2016). Motivated by previous outcomes, Dong et al. (2020b) proposed the concept of two types of shocks, where they studied two time-based replacement policies involving an age replacement policy and a block replacement policy for a fault tolerant system which is subject to degradation and two types of shocks.

In this current research, the first contribution is that two types of shocks are considered upon arrival of a random shock, where the minor shock can be removed by a minimal repair and the major shock interacts with the dominant degradation process (Chang, 2021a; Chang, 2021b, Li, et al., 2022). The closed-form reliability function for a stochastically deteriorating system embedded with two types of shocks is constructed based on the degradation-threshold-failure (DTS) modelling framework. The second contribution is that a bivariate proactive replacement policy is further put forward, where the system is replaced before failure at planned time T ($0 < T \leq \infty$), or when the numbers of minimal repairs reach N ($N = 1, 2, \cdots$), whichever takes place first. The proposed PM model is a proactive policy as it is a model-based approach, in which two special cases are investigated analytically in light of the renewal reward theorem. A numerical example is examined to demonstrate the effectiveness of the proposed maintenance policy.

The outline of the remainder of this paper is organized as follows. Section 2 lists some basic assumptions and states the problem. System reliability function is derived in Section 3 based on the age- and state-dependent competing risks model, in which two types of shocks is incorporated. The bivariate proactive replacement model is proposed in Section 4, and along with which, two special cases are investigated analytically. An illustrative example is designed in Section 5 to examine the theoretical results. Finally, Section 6 concludes the whole research.

2. Problem statement and basic assumptions

This present paper considers a single unit system which is subject to a pure degradation process $X(t)$ and a shock process $\{N(t), t \geq 0\}$, where the two processes are mutually dependent. In the following, some basic assumptions are firstly listed.

(1) The pure degradation process $X(t)$ is described with a general degradation path model $X(t; \mathbf{\Theta}) = \beta\eta(t; \mathbf{\Theta})$, in which β is a random variable representing the variation from unit to unit following a certain distribution with its cumulative distribution function (CDF) $F_\beta(x) = \Pr\{\beta \leq x\}$, $\eta(\cdot)$ is a deterministic path function, e.g., a linear degradation path (Peng et al., 2010), a logistic degradation path (Peng and Feng, 2013), an exponential functional form (Elwany et al., 2011), etc., and $\mathbf{\Theta}$ is a vector containing a set of random parameters representing the influence of uncertainties for all units.

(2) Environmental shocks arrive on the stochastically deteriorating system following an NHPP $\{N(t), t \geq 0\}$ with an intensity function $\lambda(t)$. To incorporate the degradation-shock dependence, $\lambda(t)$ is assumed to be a linear function of the pure degradation amount $X(t)$, that is, $\lambda(t) = \lambda_0 + \alpha X(t)$ (Fan et al., 2017), where λ_0 denotes the initial intensity of the NHPP, and α ($\alpha \geq 0$) is the degradation-shock influence factor representing the effect of the current degradation level $X(t)$ on random shocks.

(3) Two types of shocks are formed upon arrival of a random shock, i.e., the minor shock resulting in minor damage which can be removed by a minimal repair, and the major shock which contributes to system failure. To be specific, an external shock develops into a minor shock with a time-dependent probability $p(t)$ ($0 < p(t) < 1$), and it forms a major shock with another time-dependent probability $q(t)$, where $p(t) + q(t) \equiv 1$.

Let $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ denote the arrival numbers of minor shocks and major shocks until t, respectively, then, $\{N_1(t), t \geq 0\}$ and

$\{N_2(t), t \geq 0\}$ are two independent non-homogeneous Poisson processes with intensity functions $p(t)\lambda(t)$ and $q(t)\lambda(t)$, respectively (Dong et al., 2020b; Sheu et al., 2015). In addition, denoting $\Lambda_1(t)$ and $\Lambda_2(t)$ as the mean numbers of minor shocks and major shocks in time interval $[0, t]$, respectively, we have

$$\begin{aligned}\Lambda_1(t) &= \mathbb{E}[N_1(t)] = \int_0^t p(u)\lambda(u)\mathrm{d}u \\ &= \int_\beta \int_{\mathbf{\Theta}} \int_0^t p(u)[\lambda_0 + \alpha x \eta(u; \boldsymbol{\theta})] f_\beta(x) f_{\mathbf{\Theta}}(\boldsymbol{\theta}) \mathrm{d}u \mathrm{d}\boldsymbol{\theta} \mathrm{d}x, \end{aligned} \quad (1)$$

in which $f_\beta(x) = \mathrm{d}F_\beta(x)/\mathrm{d}x$ and $f_{\mathbf{\Theta}}(\boldsymbol{\theta})$ is the probability density function (PDF) of the vector $\mathbf{\Theta}$ given that $\mathbf{\Theta} = \boldsymbol{\theta}$, and

$$\begin{aligned}\Lambda_2(t) &= \mathbb{E}[N_2(t)] \\ &= \int_\beta \int_{\mathbf{\Theta}} \int_0^t q(u)[\lambda_0 + \alpha x \eta(u; \boldsymbol{\theta})] f_\beta(x) f_{\mathbf{\Theta}}(\boldsymbol{\theta}) \mathrm{d}u \mathrm{d}\boldsymbol{\theta} \mathrm{d}x. \end{aligned} \quad (2)$$

(4) Denote Y_i $(i = 1, 2, \cdots)$ as the damage magnitude caused by the ith major shock. $Y_1, Y_2, \cdots$ are assumed to be independent and identically distributed with CDF $F_Y(y) = \Pr\{Y_i \leq y\}$. As the process $\{N_2(t), t \geq 0\}$ for the arrival numbers of major shocks follows an NHPP, the total resulting damages caused by major shocks are a compound Poisson process (CPP) and can be recorded as

$$S(t) = \begin{cases} 0, N_2(t) = 0 \\ \sum\limits_{i=1}^{N_2(t)} Y_i, N_2(t) > 0 \end{cases}. \quad (3)$$

