

Similarity-Trust Network For Clustering Based Consensus Group Decision Making Model

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Trust relation, as defined in Social Network Analysis (SNA), is one of the recent notions considered in decision making. This inspired our integration of trust relation in constructing a similarity-trust network. Similarity of experts' preferences is analyzed inclusively with trust relation by defining a new combination function of both attributes. The agglomerative hierarchical clustering approach is applied to group experts into sub-clusters based on the constructed similarity-trust degrees. The centrality concept from SNA is then used to determine the expert's similarity-trust centrality index, which is the basis for the construction of a new aggregation operator, similarity-trust centrality (STC-IOWA) operator, to fuse the individual experts' preferences into a collective one, from which the consensus solution is derived. An analysis of results with different levels of trust degree is carried out. We show that this new idea is promising and relevant to be used in solving certain consensus group decision making problems.

KEYWORDS

trust, similarity of preferences, clustering, aggregation operator, consensus group decision making

1 | INTRODUCTION

Decision-making problems involving collective individuals have raised interest among mathematicians, psychologists, sociologists, among others. In such decision-making problems, known as group decision-making (GDM) problem, the aim is to find a solution that the individuals in the group agree with or have a specified group level of consensus [1]. This problem becomes challenging when the individuals in the group have different level of knowledge, experience, and expertise.

At the beginning of a GDM process, experts' preferences or evaluations over alternatives must be collected. One of the well-known representation formats used to achieve this is the reciprocal fuzzy preference relation, which is based on pair-wise comparison of the alternatives. From this information, the closeness of experts' preferences can be assessed using similarities or distance functions [2]. Gonzalez-Arteaga et al. [3] proposed converting the preference relations into vectors to compute the correlation experts' preferences while Zhang et al. [4] introduced the comparative linguistic expression preference relations to represent uncertain opinions of experts in GDM. The similarity measure of experts' preferences has been frequently used to evaluate the consensus level in the consensus process [5]. The consensus procedure involves a group of experts, and the aggregation of their individual preferences into a group preference from which the final decision is made. Prominent functions in aggregating the preferences of the experts are the ordered weighted averaging (OWA) operator [6] and its extended version, the induced ordered weighted averaging (IOWA) operator [7], which have been extensively implemented by researchers in their work [8, 9, 10, 11] [12].

Currently, on the one hand, there is a lot of research interest on GDM problems involving an enormous number of experts, where consensus reaching process may be difficult to interpret or understand. An approach to address this problems is to apply classification methods to divide the experts into a few subgroups or clusters since this facilitates evaluating the consensus level and determining the level of contribution for each expert toward the group consensus [13, 14]. Kamis et al. [15] used the agglomerative hierarchical clustering method to partition the group of experts into a few subgroups and proposed computing experts' consensus levels within their cluster before aggregating all the clusters' consensus levels to get the group consensus level. On the other hand, studies on social network group decision-making (SNGDM) are continuously developed to help experts with social relationships to reach sufficient consensus state [16, 17, 18, 19]. Since each individual has difference background and interests, the element of trust within the group is a critical component that might affect the discussion and interaction process. Recently, some researchers ([20], [5], [21], [22], [23]) have taken into consideration the integration of the trust relation from Social Network Analysis (SNA) with group decision making paradigm. Within this framework, if experts provide trust statements toward the other experts in the network, then their interpersonal trust relationship can be examined and decision makers can be supported within GDM procedures, such as the aggregation of preferences [24] and the estimation of unknown values in preference relations [25], and during feedback processes [26].

