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Probabilistic Decoupling Control for Stochastic Non-linear Systems using EKF-based Dynamic Set-point Adjustment

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***Abstract*—In this paper, a novel decoupling control scheme is presented for a class of stochastic non-linear systems by estimation-based dynamic set-point adjustment. The loop control layer is designed using PID controller where the parameters are fixed once the design procedure is completed, which can be considered as an existing control loop. While the compensator is designed to achieve output decoupling in probability sense by a set-point adjustment approach based on the estimated states of the systems using extended Kalman filter. Based upon the mutual information of the system outputs, the parameters of the set-point adjustment compensator can be optimised. Using this presented control scheme, the analysis of stability is given where the tracking errors of the closed-loop systems are bounded in probability one. To illustrate the effectiveness of the presented control scheme, one numerical example is given and the results show that the systems are stable and the probabilistic decoupling is achieved simultaneously.**

I. Introduction

Modern industrial processes become more and more complex with the developments of the various high-level technological requirements. In practice, PID control strategy is still widely used due to the simple structure which brings the operators a lot of benefits [1]. For example, the parameter tuning of the PID controller is rapid and convenient for new operators.

Although a lot of existing control systems are based on PID control loops in practice, it is very difficult to 'plug in' the decoupling design based on the existing control loops once the system designs are completed. In particular, some controller is realised by analogue circuits and packaged with the actuators which implies that the parameters of the existing control loops are impossible to change once the design procedure is completed [2]. On the other hand, most of the traditional decoupling designs are based on the deterministic system models, however the accuracy of the model and the random noises affect the performances of the systems [3]. Thus, the decoupling problem is hardly solved for the stochastic systems with the existing control loop which means that it is significant to present a novel decoupling control scheme for the stochastic systems with existing control loops.

Motivated by the operational optimal control scheme [4], an extra compensating loop can be added onto the existing control loop which is similar to the cascade control [5]. In this way, the additional loop dynamically re-adjust the set-point of the existing control loop to compensate the residuals even if the existing control loop, such as PID controller, cannot satisfy the design requirements perfectly. Since the existing decoupling control methods [6] would be deteriorated because of the dependent noises, the probabilistic decoupling [7], [8] has been presented as an extension of the traditional decoupling while the system outputs can be considered as the random variables. In other words, the decoupling can be achieved if the system outputs are independent in probability sense. Based upon this idea, the objective of this paper is to design an optimal set-point adjustment loop in order to achieve probabilistic decoupling for the investigated stochastic non-linear systems.

It is ideal to design this extra loop based on full states of the systems which include more information than system outputs. However, most of the system states are unmeasurable which means the states of the systems cannot be used directly. Alternatively, various filter design methods have been developed since the Kalman filter was presented [9], for example, the extended Kalman filter (EKF) was presented in [10] for non-linear stochastic systems. Similar to the PID control strategy, EKF is selected to use in this paper due to the simple parametric structure.

In particular, a novel control scheme is presented with double-loop design while the existing control loop would be never changed once the parameters are selected and the compensating loop is design to achieve the decoupling design using EKF-based set-point adjustment. Based on the assumptions, the stability of the closed-loop stochastic system is analysed in probability one, and then the decoupling problems of stochastic system outputs are considered by date-based approach where the kernel density estimation (KDE) [11] is used to approximate the statistical properties of the system outputs. Moreover, the optimal parameters of the presented

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control algorithm can be obtained by constrained optimisation approach. Notice that this control scheme can be potentially extended to tracking performance enhancement problem while the related results can be found in [12], [13]. As a summary, a novel decoupling control strategy is presented for stochastic non-linear systems with existing PID control loops.

II. Preliminaries

A. Formulation

Basically, the investigated system can be described using multivariate non-linear difference equation subjected to additive Gaussian noise. For the i-th subsystem, the model can be given by

$$x_{i,k+1} = f_i\left(x_{i,k}, u_{i,k}\right) + G_i w_{i,k} \tag{1a}$$

$$y_{i,k} = C_i x_{i,k} + D_{i,k} v_{i,k} \tag{1b}$$

where $1 \leq i \leq N$ denotes the subsystem index. $x_i \in R^{n_i}$, $y_i \in R^{m_i}$ and $u_i \in R^{s_i}$ are the state, output and control input of the i-th subsystem, respectively. While n_i, m_i and s_i are the associated dimensions of the i-th subsystem. $w_i \in R^{p_i}$ and $v_i \in R^{q_i}$ are the zero-mean vector-valued Gaussian noises. Matrices G_i, C_i and D_i are of appropriate dimensions. $f_i : R^{n_i} \times R^{s_i} \to R^{n_i}$ are real non-linear functions. Notice that the distributions of the system states and outputs are non-Gaussian even if the external noises obey Gaussian distribution.