(5) System failure is defined as the time instantly the total degradation amount $M(t)$ reaches the failure threshold L for the first time, where L $(L > 0)$ is a constant and stipulated by industrial standards in advance. Denote the random variable τ_L as system failure time, i.e.,

$$\tau_L = \inf\{t \in \mathbb{R}^+ | M(t) \geq L\}.$$

(6) A minimal repair is arranged for each minor shock, which does not change the degradation rate of the system and spends maintenance cost c_m. Proactive replacement behaviors are adopted at planned time T $(0 < T \leq \infty)$ with cost c_T, or at the Nth $(N = 1, 2, \cdots)$ minor shock with cost c_N, whichever takes place first. It is more reasonable that $c_N > c_m$. If the system fails due to degradation failure, it remains in the failed state for the time interval from system failure to the next proactive replacement, i.e., system failure is not self-announcing. c_d is the downtime cost per unit of time elapsed between system failure and the adjacent replacement. All preparation times for maintenance behaviors are negligible, and all maintenance actions are perfectly adopted.

3. Reliability modelling under the age- and state-dependent model

In this section, several reliability models considering the dependence between the pure degradation process and the shock process are briefly reviewed, as well as their advantages and disadvantages, where the effective number of shocks $N_2(t) > 0$. Then, the closed-form system reliability function is analyzed based on the age- and state-dependent model because of its merits.

(1) In the first model, the total degradation amount $M(t)$ is assumed to be composed of the pure degradation level $X(t)$ and the resulting damages $S(t)$ caused by shocks (Peng et al., 2010). That is,

$$M(t) = X(t)+S(t) = \beta\eta(t;\mathbf{\Theta}) + \sum_{i=1}^{N_2(t)} Y_i. \tag{4}$$

This dependence relationship is simple because the shock damages are standardly cumulated into the current degradation level and the degradation rate is not disturbed by external shocks. This model is mostly investigated by later researchers because of its convenience to implement and consideration of failure physics.

(2) In order to capture the extra effect of the degradation rate acceleration caused by random shocks, another DCFP model is developed (Wang and Pham, 2012), in which the cumulative degradation amount $M(t)$ is represented as

$$M(t) = \widetilde{X}(t) + S(t) = \beta\eta(t\mathrm{e}^{G(t;\boldsymbol{\gamma})};\mathbf{\Theta}) + \sum_{i=1}^{N_2(t)} Y_i, \tag{5}$$

where $G(t;\boldsymbol{\gamma}) = \gamma_1 N_2(t) + \gamma_2 \sum_{i=1}^{N_2(t)} Y_i$. It is evident that random shocks accelerate the degradation rate, and at the same time, cumulate the current degradation level. Usually, $\gamma_1 \geq 0$, reflecting that the degradation rate is more likely to increase with the shock number $N_2(t)$, and $\gamma_2 \geq 0$, representing that the cumulative shock damages $\sum_{i=1}^{N_2(t)} Y_i$ also contribute to the acceleration of system degradation rate.

(3) A state function $f(X(t))$ is incorporated into the cumulated shock loads $S(t)$ in the model shown in (4), and correspondingly, the total degradation amount $M(t)$ is denoted as

$$M(t) = X(t)+f(X(t))S(t) = \beta\eta(t;\mathbf{\Theta}) + f(X(t)) \sum_{i=1}^{N_2(t)} Y_i, \tag{6}$$

where $f(X(t))$ represents the effects that natural degradation process impacts on the cumulative damage of random shocks (Wang et al., 2020b). The disadvantage of (6) is that system degradation rate is not disturbed by external shocks, which is similar to that of (4) and usually unrealistic in practice.

(4) An age- and state-dependent competing risks model embedded with random shocks is proposed (Wang et al., 2020c), in which the impacts of the current degradation state on the overall degradation process are incorporated. The total degradation

amount becomes

$$M(t) = \beta\eta(te^{f_1(M(t))N_2(t)+f_2(M(t))\sum_{i=1}^{N_2(t)} Y_i}; \mathbf{\Theta}) + f_3(M(t))\sum_{i=1}^{N_2(t)} Y_i, \tag{7}$$

in which $f_1(M(t))$, $f_2(M(t))$, and $f_3(M(t))$ are three coefficients associated with $M(t)$. More specifically, $f_1(M(t))$ represents the influence of random shock number on system degradation rate, $f_2(M(t))$ reflects the influence of shock damages on system degradation rate, and $f_3(M(t))$ indicates the effect of random shocks on the current degradation amount $M(t)$.

The age- and state-dependent competing risks model in (7) is a comprehensive concept that considers both the age dependence and the state dependence of a system in degradation modelling. Meanwhile, it is viewed as the generalization of existing dependence models from (4) to (6). In this current study, the dependence model in (7) serves as the basis for reliability modelling because of its advantages.