Although a lot of concepts and notions has been introduced, such as trust statement, trust socio-matrix and trust social network in SNA, there are limited studies that focus on the trust relationship within a group of experts from a decision-making perspective. After the experts provide their preference and trust statement, it is very important for the decision-makers to thoroughly analyze the information. However, there are difficulties in visualizing the closeness of experts' preference and trust degree. Naturally, an expert's opinion will bear resemblance to the opinion of other experts that he/she trusts. An expert that is highly trusted will influence other experts' evaluation on the alternatives. This most influential expert is considered to be the most important expert in the group or network. The main aim of this study is to propose a new clustering based consensus group decision making model by considering trust relation defined in SNA. In general, these novel knowledge contributions are focused on the integration and impact of trust

relation in SNGDM framework. Specifically, we: 1) introduce a new combination function, where trust relation is integrated into the preference similarity network, 2) propose a new similarity-trust network to visualize the closeness of preferences and trust relationship between experts, 3) measure the centrality index in the similarity-trust network to identify the most important expert in the network, and 4) formulate a particular IOWA, STC-IOWA, operator to aggregate all individual experts' preferences into a collective one according to the most important and trusted expert.

This paper is organized as follows. In Section 2, the similarity, trust and similarity-trust measures are introduced. In Section 3, a new similarity-trust network is visualized and its corresponding centrality concept is presented. The integration of the similarity-trust network in clustering based consensus GDM (CGDM) is then demonstrated and the resolution phase is performed in Section 4 and 5, respectively. In Section 6, a comparative analysis of results is conducted and finally, the conclusion is presented in Section 7.

2 | DEVELOPMENT OF SIMILARITY-TRUST MEASURE

This section provides concepts, definitions, elaborations and examples related to the measurement of experts' preference similarity and trust. The introduction of our novel functions, called symmetrized trust socio-matrix and similarity-trust measure, are also presented. For the purpose of demonstrating all procedures involve in the methodology, related examples are provided in each section.

2.1 | Similarity of Experts' Preferences

Let $E = \{e^1, e^2, \dots, e^m\}$ be the group of experts, and $X = \{x_1, x_2, \dots, x_n\}$ ($n > 2$) the finite set of alternatives. Experts' preferences or opinions over the set of alternatives are represented using reciprocal preference relations, as defined below:

Definition 1 (Reciprocal Preference Relation [27]) A reciprocal preference relation on X is a fuzzy binary relation P , where $P(x_i, x_j) = p_{ij} \in [0, 1]$ represents the preference intensity of alternative x_i over alternative x_j , verifying $p_{ij} + p_{ji} = 1, \forall x_i, x_j \in X$.

According to Definition 1, an expert not only performs his/her preference on alternative x_i over x_j , but also establish the intensity of preference in terms of the value of p_{ij} . The higher p_{ij} , the more preferred is alternative x_i when compared against alternative x_j . The associated semantic for the unit interval reciprocal preference relation is as follows:

$$p_{ij} = \begin{cases} 0 & \text{if } x_j \text{ is completely preferred to } x_i \\ p_{ij} \in]0, 0.5[& \text{if } x_j \text{ is preferred to } x_i \\ 0.5 & \text{if } x_i \text{ and } x_j \text{ are equally preferred (indifference)} \\ p_{ij} \in]0.5, 1[& \text{if } x_i \text{ is preferred to } x_j \\ 1 & \text{if } x_i \text{ is completely preferred to } x_j \end{cases} \quad (1)$$

The set of reciprocal preference relations on X is symbolised by $\mathbb{P}_{n \times n}$. Notice that mathematically, a reciprocal preference relation can also be represented by means of a column vector, which is known as its *intensity preference vector* [3].

Definition 2 (Intensity Preference Vector) The intensity preference vector of a reciprocal preference relation $P = (p_{ij})_{n \times n} \in \mathbb{P}_{n \times n}$ is the vector of dimension $n(n-1)/2$, $V \in \mathbb{R}^{n(n-1)/2}$, with components the elements above its main diagonal:

$$V = (p_{12}, p_{13}, \dots, p_{1n}, p_{23}, \dots, p_{2n}, \dots, p_{(n-1)n}) = (v_1, v_2, \dots, v_r, \dots, v_{n(n-1)/2}). \tag{2}$$

The reciprocity property allows the use of the preference values below the main diagonal of P as components of its intensity preference vectors.