In practice, most of the controller design approaches are based on linear model around the known equilibrium points. Without loss of generality, the model for the subsystems can be restated as

$$x_{i,k+1} = A_i x_{i,k} + B_i u_{i,k} + g_{i,k}\left(x_{i,k}, u_{i,k}\right) + G_i w_{i,k} \tag{2a}$$

$$y_{i,k} = C_i x_{i,k} + D_{i,k} v_{i,k} \tag{2b}$$

where $g_i : R^{n_i} \times R^{s_i} \to R^{n_i}$ are unknown non-linear functions which represents the unmodelled dynamics. Since the approximated equilibrium points are denoted as (x_i^*, u_i^*), the coefficient matrices for the i-th subsystems are calculated by linearisation as follows:

$$\{A_i, B_i\} = \left\{ \frac{\partial f_i\left(x_i, u_i\right)}{\partial x_i}, \frac{\partial f_i\left(x_i, u_i\right)}{\partial u_i} \right\} \Bigg|_{x_i = x_i^*, u_i = u_i^*} \tag{3}$$

In order to simplify the expression, all the subsystems can be rewritten into a composite system as follows:

$$x_{k+1} = A x_k + B u_k + g_k\left(x_k, u_k\right) + G w_k \tag{4a}$$

$$y = C x_k + D v_k \tag{4b}$$

where x, y, u, w, v are compact vectors defined as $x = \left[x_1^T, \ldots, x_N^T\right]^T$, $y = \left[y_1^T, \ldots, y_N^T\right]^T$, $u = \left[u_1^T, \ldots, u_N^T\right]^T$, $w = \left[w_1^T, \ldots, w_N^T\right]^T$, $v = \left[v_1^T, \ldots, v_N^T\right]^T$, while A, B, C, D, G are corresponding system matrices which can be written as $A = diag\{A_1, \ldots, A_N\}$, $B = diag\{B_1, \ldots, B_N\}$, $C = diag\{C_1, \ldots, C_N\}$, $G = diag\{G_1, \ldots, G_N\}$, $D = diag\{D_1, \ldots, D_N\}$. The unknown vector-valued function term is given by $g(x, u) = \left[g_1^T(x_1, u_1), \ldots, g_N^T(x_N, u_N)\right]^T$.

Based on the linear part of the composite model, the loop controller can be designed using the error of loop control $e_{\bar{k}}$. Unfortunately, the system output y_k cannot be analysed separately due to the effect of the non-linear term. Therefore, the set-point of the loop control $r_{\bar{k}}$ should be adjusted by the residual ε_k. Particularly, $e_{\bar{k}}$ and ε_k are described respectively as

$$e_{\bar{k}} = r_{\bar{k}} - y_{\bar{k}}, \varepsilon_k = y_k^* - y_k \tag{5}$$

where y_k^* denotes the ideal reference signal and $\bar{k}$ stands for the faster discretization operation than sampling instant k because the dynamic of the set-point should always be slower than the control loop. In other words, the loop controller should be designed based on the shorter sampling time. The coefficient matrices A, B, C, D and G should also rewritten as $A_{\bar{k}}$, $B_{\bar{k}}$, $C_{\bar{k}}$, $D_{\bar{k}}$ and $G_{\bar{k}}$ although $y_{\bar{k}}$ and y_k are still equivalent.