The coefficients $f_1(M(t))$ and $f_2(M(t))$ are assumed to be linear logarithmic functions of $M(t)$, and the coefficient $f_3(M(t))$ is directly proportional to $M(t)$. More specifically, following the assumptions with Wang et al. (2020c), we have $f_1(M(t)) = a_1\ln(M(t)) + b_1$, $f_2(M(t)) = a_2\ln(M(t)) + b_2$, and $f_3(M(t)) = a_3M(t)$, where $a_1 \geq 0$, $a_2 \geq 0$, and $a_3 \geq 0$. Then, (7) becomes

$$M(t) = \beta\eta(te^{[a_1\ln(M(t))+b_1]N_2(t)+[a_2\ln(M(t))+b_2]\sum_{i=1}^{N_2(t)} Y_i}; \mathbf{\Theta}) + a_3M(t)\sum_{i=1}^{N_2(t)} Y_i. \tag{8}$$

For a clear illustration, the closed-form reliability function is derived for the case where system degradation path is a linear function under the modelling framework of DTS theory. Assume that $\eta(t; \mathbf{\Theta}) = t$, i.e., $X(t; \mathbf{\Theta}) = \beta t$, then, a logarithmic form of $M(t)$ is

$$\ln(M(t)) = \frac{\ln(\beta t) + b_1N_2(t) + b_2\sum_{i=1}^{N_2(t)} Y_i - \ln\left(1 - a_3\sum_{i=1}^{N_2(t)} Y_i\right)}{1 - a_1N_2(t) - a_2\sum_{i=1}^{N_2(t)} Y_i}. \tag{9}$$

Therefore, system reliability function $R(t)$ is derived as

$$
\begin{aligned}
&R(t)\\
&=\Pr\{\tau_L>t\}\\
&=\Pr\{M(t)\le L\,|N_2(t)=0\}\Pr\{N_2(t)=0\}+\sum_{k=1}^{\infty}\Pr\{\ln(M(t))\le \ln L\,|N_2(t)=k\}\Pr\{N_2(t)=k\}\\
&=\Pr\{\beta t\le L\}\mathrm{e}^{-\Lambda_2(t)}\\
&+\sum_{k=1}^{\infty}\Pr\left\{\frac{\ln(\beta t)+b_1k+b_2\sum_{i=1}^{k}Y_i-\ln\left(1-a_3\sum_{i=1}^{k}Y_i\right)}{1-a_1k-a_2\sum_{i=1}^{k}Y_i}\le \ln L\right\}\frac{[\Lambda_2(t)]^k}{k!}\mathrm{e}^{-\Lambda_2(t)}\\
&=F_\beta\left(\frac{L}{t}\right)\mathrm{e}^{-\Lambda_2(t)}+\sum_{k=1}^{\infty}\int_0^{\ln L}F_\beta\left(\frac{\mathrm{e}^{\ln L(1-a_1k-a_2z)-b_1k-b_2z-\ln(1-a_3z)}}{t}\right)\mathrm{d}F_Y^{(k)}(z)\\
&\times\frac{[\Lambda_2(t)]^k}{k!}\mathrm{e}^{-\Lambda_2(t)},
\end{aligned}
\tag{10}
$$

in which $F_Y^{(k)}(z)$ is the k-fold Stieltjes convolution of $F_Y(z)$, i.e., $F_Y^{(k)}(z)=\Pr\{Y_1+Y_2+\cdots+Y_k\le z\}$. Under the assumption that $X(t;\mathbf{\Theta})=\beta t$, $\Lambda_2(t)$ in (2) becomes

$$
\Lambda_2(t)=\int_\beta\int_0^t q(u)(\lambda_0+\alpha xu)\mathrm{d}u\mathrm{d}x=\int_0^t q(u)(\lambda_0+\alpha\mu_\beta u)\mathrm{d}u, \tag{11}
$$

where $\mu_\beta=\int_\beta x\mathrm{d}\Pr\{\beta\le x\}=\int_\beta xf_\beta(x)\mathrm{d}x$.

Thus, the closed-form system reliability function in (10) is further developed into

$$
\begin{aligned}
&R(t)\\
&=F_\beta\left(\frac{L}{t}\right)\mathrm{e}^{-\int_0^t q(u)(\lambda_0+\alpha\mu_\beta u)\mathrm{d}u}+\sum_{k=1}^{\infty}\int_0^{\ln L}F_\beta\left(\frac{\mathrm{e}^{\ln L(1-a_1k-a_2z)-b_1k-b_2z-\ln(1-a_3z)}}{t}\right)\mathrm{d}F_Y^{(k)}(z)\\
&\times\frac{\left[\int_0^t q(u)(\lambda_0+\alpha\mu_\beta u)\mathrm{d}u\right]^k}{k!}\mathrm{e}^{-\int_0^t q(u)(\lambda_0+\alpha\mu_\beta u)\mathrm{d}u}.
\end{aligned}
\tag{12}
$$

4. Scheduling a proactive replacement policy

This section aims to schedule a proactive replacement policy for the stochastically deteriorating system embedded with two types of shocks before its failure. As stated in Section 2, the system is proactively replaced at a planned time T, or at the Nth minimal repair, whichever occurs first. A pictorial representation of the context of system maintenance behaviors is depicted in Figure 1, in which T_N is the time epoch when the numbers of minimal repairs reach N, and $K_1, K_2, \cdots$ are the consecutive renewal cycles.

To evaluate the effectiveness of the proposed proactive maintenance policy, we focus on the cost criterion which is the expected maintenance cost in one renewal cycle. According to the regenerative property of the renewal reward theorem (Grall et al., 2002),

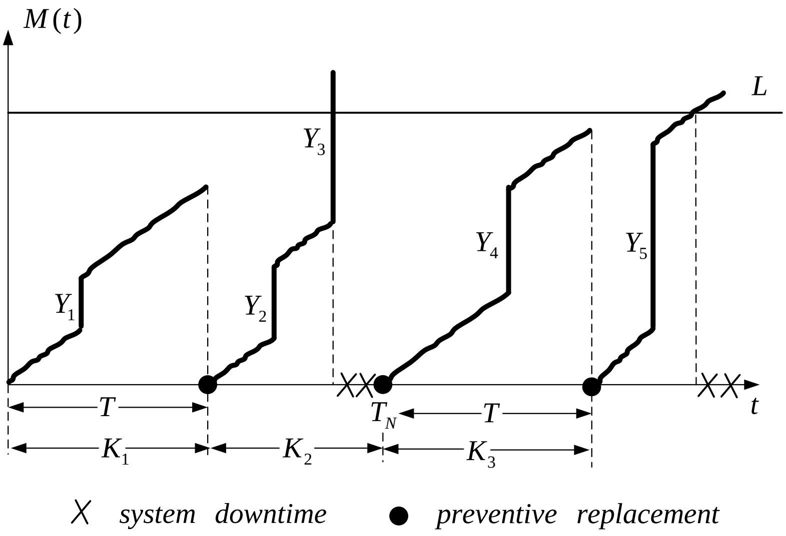

Figure 1. A possible sample path depicting system maintenance behavior.