The closeness of experts' preferences can be measured using a similarity functions. Using intensity preference vectors, the following preference similarity function is defined:

Definition 3 (Preference Similarity Measure [15]) Let $\mathbb{V} = \{V^1, V^2, \dots, V^m\}$ be the set of intensity preference vectors associated to the preferences expressed by a set of experts, E , on a set of alternatives, X . A preference similarity measure on E is a fuzzy subset of $\mathbb{V}(X) \times \mathbb{V}(X)$ with membership function $S : \mathbb{V}(X) \times \mathbb{V}(X) \rightarrow [0, 1]$ being reflexive ($S(V^i, V^i) = 1$) and symmetric ($S(V^i, V^j) = S(V^j, V^i)$).

It is well-known that the cosine similarity function is a special type of Pearson correlation coefficient when the mean is equal to zero. Therefore, the use of the cosine similarity allows us to measure the degree of closeness of experts' preferences. In addition, the cosine similarity also appeared quite stable in evaluating consensus regardless of the number of experts [28].

Definition 4 (Cosine Preference Similarity [15]) The cosine preference similarity between of experts e^a and e^b is

$$S^{ab} = S(V^a, V^b) = \frac{\langle V^a, V^b \rangle}{\|V^a\| \|V^b\|}, \tag{3}$$

where V^a and V^b are the preference intensity vectors of expert e^a and e^b , respectively; $\langle \cdot \rangle$ denotes the inner product and $\| \cdot \|$ is the Euclidean norm. The $m \times m$ preference similarity matrix with elements the cosine preference similarity indices associated to the set of experts E is symbolized by \tilde{S} .

Example 1 To illustrate our proposed model, a numerical example is provided throughout the paper involving eight ministers (experts), $E = \{e^1, e^2, \dots, e^8\}$, giving their opinions regarding the annual budget allocations toward six necessary sectors (alternatives), $X = \{x_1, x_2, \dots, x_6\}$: education, health care, infrastructures, security, subsidies, and transportation. It is assumed the following reciprocal preference relations are provided by the ministers:

$$P^1 = \begin{bmatrix} 1 & 0.4 & 0.2 & 0.6 & 0.7 & 0.8 \\ 0.6 & 1 & 0.1 & 0.6 & 0.9 & 0.7 \\ 0.8 & 0.9 & 1 & 0.3 & 0.1 & 0.1 \\ 0.4 & 0.4 & 0.7 & 1 & 0.5 & 0.2 \\ 0.3 & 0.1 & 0.9 & 0.5 & 1 & 0.7 \\ 0.2 & 0.3 & 0.9 & 0.8 & 0.3 & 1 \end{bmatrix} \quad P^2 = \begin{bmatrix} 1 & 0.3 & 0.3 & 0.5 & 0.6 & 0.6 \\ 0.7 & 1 & 0.4 & 0.7 & 0.2 & 0.3 \\ 0.7 & 0.6 & 1 & 0.5 & 0.4 & 0.2 \\ 0.5 & 0.3 & 0.5 & 1 & 0.6 & 0.7 \\ 0.4 & 0.8 & 0.6 & 0.4 & 1 & 0.4 \\ 0.4 & 0.7 & 0.8 & 0.3 & 0.6 & 1 \end{bmatrix} \quad P^3 = \begin{bmatrix} 1 & 0.6 & 0.6 & 0.6 & 0.1 & 0.4 \\ 0.4 & 1 & 0.3 & 0.6 & 0.3 & 0.6 \\ 0.4 & 0.7 & 1 & 0.6 & 0.1 & 0.6 \\ 0.4 & 0.4 & 0.4 & 1 & 0.7 & 0.6 \\ 0.9 & 0.7 & 0.9 & 0.3 & 1 & 0.2 \\ 0.6 & 0.4 & 0.4 & 0.4 & 0.8 & 1 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 1 & 0.3 & 0.2 & 0.5 & 0.7 & 0.7 \\ 0.7 & 1 & 0.3 & 0.7 & 0.3 & 0.5 \\ 0.8 & 0.7 & 1 & 0.7 & 0.3 & 0.3 \\ 0.5 & 0.3 & 0.3 & 1 & 0.8 & 0.7 \\ 0.3 & 0.7 & 0.7 & 0.2 & 1 & 0.6 \\ 0.3 & 0.5 & 0.7 & 0.3 & 0.4 & 1 \end{bmatrix} \quad P^5 = \begin{bmatrix} 1 & 0.6 & 0.3 & 0.6 & 0.6 & 0.7 \\ 0.4 & 1 & 0.1 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.7 & 1 & 0.3 & 0.3 & 0.2 \\ 0.4 & 0.3 & 0.7 & 1 & 0.5 & 0.2 \\ 0.4 & 0.2 & 0.7 & 0.5 & 1 & 0.7 \\ 0.3 & 0.6 & 0.8 & 0.8 & 0.3 & 1 \end{bmatrix} \quad P^6 = \begin{bmatrix} 1 & 0.3 & 0.1 & 0.5 & 0.7 & 0.6 \\ 0.7 & 1 & 0.4 & 0.7 & 0.2 & 0.4 \\ 0.9 & 0.6 & 1 & 0.5 & 0.4 & 0.2 \\ 0.5 & 0.3 & 0.5 & 1 & 0.6 & 0.7 \\ 0.3 & 0.8 & 0.6 & 0.4 & 1 & 0.4 \\ 0.4 & 0.6 & 0.8 & 0.3 & 0.6 & 1 \end{bmatrix}$$