Generally, the control inputs and the set-points can be further expressed as

$$u_{\bar{k}} = f_u\left(\bar{e}_{\bar{k}}, \bar{u}_{\bar{k}-1}\right), r_k = f_r\left(\bar{\varepsilon}_k, \bar{\hat{x}}_k, \bar{r}_{k-1}\right) \tag{6}$$

where $\hat{x}$ is the estimated state of the composite system. f_u and f_r are general real functions while variables of the functions are formulated as $\bar{e}_k = [e_k, \ldots, e_0]$, $\bar{u}_{k-1} = [u_{k-1}, \ldots, u_0]$, $\bar{\varepsilon}_k = [\varepsilon_k, \ldots, \varepsilon_0]$, $\bar{\hat{x}}_k = [\hat{x}_k, \ldots, \hat{x}_0]$ and $\bar{r}_{k-1} = [r_{k-1}, \ldots, r_0]$.

In this paper, the control objective is to develop an approach to find a function f_r which makes the system outputs independent in probability sense with a non-adjustable f_u. To achieve this design objective, the following assumptions are taken into accounts.

H1: The system model (4) is controllable and observable.

H2: The non-linear term satisfies Lipschitz condition while there exist two real positive numbers L_1 and L_2, such that

$$\left\|g_k\left(x_k, u_k\right) - g_k\left(\hat{x}_k, u_k\right)\right\| \leq L_1 \left\|\tilde{x}_k\right\| \tag{7a}$$

$$\left\|g_k\left(x_k, u_k\right)\right\| \leq L_2 \left\|x_k\right\| \tag{7b}$$

H3: The norm of the states and the estimated error are bounded by the following inequalities.

$$\|x_k\| \leq \bar{a} \|\varepsilon_k\| + \bar{b} \tag{8a}$$

$$\|\tilde{x}_{k-1}\| \leq \bar{c} \|\tilde{x}_k\| + \bar{d} \tag{8b}$$

where $\bar{a}, \bar{b}, \bar{c}$ and $\bar{d}$ are positive real numbers.

Remark 1: r_k denotes the set-point of the control loop which might be different from y_k^*. For single layer control strategies, the reference signal y_k^* is equal to the set-point r_k.

B. The Extended Kalman Filter

Based upon the stochastic system model (4), the associated EKF [14] can be applied as follow:

Definition 1: The discrete-time extended Kalman filter can be given by the following equations:

- State estimation:

$$\hat{x}_{k+1} = f\left(\hat{x}_k, u_k\right) + K_{f,k}\left(y_k - C\hat{x}_k\right) \tag{9a}$$

- Kalman gain:

$$K_{f,k} = AP_{f,k}C^T\left(CP_{f,k}C^T + R_k\right)^{-1} \tag{9b}$$

- Riccati difference equation:

$$P_{f,k+1} = AP_{f,k}A^T + Q_k - AP_{f,k}C^T K_{f,k}^T \tag{9c}$$

where Q_k and R_k are time-varying symmetric positive definite matrices with appropriate dimensions. Matrix A is defined by Eq.(3) and Eq.(4).

C. Stochastic Boundedness

Since the states of the system are estimated by EKF and the estimated error can be given by

$$\tilde{x}_k = x_k - \hat{x}_k \tag{10}$$

The error vector of the closed-loop system can be further expressed as

$$\zeta_k = \left[\tilde{x}_k^T, \varepsilon_k^T\right]^T \tag{11}$$

To analyse the error dynamic of ζ_k, the following concepts are recalled for the boundedness of stochastic processes.

Definition 2: The stochastic process ζ_k is said to be exponentially bounded in mean square sense, if there exist real numbers $\eta, \upsilon > 0$ and $0 < \vartheta < 1$ such that for $\forall k$, the following inequality holds.

$$E\left\{\|\zeta_k\|^2\right\} \leq \eta\|\zeta_0\|^2\vartheta^k + \upsilon \tag{12}$$

where $E\{\cdot\}$ and $\|\cdot\|$ denote expectation operations and norm operation, respectively.

Definition 3: The stochastic process ζ_k is said to be bounded with probability one, if the following equation holds.

$$\Pr\left\{\limsup_{k\to\infty}\|\zeta_k\| < \infty\right\} = 1 \tag{13}$$

where $\Pr\{\cdot\}$ is the operator to obtain the value of probability.

In addition, a lemma is given about the boundedness of stochastic processes.