the analytical form of the long-run maintenance cost rate is formulated. Denoting $C(t)$ as the accumulated maintenance cost until t for the system, we have

$$C_\infty = \lim_{t\to\infty} \frac{C(t)}{t} = \frac{\mathbb{E}[C(K_1)]}{\mathbb{E}[K_1]}, \tag{13}$$

in which $\mathbb{E}[C(K_1)]$ is the expected maintenance cost in the first renewal cycle K_1, and $\mathbb{E}[K_1]$ is the mean length of the first renewal cycle.

Let the random variable T_i $(i = 1, 2, \cdots, N)$ represent the waiting time until the ith major shock has occurred for the stochastically deteriorating system described in Section 3. Since the major shocks happen according to an NHPP with a mean function $\Lambda_1(t)$, the survival function of T_i is

$$\overline{F}_{T_i}(t) = \Pr\{T_i > t\} = \sum_{k=0}^{i-1} P_k(t) = \sum_{k=0}^{i-1} \frac{[\Lambda_1(t)]^k}{k!} \mathrm{e}^{-\Lambda_1(t)}, \tag{14}$$

where $\Lambda_1(t) = \int_0^t p(u)(\lambda_0 + \alpha\mu_\beta u)\mathrm{d}u$. $P_k(t)$ $(k = 0, 1, \cdots)$ is the probability that k major shocks have occurred in time interval $[0, t]$, and can be derived as

$$P_k(t) = \overline{F}_{T_{k+1}}(t) - \overline{F}_{T_k}(t) = \frac{[\Lambda_1(t)]^k}{k!} \mathrm{e}^{-\Lambda_1(t)}. \tag{15}$$

Then, the probability that the system is proactively replaced at the planned time T is

$$\Pr\{T \le T_N\} = \overline{F}_{T_N}(T), \tag{16}$$

and the probability that it is proactively replaced at the Nth minor shock is

$$\Pr\{T_N < T\} = F_{T_N}(T), \tag{17}$$

where should be noted that $\Pr\{T \leq T_N\} + \Pr\{T_N < T\} \equiv 1$.

Furthermore, the mean length of the first renewal cycle K_1 is

$$\mathbb{E}[K_1] = T\overline{F}_{T_N}(T) + \int_0^T t\mathrm{d}F_{T_N}(t) = \int_0^T \overline{F}_{T_N}(t)\mathrm{d}t. \tag{18}$$

It has been shown that the system undergoes minimal repair upon arrival of a minor shock. The mean number of minor shocks in the first renewal cycle K_1 is

$$\mathbb{E}[N_m] = \Lambda_1(T)\overline{F}_{T_N}(T) + \int_0^T \Lambda_1(t)\mathrm{d}F_{T_N}(t) = \int_0^T \overline{F}_{T_N}(t)p(t)\lambda(t)\mathrm{d}t. \tag{19}$$

The mean length of system downtime in the first renewal cycle K_1 is

$$\begin{aligned}\mathbb{E}[T_d] &= \overline{F}_{T_N}(T)\int_0^T (T-t)\mathrm{d}F(t) + \int_0^T\int_0^x (x-t)\mathrm{d}F(t)\mathrm{d}\overline{F}_{T_N}(x)\\ &= \int_0^T \overline{F}_{T_N}(t)F(t)\mathrm{d}t,\end{aligned} \tag{20}$$

in which $F(t) = \Pr\{\tau_L \leq t\} = 1 - R(t)$ is system failure distribution, and can be derived from (12). Therefore, the total maintenance cost in the first renewal cycle K_1 is

$$\begin{aligned}\mathbb{E}[C(K_1)] &= c_N F_{T_N}(T) + c_T\overline{F}_{T_N}(T) + c_m\mathbb{E}[N_m] + c_d\mathbb{E}[T_d]\\ &= c_T + (c_N - c_T)F_{T_N}(T) + c_m\int_0^T \overline{F}_{T_N}(t)p(t)\lambda(t)\mathrm{d}t + c_d\int_0^T \overline{F}_{T_N}(t)F(t)\mathrm{d}t.\end{aligned} \tag{21}$$

Therefore, (13) becomes

$$C(N,T) = \frac{c_T + (c_N - c_T)F_{T_N}(T) + c_m\int_0^T \overline{F}_{T_N}(t)p(t)\lambda(t)\mathrm{d}t + c_d\int_0^T \overline{F}_{T_N}(t)F(t)\mathrm{d}t}{\int_0^T \overline{F}_{T_N}(t)\mathrm{d}t}. \tag{22}$$

It is clear that the proposed maintenance strategy is a proactive policy because it is a model-based policy and preventive actions are adopted before system failure. Minimizing $C(N,T)$ in (22) is a bivariate optimization problem containing a discrete variable N and a continuous variable T. If $C(N,T)$ is jointly convex in (N^*, T^*), then, (N^*, T^*) is the optimal policy which minimizes $C(N,T)$. The heuristic solution to the optimized is obtained such that

$$C(N^*, T^*) = \min_N\left\{\min_T C(N,T)\right\}. \tag{23}$$

In the following parts, we seek the optimal solutions for Policy T ($N \to \infty$) and Policy N ($T \to \infty$), respectively.