$$P^7 = \begin{bmatrix} 1 & 0.7 & 0.4 & 0.6 & 0.2 & 0.6 \\ 0.3 & 1 & 0.3 & 0.7 & 0.3 & 0.8 \\ 0.6 & 0.7 & 1 & 0.6 & 0.1 & 0.6 \\ 0.4 & 0.7 & 0.4 & 1 & 0.7 & 0.6 \\ 0.8 & 0.7 & 0.9 & 0.3 & 1 & 0.2 \\ 0.2 & 0.4 & 0.4 & 0.4 & 0.8 & 1 \end{bmatrix} \quad P^8 = \begin{bmatrix} 1 & 0.4 & 0.4 & 0.3 & 0.5 & 0.6 \\ 0.6 & 1 & 0.3 & 0.7 & 0.3 & 0.4 \\ 0.6 & 0.7 & 1 & 0.7 & 0.2 & 0.3 \\ 0.7 & 0.3 & 0.3 & 1 & 0.8 & 0.7 \\ 0.5 & 0.7 & 0.8 & 0.2 & 1 & 0.6 \\ 0.4 & 0.6 & 0.7 & 0.3 & 0.4 & 1 \end{bmatrix}$$

The preference relations are transformed into preference intensity vectors. For instance, the preference intensity vector for e^1 is formed with the elements above the main diagonal of P^1 , which are highlighted in boldface:

$$V^1 = (0.4, 0.2, 0.6, 0.7, 0.8, 0.1, 0.6, 0.9, 0.7, 0.3, 0.1, 0.1, 0.5, 0.2, 0.7).$$

Similarly, preference intensity vectors V^2, \dots, V^8 are constructed. Applying Definition 4, the preference similarity matrix on E becomes:

$$\tilde{S} = \begin{bmatrix} 1 & 0.833 & 0.762 & 0.877 & 0.973 & 0.844 & 0.810 & 0.846 \\ 0.833 & 1 & 0.873 & 0.979 & 0.871 & 0.991 & 0.886 & 0.966 \\ 0.762 & 0.873 & 1 & 0.875 & 0.805 & 0.844 & 0.984 & 0.908 \\ 0.877 & 0.979 & 0.875 & 1 & 0.890 & 0.983 & 0.900 & 0.983 \\ 0.973 & 0.871 & 0.805 & 0.890 & 1 & 0.866 & 0.835 & 0.876 \\ 0.844 & 0.991 & 0.844 & 0.983 & 0.866 & 1 & 0.876 & 0.952 \\ 0.810 & 0.886 & 0.984 & 0.900 & 0.835 & 0.876 & 1 & 0.914 \\ 0.846 & 0.966 & 0.908 & 0.983 & 0.876 & 0.952 & 0.914 & 1 \end{bmatrix}$$

2.2 | Trustworthiness Among Experts

In certain GDM process, the element of trust plays a vital role in visualising the experts relationship and should also be taken into account as a reliable information to be utilized in its resolution. The concept of trust defined by Wu et al. [29] is adapted to be used in our proposed framework.