Lemma 4: For stochastic process ζ_k, assume there is a stochastic process $V_k(\zeta_k)$ as well as real positive numbers $\overline{v}, \underline{v}, \mu, \alpha, \beta > 0$ and $0 < \alpha + \beta \leq \underline{v}$, such that

$$\underline{v}\,\|\zeta_k\|^2 \leq V_k(\zeta_k) \leq \overline{v}\|\zeta_k\|^2 \tag{14}$$

and

$$E\left\{V_{k+1}(\zeta_{k+1})|_{\zeta_k}\right\} \leq \alpha\|\zeta_k\|^2 + \beta\|\zeta_k\| + \mu \tag{15}$$

are fulfilled for every solution of stochastic process ζ_k. Then ζ_k is bounded with probability one. Moreover it is also exponentially bounded in mean square sense, which implies that for $\forall k \geq 0$, we have

$$E\left\{\|\zeta_k\|^2\right\} \leq \frac{\overline{v}}{\underline{v}}E\left\{\|\zeta_0\|^2\right\}\left(\frac{\alpha+\beta}{\underline{v}}\right)^k + \frac{\beta+\mu}{\underline{v}-\alpha-\beta} \tag{16}$$

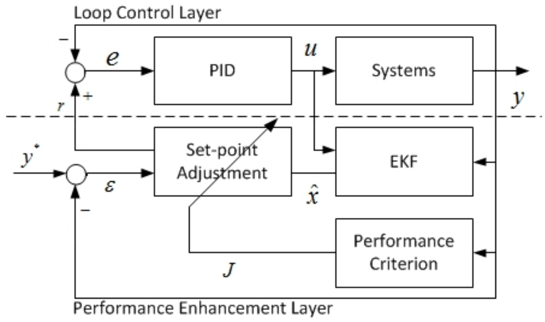

Fig. 1. The schematic diagram for the control scheme

III. Control Scheme by Set-point Adjustment

The control scheme is presented in this section where the discrete-time PID controllers are obtained for subsystem loops and the set-point signals are designed using the estimated states. The schematic diagram is given by Fig. 1.

A. Loop Control Design

Denoting $K = [K_P, K_I, K_D]$ and $z_{\bar{k}} = \sum_{i=0}^{k} e_{\bar{k}}$, the PID controller can be given as follow:

$$u_{\bar{k}} = K\left[e_{\bar{k}}^T, z_{\bar{k}}^T, e_{\bar{k}}^T - e_{\bar{k}-1}^T\right]^T \tag{17a}$$

$$z_{\bar{k}} = z_{\bar{k}-1} + e_{\bar{k}} \tag{17b}$$

where the elements of matrix K are the parameters of the controller. Next, the reasonable parameters can be selected using the system model (4).

Defining $\theta_{\bar{k}} = \left[x_{\bar{k}}^T, z_{\bar{k}}^T, x_{\bar{k}-1}^T\right]^T$ as a new state vector, then the dynamic of $\theta_{\bar{k}}$ can be stated by a state-space model as

$$\theta_{\bar{k}+1} = A_d\theta_{\bar{k}} + B_d r_k + G_d w_{\bar{k}} + E_d g_k(x_k, u_k) \tag{18a}$$

$$y_{\bar{k}} = C_d\theta_{\bar{k}} + D_d v_{\bar{k}} \tag{18b}$$

where $E_d = \left[\overline{1}, 0, 0\right]^T$ where both $\overline{1}$ and 0 in this equation are vectors. Other coefficient matrices can be expressed by $A_d = \bar{A} + \bar{B}K\bar{C}$, $B_d = [B_{\bar{k}}K_P, I, 0]^T$, $C_d = [C_{\bar{k}}, 0, 0]$, $D_d = [D_{\bar{k}}, 0, 0]^T$. $G_d = [G_{\bar{k}}, 0, 0]^T$, while

$$\bar{A} = \begin{bmatrix} A_{\bar{k}} & 0 & 0 \\ -C_{\bar{k}} & I & 0 \\ I & 0 & 0 \end{bmatrix} \bar{B} = \begin{bmatrix} B_{\bar{k}} \\ 0 \\ 0 \end{bmatrix} \bar{C} = \begin{bmatrix} -C_{\bar{k}} & 0 & 0 \\ 0 & I & 0 \\ -C_{\bar{k}} & 0 & C_{\bar{k}} \end{bmatrix} \tag{19}$$

The parameters can be chosen in the ideal situation ignoring non-linear term. Thus, the loop control model can be simplified as

$$\theta_{\bar{k}+1} = A_d\theta_{\bar{k}} + \bar{d}_{1,\bar{k}} \tag{20a}$$

$$y_{\bar{k}} = C_d\theta_{\bar{k}} + \bar{d}_{2,\bar{k}} \tag{20b}$$

where $\bar{d}_{1,\bar{k}} = B_d y_{\bar{k}}^* + G_d w_{\bar{k}}$ and $\bar{d}_{2,\bar{k}} = D_d v_{\bar{k}}$ are considered as the exogenous disturbances.