4.1. *Policy T*

When the system is proactively replaced only at planned time T, i.e., $N \to \infty$, the maintenance policy is called Policy T in this current study. Then, the expected maintenance cost rate in (22) becomes

$$\begin{aligned} C_N(T) &= \lim_{N\to\infty} C(N,T) \\ &= \frac{c_T + c_m \int_0^T p(t)\lambda(t)\mathrm{d}t + c_d \int_0^T F(t)\mathrm{d}t}{T} \\ &= \frac{c_T + c_m \int_0^T p(t)(\lambda_0 + \alpha\mu_\beta t)\mathrm{d}t + c_d \int_0^T F(t)\mathrm{d}t}{T}. \end{aligned} \tag{24}$$

The aim is to find an optimal T^* which minimizes $C_N(T)$. Differentiating $C_N(T)$ in (24) with respect to T and setting it equal to zero, we have the optimal T^* satisfies

$$c_m\left[Tp\left(T\right)\left(\lambda_0 + \alpha\mu_\beta T\right) - \int_0^T p\left(t\right)\left(\lambda_0 + \alpha\mu_\beta t\right)\mathrm{d}t\right] + c_d\left[TF\left(T\right) - \int_0^T F(t)\mathrm{d}t\right] = c_T. \tag{25}$$

Denote the left-hand side of (25) as $\psi(T)$, that is, $\psi(T) = c_m\left[Tp(T)(\lambda_0 + \alpha\mu_\beta T) - \int_0^T p(t)(\lambda_0 + \alpha\mu_\beta t)\mathrm{d}t\right] + c_d\left[TF(T) - \int_0^T F(t)\mathrm{d}t\right]$. Differentiating $\psi(T)$ with respect to T, we have

$$\frac{\mathrm{d}\psi(T)}{\mathrm{d}T} = c_m\left[\lambda_0 T\frac{\mathrm{d}p(T)}{\mathrm{d}T} + \alpha\mu_\beta T^2 + p(T)\alpha\mu_\beta T\right] + c_d T\frac{\mathrm{d}F(T)}{\mathrm{d}T}.$$

Setting that the probability $p(t)$ does not decrease with t, i.e., $\mathrm{d}p(t)/\mathrm{d}t \geq 0$, we judge $\mathrm{d}\psi(t)/\mathrm{d}t > 0$, illustrating that $\psi(T)$ increases strictly with T. Therefore, an optimum T^* for Policy T which minimizes $C_N(T)$ is as follows:

(1) If $\psi(\infty) = \lim_{T\to\infty} \psi(T) > c_T$, there exists an optimal T^* which minimizes $C_N(T)$ satisfying (25). Policy T under this condition is perfectly adopted for the deteriorating system and the corresponding maintenance cost rate is

$$C_N(T^*) = c_m p(T^*)(\lambda_0 + \alpha\mu_\beta T^*) + c_d T^* F(T^*). \tag{26}$$

(2) If $\psi(\infty) \leq c_T$, the optimal $T^* = \infty$, that is, Policy T is not yet adopted.

4.2. *Policy N*

When the system is proactively replaced only at the Nth minimal repair, i.e., $T \to \infty$, the maintenance policy is called Policy N in this current study. Then, the expected maintenance cost rate in (22) becomes

$$\begin{aligned} C_T(N) &= \lim_{T\to\infty} C(N,T) \\ &= \frac{c_N + c_m \int_0^\infty \overline{F}_{T_N}(t)p(t)\lambda(t)\mathrm{d}t + c_d \int_0^\infty \overline{F}_{T_N}(t)F(t)\mathrm{d}t}{\int_0^\infty \overline{F}_{T_N}(t)\mathrm{d}t}. \end{aligned} \tag{27}$$

Forming the inequalities $C_T(N+1) \geq C_T(N)$, we have the optimal N^* which minimizes $C_T(N)$ satisfying

$$\phi_T(N) \geq c_N, \tag{28}$$

in which,

$$\begin{aligned}
&\phi_T(N) \\
&= c_m \left[\frac{\int_0^\infty \overline{F}_{T_N}(t)\mathrm{d}t \int_0^\infty P_N(t)p(t)\lambda(t)\mathrm{d}t}{\int_0^\infty P_N(t)\mathrm{d}t} - \int_0^\infty \overline{F}_{T_N}(t)p(t)\lambda(t)\mathrm{d}t \right] \\
&+ c_d \left[\frac{\int_0^\infty \overline{F}_{T_N}(t)\mathrm{d}t \int_0^\infty P_N(t)F(t)\mathrm{d}t}{\int_0^\infty P_N(t)\mathrm{d}t} - \int_0^\infty \overline{F}_{T_N}(t)F(t)\mathrm{d}t \right] \\
&= c_m \left[\frac{\sum_{k=0}^{N-1} \int_0^\infty P_k(t)\mathrm{d}t \int_0^\infty P_N(t)p(t)(\lambda_0 + \alpha\mu_\beta t)\mathrm{d}t}{\int_0^\infty P_N(t)\mathrm{d}t} - \sum_{k=0}^{N-1} \int_0^\infty P_k(t)p(t)(\lambda_0 + \alpha\mu_\beta t)\mathrm{d}t \right] \\
&+ c_d \left[\frac{\sum_{k=0}^{N-1} \int_0^\infty P_k(t)\mathrm{d}t \int_0^\infty P_N(t)F(t)\mathrm{d}t}{\int_0^\infty P_N(t)\mathrm{d}t} - \sum_{k=0}^{N-1} \int_0^\infty P_k(t)F(t)\mathrm{d}t \right],
\end{aligned}$$

where $P_k(t)$ has been derived in (15).