Definition 5 (Trust Relation) A trust relation, T , on $E \times E$ is a mapping $T : E \times E \rightarrow [0, 1]$, where $T(e_i, e_j) = t_{ij}$ represents the trust degree of expert e_i towards expert e_j , that is reflexive $t_{ii} = 1 \forall i$. The associated semantics of a trust relation is:

$$t_{ij} = \begin{cases} 1 & \text{if } e_i \text{ completely trust } e_j \\ t_{ij} \in (0, 1) & \text{if } e_i \text{ proportionally trust } e_j \\ 0 & \text{if } e_i \text{ completely distrust } e_j \end{cases} \quad (4)$$

The higher t_{ij} , the higher the trustworthiness of expert e_i towards expert e_j . Experts' trust relations can be represented as a trust sociomatrix, $T = (t_{ij})_{m \times m}$.

Example 2 (Continuation of Example 1) It is assumed that the trust sociomatrix of the set of ministers is:

$$T = \begin{bmatrix} 1 & 0.64 & 0.59 & 0.80 & 0.91 & 0.67 & 0.69 & 0.71 \\ 0.78 & 1 & 0.81 & 0.83 & 0.82 & 0.73 & 0.88 & 0.84 \\ 0.64 & 0.74 & 1 & 0.83 & 0.69 & 0.70 & 0.81 & 0.85 \\ 0.77 & 0.83 & 0.72 & 1 & 0.76 & 0.85 & 0.81 & 0.91 \\ 0.98 & 0.72 & 0.66 & 0.84 & 1 & 0.73 & 0.74 & 0.78 \\ 0.81 & 0.97 & 0.76 & 0.83 & 0.82 & 1 & 0.86 & 0.97 \\ 0.68 & 0.71 & 0.90 & 0.83 & 0.69 & 0.70 & 1 & 0.81 \\ 0.76 & 0.86 & 0.81 & 0.83 & 0.78 & 0.85 & 0.88 & 1 \end{bmatrix}.$$

2.3 | A New Similarity-Trust Measure

At this stage, a new function that integrates preference similarity and trust relation is proposed. Since each of the experts established their directed trust degree towards the other experts, the trust sociomatrix formed is not necessarily symmetric. Thus, a symmetrization approach is implemented to convert the trust relation into a symmetric relation. Recall that a square matrix M can be decomposed into a symmetric matrix, $\frac{1}{2}(M + M^T)$, and a skew-symmetric matrix, $\frac{1}{2}(M - M^T)$. The symmetric part of the decomposition is called the weighted average transpose of a matrix [30].

Definition 6 (Symmetrized Trust Sociomatrix) Let $T = (t_{ij})_{m \times m}$ be the trust sociomatrix of a set of expert E . The corresponding symmetrized trust sociomatrix is:

$$\Gamma = \left(\frac{T + T^T}{2} \right) = \left(\frac{t_{ij} + t_{ji}}{2} \right) = (\tilde{t}_{ij})_{m \times m}. \quad (5)$$