The reasonable selection of the parameters can be determined by the following proposition which is similar to the design approach in [15].

Proposition 5: The systems presented by model (18) are stable in ideal situation (19). If the parameter matrix $K = W^{-1}Y$ is deigned by the following linear matrix inequality (LMI):

$$\begin{bmatrix} -M & M\bar{A} + \bar{B}Y\bar{C} \\ \left(M\bar{A} + \bar{B}YC\right)^T & -(1-\bar{\alpha})M \end{bmatrix} < 0 \quad (21)$$

where M is a symmetric positive definite matrix, $Y = WK$, $M\bar{B} = \bar{B}W$ and $\bar{\alpha} \geq 0$ denotes the decay rate.

Proof of Proposition 5: The proof is similar to the results in [15], therefore, we omit it here. ■

B. Decoupling Layer Design

The subsystems cannot work in the ideal situation. In particular, the parameters cannot be changed once they are fixed in practice. In other words, the subsystems can be treated as the existing control loops. In these existing control loops, only the set-points can be adjusted and the existing control loops can be treated as a new system with slow sampling rate. In this paper, we assume C_dB_d is invertible, then the dynamic set-point vector is designed as follows:

$$r_k = (C_dB_d)^{-1}\left(y^*_{k+1} - C_dA_d\hat{\theta}_k - \Theta_1\varepsilon_k - \Theta_2\Lambda_k\right) \quad (22a)$$

$$\Lambda_k = \kappa_1\Lambda_{k-1} + \kappa_2\varepsilon_{k-1} \quad (22b)$$

where Λ is weighted integrator, Θ_1 and Θ_2 are the design parameters and $\hat{\theta}_k = \left[\hat{x}^T_k, z^T_k, \hat{x}^T_{k-1}\right]^T$ is the estimated state vector for θ_k. κ_i denotes the real positive number where $0 < \kappa_i < 1$.

IV. Convergence Analysis in Probability Sense

First of all, the error dynamics of the estimated states are described by

$$\tilde{x}_{k+1} = (A - K_fC)\tilde{x}_k + Gw_k - K_fDv_k + g_k(x_k, u_k) - g_k(\hat{x}_k, u_k) \quad (23)$$

Next, the residual dynamic can be given as

$$\varepsilon_{k+1} = y^*_{k+1} - C_dA_d\theta_k - C_dB_dr_k - C_dG_dw_k - D_dv_k - C_dE_dg_k(x_k, u_k) \quad (24)$$

Notice that the vector term $C_dA_d\theta_k$ can be restated by

$$C_dA_d\theta_k = \Pi_1x_k + \Pi_2z_k + \Pi_3x_{k-1} \quad (25)$$

where $\Pi_1 = -C(BCK_D - A + BCK_P)$, $\Pi_2 = CBK_I$ and $\Pi_3 = CBCK_D$.

Substituting the dynamic set-point adjustment law (22) and Eq. (25) to Eq. (24), we have

$$\varepsilon_{k+1} = \Theta_1\varepsilon_k + \Theta_2\Lambda_k - \Pi_1\tilde{x}_k - \Pi_3\tilde{x}_{k-1} - C_dG_dw_k - D_dv_k - C_dE_dg_k(x_k, u_k) \quad (26)$$

Thus, the generalized error dynamic system can be formulated as the following equation.

$$\zeta_{k+1} = A_f\zeta_k + d_k + s_k \quad (27)$$

where

$$A_f = \begin{bmatrix} A - K_fC & 0 & 0 \\ -\Pi_1 & \Theta_1 & \Theta_2 \\ 0 & \kappa_2 & \kappa_1 \end{bmatrix} \quad (28a)$$

$$d_k = \begin{bmatrix} Gw_k - K_fDv_k \\ -C_dG_dw_k - D_dv_k \\ 0 \end{bmatrix} \quad (28b)$$

$$s_k = \begin{bmatrix} g_k(x_k, u_k) - g_k(\hat{x}_k, u_k) \\ -\Pi_3\tilde{x}_{k-1} - C_dE_dg_k(x_k, u_k) \\ 0 \end{bmatrix} \quad (28c)$$

and $\zeta_k = \left[\tilde{x}^T_k, \varepsilon^T_k, \Lambda^T_k\right]^T$.