From the fact that

$$\begin{aligned}
&\phi_T(N+1) - \phi_T(N) \\
&= c_m \sum_{k=0}^{N} \int_0^\infty P_k(t)\mathrm{d}t \left[\frac{\int_0^\infty P_{N+1}(t)p(t)(\lambda_0 + \alpha\mu_\beta t)\mathrm{d}t}{\int_0^\infty P_{N+1}(t)\mathrm{d}t} - \frac{\int_0^\infty P_N(t)p(t)(\lambda_0 + \alpha\mu_\beta t)\mathrm{d}t}{\int_0^\infty P_N(t)\mathrm{d}t} \right] \\
&+ c_d \sum_{k=0}^{N} \int_0^\infty P_k(t)\mathrm{d}t \left[\frac{\int_0^\infty P_{N+1}(t)F(t)\mathrm{d}t}{\int_0^\infty P_{N+1}(t)\mathrm{d}t} - \frac{\int_0^\infty P_N(t)F(t)\mathrm{d}t}{\int_0^\infty P_N(t)\mathrm{d}t} \right],
\end{aligned}$$

an optimum N^* for Policy N which minimizes $C_T(N)$ is as follows:

(1) If $\phi_T(N)$ increases strictly to $\phi_T(\infty)$ and $\phi_T(\infty) > c_N$, then there exists an optimal N^* $(1 \leq N^* < \infty)$ which minimizes $C_T(N)$ in (27). Policy N under this condition is perfectly adopted for the deteriorating system.

(2) If $\phi_T(\infty) \leq c_N$, the optimal $N^* = \infty$, that is, Policy N is not yet adopted.

5. An illustrative example

In this section, an illustrative example is designed to validate the theoretical results in terms of previous literatures. To keep the consistence, some parameters are the same with them in Wang and Pham (2012).

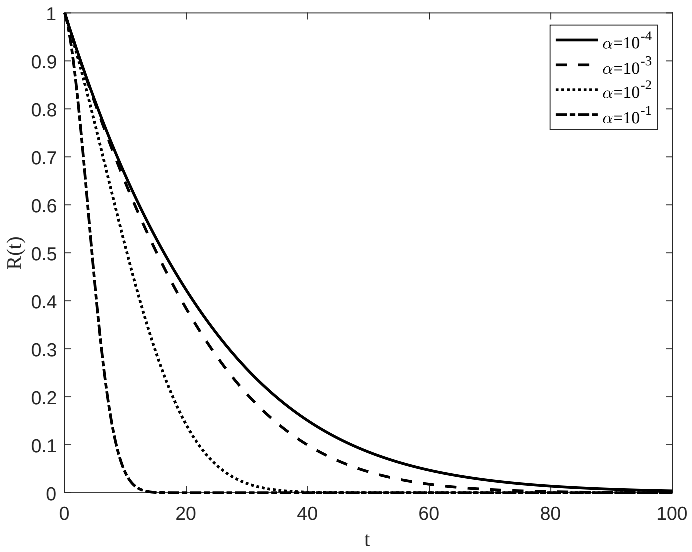

Figure 2. System reliability function under different degradation-shock influence factor α.

5.1. *System description*

Consider a system subject to a degradation process and random shocks. The degradation process is described with a general degradation path model $X(t) = \beta t$, in which β is Weibull distributed with CDF $F_\beta(x) = 1 - \exp[-(x/\mu)^\kappa]$ for $\mu > 0$ and $\kappa > 0$, where $\mu = 0.8$ and $\kappa = 1$. The individual shock load Y_i $(i = 1, 2, \cdots)$ resulting from major shocks towards the degradation path $X(t)$ follows a normal distribution, that is, $Y_i \sim \mathcal{N}(\mu_Y, \sigma_Y^2)$, in which $\mu_Y = 5$ and $\sigma_Y = 3$. The initial intensity of the random shock is $\lambda_0 = 1/15$, with a probability $p(t) = 1 - \exp(-\omega t)$ it becomes a minor shock, where $\omega = 0.0002$, and with a probability $q(t) \equiv 1 - p(t) = \exp(-0.0002t)$ it becomes a major shock. The failure threshold L is assumed to be a constant, that is, $L = 100$.

Setting the parameters $a_1 = 0.05$, $a_2 = 0.008$, $a_3 = 0.001$, $b_1 = 0$, and $b_2 = 0$ (Wang et al., 2020d), we have system reliability function shown in Fig. 2, where four values of degradation-shock influence factor are exhibited, i.e., $\alpha = 10^{-4}$, $\alpha = 10^{-3}$, $\alpha = 10^{-2}$, and $\alpha = 10^{-1}$, respectively.

As can be seen from Figure 2, system reliability $R(t)$ becomes lower and lower with the increase of α, illustrating that the deteriorating system is more vulnerable to fail when the dependence of the degradation process onto the shock process increases. For a clear illustration, Figure 3 plots the corresponding failure probability functions.

5.2. *Optimal maintenance policies*

Setting the maintenance costs $c_T = 80K\$$, $c_N = 100K\$$, $c_m = 20K\$$, and $c_d = 1K\$$, we have the maintenance cost rate $C_N(T)$ for Policy T under different degradation-shock influence factor α shown in Figure 4.

From Figure 4, we know that T^* exists for all α. At the same time, all T^* satisfies (25) for different α. Resorting to the software MATLAB, we give T^* and its corresponding $C_N(T^*)$ for different α, which is shown in Table 1.

From Table 1, we can see that the optimal T^* decreases with α while its resulting

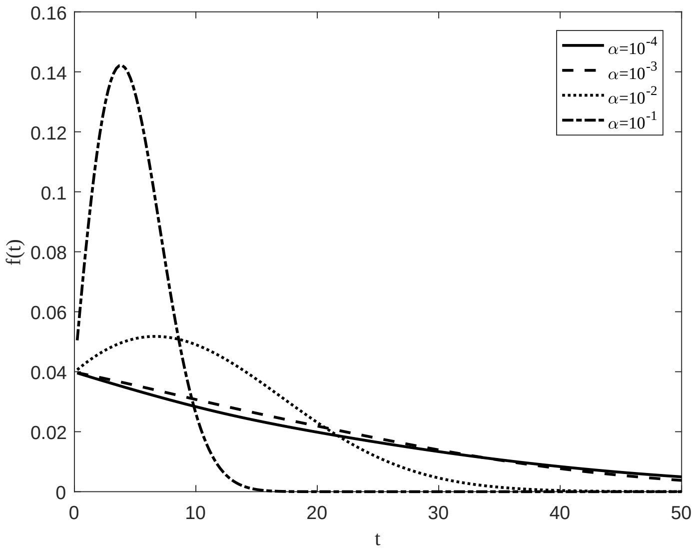

Figure 3. System failure density function under different degradation-shock influence factor α.

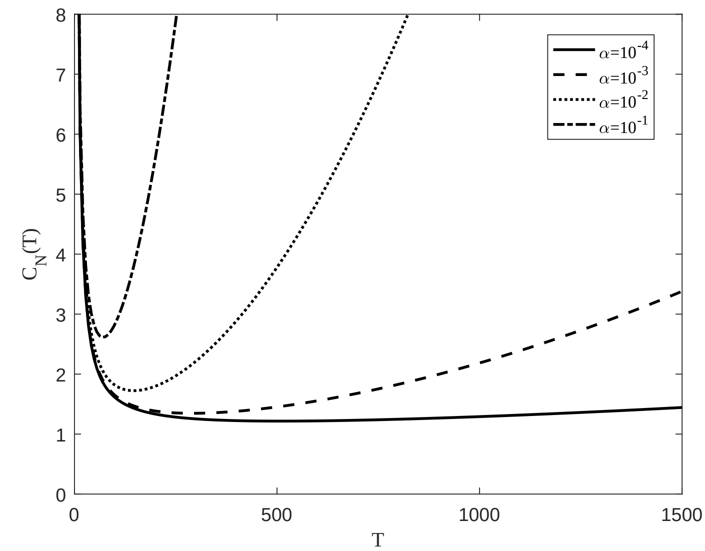

Figure 4. Expected maintenance cost rate for Policy T under different degradation-shock influence factor α.

Table 1. T^* and $C_N(T^*)$ for Policy T under different α.

α	10^{-4}	10^{-3}	10^{-2}	10^{-1}
T^*	542.7	304.8	145.5	69.28
$C_N(T^*)$	1.217	1.347	1.724	2.614

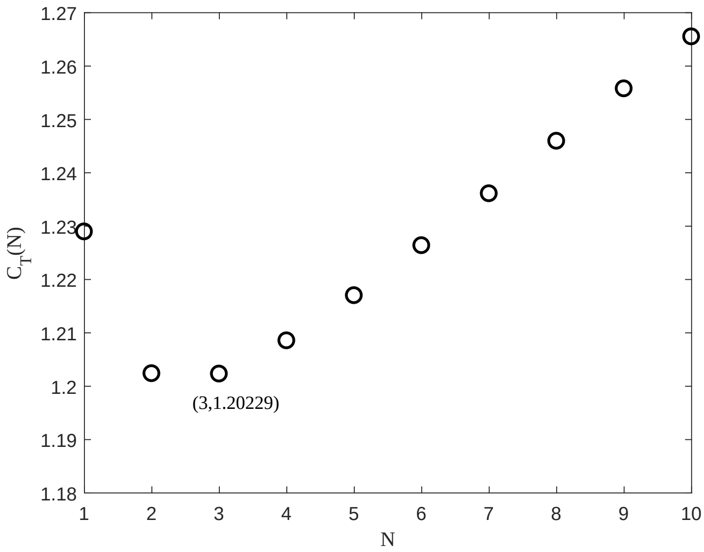

Figure 5. Expected maintenance cost rate for Policy N with $\alpha = 10^{-4}$.

Table 2. N^* and $C_T(N^*)$ for Policy N under different α.

α	10^{-4}	10^{-3}	10^{-2}	10^{-1}
N^*	3	2	2	2
$C_T(N^*)$	1.202	1.319	1.634	2.107

$C_N(T^*)$ increases with α. The reason is that the dependence between the degradation process and the shock process increases with α from the fact $\lambda(t) = \lambda_0 + \alpha X(t)$, causing that system reliability $R(t)$ decreases with α and an earlier replacement time should be arranged to minimize the average cost rate. Therefore, for Policy T, a shorter replacement cycle T^* is adopted along with a higher maintenance cost rate $C_N(T^*)$ when the degradation-shock influence factor α is relatively larger.

Then, we fix the degradation-shock influence factor $\alpha = 10^{-4}$ and plot the maintenance cost rate $C_T(N)$ for Policy N in Figure 5 at the same cost parameters $c_T = 80K\$$, $c_N = 100K\$$, $c_m = 20K\$$, and $c_d = 1K\$$.

It can be clearly seen from Figure 5 that the optimal $N^* = 3$ with its corresponding maintenance cost rate $C_T(N^*) = 1.202$ for Policy N at the same cost parameters $c_T = 80K\$$, $c_N = 100K\$$, $c_m = 20K\$$, and $c_d = 1K\$$, where the degradation-shock influence factor α is fixed as 10^{-4}. Similar to Policy T, we vary α and research the tendencies of N^* and $C_T(N^*)$. The results are listed in Table 2.

The reason is similar to that for Policy T. System reliability $R(t)$ decreases with α, and an earlier replacement policy should be implemented to minimize the average cost rate. The results in Table 2 shows that the discrete N^* has a decreasing tendency with the increase of α and $C_T(N^*)$ increases with α. Therefore, a smaller number N^* of minimal repair is suggested along with a higher maintenance cost rate $C_T(N^*)$ for Policy N when the degradation-shock influence factor α is relatively larger.