Before presenting the main result of the convergence analysis, the following lemmas are given at first.

Lemma 6: Suppose A is a real non-singular matrix, there exist a real positive definite matrix P and a real constant $\alpha > 0$ such that

$$A^TP^{-1}A \leq \alpha P^{-1} \quad (29)$$

Moreover, if $\|A\| < 1$, the constant $0 < \alpha < 1$ exists.

Lemma 7: Suppose P is a real positive definite matrix, $\underline{p}$ is the infimum of P. Based on the error dynamic (27), the following inequalities holds.

$$s^T_kP^{-1}(2A_f\zeta_k + s_k) \leq \frac{1}{\underline{p}}\left(N_1\|\zeta_k\|^2 + N_2\|\zeta_k\| + M^2_3\right) \quad (30a)$$

$$E\left\{d^T_kP^{-1}d_k\right\} \leq \frac{\delta^2}{\underline{p}} \quad (30b)$$

where

$$N_1 = (M_1 + M_2)(2\|A_f\| + M_1 + M_2) \quad (31a)$$

$$N_2 = 2M_3(\|A_f\| + M_1 + M_2) \quad (31b)$$

$$\delta = (\|G\| + \|C_dG_d\|)E\{\|w_k\|\} + (\|K_fD\| + \|D_d\|)E\{\|v_k\|\} \quad (31c)$$

while

$$M_1 = L_1 + \bar{c}\|\Pi_3\| \quad (32a)$$

$$M_2 = \bar{a}L_2\|C_dE_d\| \quad (32b)$$

$$M_3 = \bar{d}\|\Pi_3\| + \bar{b}L_2\|C_dE_d\| \quad (32c)$$

Based on Eq. (27) and the lemmas above, the theorem is obtained as the main result of the convergence analysis.

Theorem 8: The stochastic systems (1) with control inputs (17) and dynamic set-points (22) are stable in probability one if the following condition meet. Moreover, the outputs of the systems are stable in mean square sense.

$$0 < \|A_f\| \leq \frac{1 - \alpha - L^2_1 - \bar{\delta}_1}{2\left(L_1 + \bar{\delta}_2\right)} < 1 \quad (33)$$

where

$$\bar{\delta}_1 = \bar{c}\left(\bar{c} + 2\bar{d}\right)\|\Pi_3\|^2 + 2\left(\bar{c} + \bar{d}\right)L_1\|\Pi_3\| + \left(\bar{a} + 2\bar{b}\right)L^2_2\|C_dE_d\|^2 + 2\left(\bar{a} + \bar{b}\right)L_1L_2\|C_dE_d\| + 2\left(\bar{a}\bar{c} + \bar{b}\bar{c} + \bar{a}\bar{d}\right)L_2\|\Pi_3\|\|C_dE_d\| \quad (34a)$$

$$\bar{\delta}_2 = \left(\bar{c} + \bar{d}\right)\|\Pi_3\| + \left(\bar{a} + \bar{b}\right)L_2\|C_dE_d\| \quad (34b)$$

Proof of Theorem 8: Using Lemma 6, a positive definite matrix P is obtained and the Lyapunov function candidate can be chosen as

$$V_{k+1}(\zeta_{k+1}) = \zeta_{k+1}^T P^{-1} \zeta_{k+1} \tag{35}$$

Then we have,

$$\begin{aligned} &V_{k+1}(\zeta_{k+1}) && (36)\\ &= (A_f\zeta_k + d_k + s_k)^T P^{-1} (A_f\zeta_k + d_k + s_k)\\ &= \zeta_k^T A_f^T P^{-1} A_f \zeta_k + 2d_k^T P^{-1}(A_f\zeta_k + s_k)\\ &+ s_k^T P^{-1}(2A_f\zeta_k + s_k) + d_k^T P^{-1} d_k && (37)\end{aligned}$$