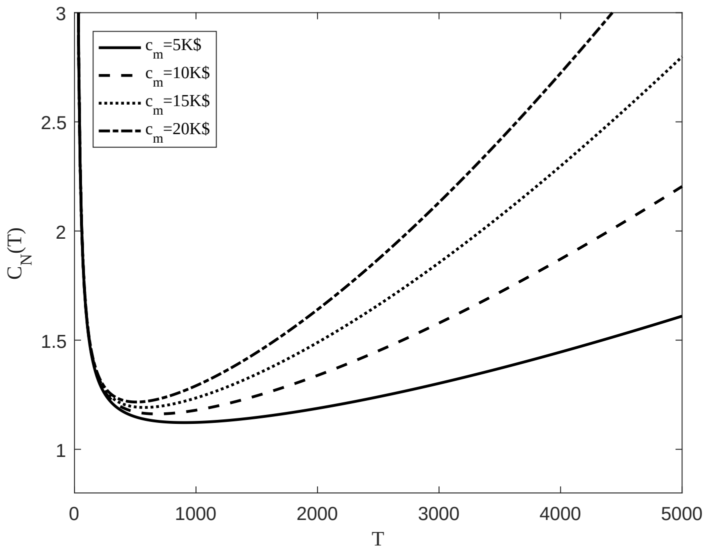

Figure 6. Expected maintenance cost rate for Policy T under different c_m.

Table 3. T^*, $C_N(T^*)$ and N^*, $C_T(N^*)$ for Policy T and Policy N under different c_m.

c_m	$5K\$$	$10K\$$	$15K\$$	$20K\$$
T^*	935.3	762.1	681.3	542.7
$C_N(T^*)$	1.122	1.163	1.195	1.217
N^*	8	4	3	3
$C_T(N^*)$	1.105	1.144	1.174	1.202

5.3. *Sensitivity analysis*

Sensitivity analysis is an important part of any decision making process. Maintenance policymakers like to be informed about the results of any changes in sensitive parameters. In this part, we vary the value of each minimal repair c_m from $5K\$$ to $20K\$$ with an increasing cost step $5K\$$. Figure 6 and Figure 7 show the optimal solutions for Policy T and Policy N under different c_m, respectively, in which $c_T = 80K\$$, $c_N = 100K\$$, $c_d = 1K\$$, and $\alpha = 10^{-4}$.

According to Section 4, the optimal T^*, $C_N(T^*)$ and N^*, $C_T(N^*)$ under different c_m are listed in Table 3 for Policy T and Policy N, respectively.

From Table 3, we can see that both T^* and N^* decrease with the minimal cost c_m, while on the contrary, both $C_N(T^*)$ and $C_T(N^*)$ increase with c_m. The reason is that the numerator of (2[illegible]he total maintenance cost in the first renewal cycle, increases with c_m, ~~and to~~ minimize the average cost rate, an earlier replacement policy should be arranged. Therefore, both T^* and N^* decrease with the minimal cost c_m while $C_N(T^*)$ and $C_T(N^*)$ increase with c_m. The variation tendencies for Policy T and Policy N are consistent.

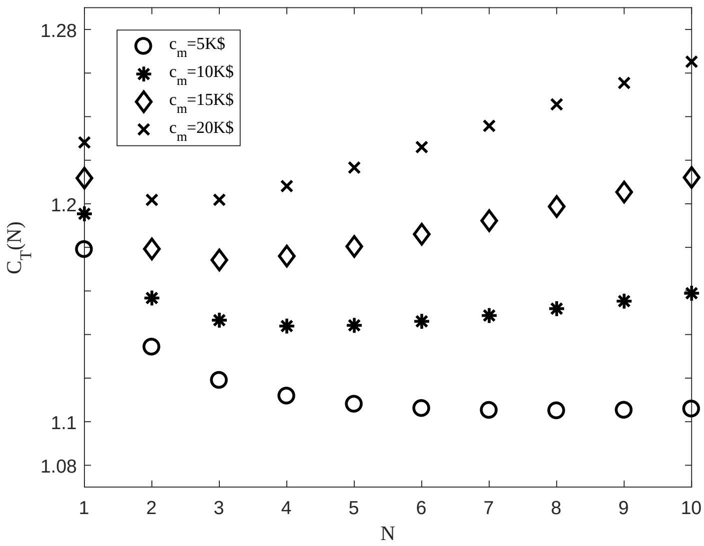

Figure 7. Expected maintenance cost rate for Policy N under different c_m.

6. Conclusions

In this paper, a proactive replacement maintenance strategy for a stochastically deteriorating system with random shocks is scheduled, in which the system is proactively replaced at a planned time, or at an optimal number of minimal repairs, whichever takes place first. Different from previous researches, two types of shocks are incorporated upon arrival of an external shock, where a minor shock is formed with a time-dependent probability, and a major shock is generated with another time-dependent probability. The closed-form reliability function is constructed based on an age- and state-dependent competing risks model embedded with two types of shocks, including the consideration of the impacts from the current degradation state in regard to the effects that the major shocks have on the pure degradation process, which is described with a general degradation path model. Afterwards, a bivariate proactive replacement policy containing a discrete optimization variable and a continuous optimization variable is formulated, and the optimal solutions for two special cases are settled analytically in terms of the renewal reward theorem. A numerical example is reexamined to validate the theoretical results, and the sensitivities of cost parameters are analyzed.

The conclusions show that two types of shocks and mutual dependence between the degradation process and the shock process are important factors contributing to system reliability and the optimal maintenance strategy. Further researches will emphasize on the operability of the proactive maintenance model and its introduction into more industrial applications in all sorts of production systems.

Disclosure statement

The authors report there are no competing interests to declare.

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