Taking the conditional expectation, it is shown that $E\left\{ d_k^T P^{-1}(A_f\zeta_k + s_k)\big|_{\zeta_k}\right\}$ vanishes. Based upon Lemma 6 and Lemma 7 the following inequality is obtained.

$$\begin{aligned} &E\left\{ V_{k+1}(\zeta_{k+1})\big|_{\zeta_k}\right\}\\ &\le \alpha V_k(\zeta_k) + \frac{1}{\underline{p}}\left(N_1\|\zeta_k\|^2 + N_2\|\zeta_k\| + M_3^2\right) + \frac{\delta^2}{\underline{p}}\\ &\le \frac{\alpha}{\underline{p}}\|\zeta_k\|^2 + \frac{1}{\underline{p}}\left(N_1\|\zeta_k\|^2 + N_2\|\zeta_k\| + M_3^2\right) + \frac{\delta^2}{\underline{p}}\\ &\le \frac{\alpha + N_1}{\underline{p}}\|\zeta_k\|^2 + \frac{N_2}{\underline{p}}\|\zeta_k\| + \frac{M_3^2 + \delta^2}{\underline{p}} && (38)\end{aligned}$$

where $\underline{p}$ is the infimum of P.

Based upon the condition of Lemma 4, the generalized error dynamic system (27) is bounded with probability one if the following inequality holds.

$$\frac{\alpha + N_1}{\underline{p}} + \frac{N_2}{\underline{p}} \le \frac{1}{\overline{p}} \tag{39}$$

where $\overline{p}$ is the supremum of P.

Without loss of generality, this inequality can be restated by

$$\alpha + N_1 + N_2 \le 1 \tag{40}$$

Substituting Eq.(31) and Eq.(32), inequality (39) can be rewritten as (33) while the proof is completed. ■

V. Parametric Optimization

The parameter optimization is not a unconstrained optimization problem since the design parameter Θ_i has to be chosen within a set which satisfied the condition of Theorem 8. Based on the structure of A_f and condition (33), the constraint condition can be given as

$$\|\Theta_1 + \Theta_2\| \le \Theta_0 \tag{41}$$

while

$$\Theta_0 = \frac{1 - \alpha - L_1^2 - \bar{\delta}_1}{2\left(L_1 + \bar{\delta}_2\right)} - \|A - K_f C\| - \|\Pi_1\| - \|\kappa_1\| - \|\kappa_2\| \tag{42}$$

In order to obtain the optimal parameters, the performance criterion can be given combining the Cauchy-Schwarz quadratic mutual information (CSQMI) [16] and KDE.

$$\begin{aligned} J_k = \min_{\Theta} &\left(\frac{1}{N^2}\sum_{i,j=1}^{N}\prod_{dim=1}^{n}\hat{V}_{dim}(i,j)\right)\\ &+ \prod_{dim=1}^{n}\hat{V}_{dim} - 2\left(\frac{1}{N}\sum_{i=1}^{N}\prod_{dim=1}^{n}\hat{V}_{dim}(i)\right) && (43)\end{aligned}$$

where $dim = 1, \ldots, n$ stands for the index of the system outputs and

$$\hat{V}_{dim}(i,j) = G_{\sqrt{2}\Sigma}(y_{i,k} - y_{j,k}) \tag{44a}$$

$$\hat{V}_{dim}(i) = \frac{1}{N}\sum_{j=1}^{N}\hat{V}_{dim}(i,j) \tag{44b}$$

$$\hat{V}_{dim} = \frac{1}{N}\sum_{i=1}^{N}\hat{V}_{dim}(i) \tag{44c}$$

where $G(\cdot)$ is Gaussian kernel function. Therefore, the selection of design parameters can be transformed to a constrained optimisation problem formulated by (43) and (41).

VI. Numerical Example

To verify the performance enhancement of probabilistic decoupling, the following MIMO discrete-time stochastic systems are considered.

$$\begin{aligned} x_{k+1} &= \begin{bmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ -0.5 & -0.6 & -0.7 \end{bmatrix} x_k + \begin{bmatrix} 0 & 1\\ 1 & 0\\ 1 & 1 \end{bmatrix} u_k\\ &+ 0.3\sin x_k + w_k\\ y &= \begin{bmatrix} 0.3 & 0.2 & -0.5\\ 1.5 & -1 & -0.5 \end{bmatrix} x_k + v_k \end{aligned}$$

where the ideal reference vector y^* is equal to 5, sampling time in terms of $\bar{k}$ is 0.01s and the w_k, v_k are Gaussian noises.

Using the presented design algorithm, the results are given by figures in this section: Figure 2 shows the mutual information of the system tracking errors while the mutual information is attenuated which indicates that the coupling among the system outputs are reduced in probability sense. Notice that the mutual information cannot be estimated in the first 2 second due to the lack of data. The PDFs of the system outputs are given by Figure 3 and Figure 4, respectively, which imply the stability of the systems are guaranteed. Furthermore, Figure 5 shows the designed dynamic set-point r_1 and r_2 with sampling time 1s. All of these results verify the effectiveness of the presented algorithm.

VII. Conclusions

This paper presents a novel decoupling design for a class of stochastic non-linear systems. Particularly, a dynamic set-point adjustment compensator is designed based on EKF. The control scheme can be divided into two layers: the loop control layer can be considered as the existing control loop and standard PID controller has been design in this paper; and the probabilistic decoupling loop is designed to adjust the set-point of the PID loop by the estimated system states. The stability analysis shows that all of the parameters within the stable parameter set can guarantee the closed-loop systems bounded in probability one. Using the mutual information based performance criterion, the design parameters can be searched within the stable parameter set by constrained optimization and the optimums can be obtained which is validated by the numerical example.

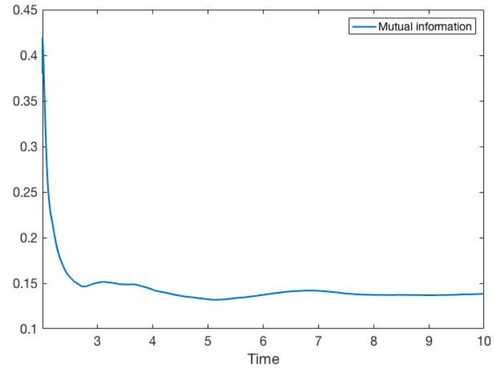

Fig. 2. The curve of performance criterion J_k for the system tracking errors

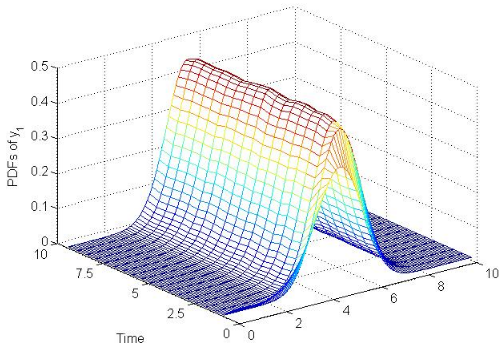

Fig. 3. The PDFs of the system output y_1

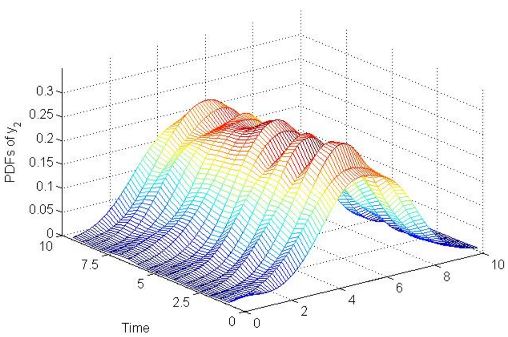

Fig. 4. The PDFs of the system output y_2

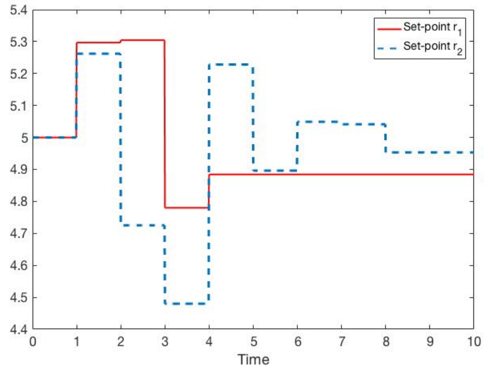

Fig. 5. The dynamic set-point r_1 and r_2 for the system output y_1 and y_2

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