Example 3 (Continuation of Example 2)) The symmetrized trust sociomatrix, Γ , of Trust sociomatrix, T , is:

$$\Gamma = \begin{bmatrix} 1 & 0.71 & 0.62 & 0.78 & 0.95 & 0.74 & 0.68 & 0.73 \\ 0.71 & 1 & 0.77 & 0.83 & 0.77 & 0.85 & 0.79 & 0.85 \\ 0.62 & 0.77 & 1 & 0.78 & 0.67 & 0.73 & 0.86 & 0.83 \\ 0.78 & 0.83 & 0.78 & 1 & 0.80 & 0.84 & 0.82 & 0.87 \\ 0.95 & 0.77 & 0.67 & 0.80 & 1 & 0.77 & 0.72 & 0.78 \\ 0.74 & 0.85 & 0.73 & 0.84 & 0.77 & 1 & 0.78 & 0.91 \\ 0.68 & 0.79 & 0.86 & 0.82 & 0.72 & 0.78 & 1 & 0.84 \\ 0.73 & 0.85 & 0.83 & 0.87 & 0.78 & 0.91 & 0.84 & 1 \end{bmatrix}.$$

Symmetrized trust sociomatrix allows the integration of closeness of preferences and trustworthiness between experts into a similarity-trust measure:

Definition 7 (Similarity-trust Measure) The linear combination of the preference similarity matrix, $\tilde{S} = (S_{ij})_{m \times m}$, and the

symmetrized trust sociomatrix, $\Gamma = (\tilde{t}_{ij})_{m \times m}$, of the set of experts E

$$ST_{ij}^{\beta} = \beta S_{ij} + (1 - \beta) \tilde{t}, \quad \beta \in [0, 1] \quad (6)$$

is reflexive and symmetric, and it is called a similarity-trust measure on E . The set $ST^{\beta} = (ST_{ij}^{\beta})$ is the similarity-trust matrix (with parameter β).

Different values of β show the weightage of similarity and trust required in the decision making process. The higher the value of β , the higher the weightage of preference similarity and the lower the weightage of trust. If $\beta = 0.5$, preference similarity and trust have the same weight in the resolution process. If $\beta = 1$, then only preference similarity index is considered, while if $\beta = 0$, then trust becomes the sole element taken into account. To simplify notation, unless absolutely necessary, the parameter β will not be written when referring to the similarity-trust matrix or its elements.

Example 4 (Continuation of Example 2)) The similarity-trust matrix $ST^{0.5}$ is

$$ST^{0.5} = \begin{bmatrix} 1 & 0.77 & 0.69 & 0.83 & 0.96 & 0.79 & 0.75 & 0.79 \\ 0.77 & 1 & 0.82 & 0.91 & 0.82 & 0.92 & 0.84 & 0.91 \\ 0.69 & 0.82 & 1 & 0.83 & 0.74 & 0.79 & 0.92 & 0.87 \\ 0.83 & 0.91 & 0.83 & 1 & 0.85 & 0.91 & 0.86 & 0.93 \\ 0.96 & 0.82 & 0.74 & 0.85 & 1 & 0.82 & 0.78 & 0.83 \\ 0.79 & 0.92 & 0.79 & 0.91 & 0.82 & 1 & 0.83 & 0.93 \\ 0.75 & 0.84 & 0.92 & 0.86 & 0.78 & 0.83 & 1 & 0.88 \\ 0.79 & 0.91 & 0.87 & 0.93 & 0.83 & 0.93 & 0.88 & 1 \end{bmatrix}.$$

3 | REPRESENTATION OF SIMILARITY-TRUST NETWORK AND CENTRALITY

This section introduces a novel similarity-trust network representation and the emergence of centrality measure.

3.1 | A New Similarity-Trust Network

Symmetry of ST means that element ST_{ij} can be used as the the weight attached to an undirected edge between vertex i and vertex j of a network. Thus, an undirected weighted similarity-trust network can be constructed that consist of a set of vertexes, $E = \{e^1, e^2, \dots, e^m\}$, a set of edges between vertexes, $L = \{l_{12}, l_{13}, \dots, l_{1m}, l_{23}, \dots, l_{2m}, \dots, l_{(m-1)m}\}$, and a set of weights attached to the set of edges, $ST = \{ST_{12}, ST_{13}, \dots, ST_{1m}, ST_{23}, \dots, ST_{2m}, \dots, ST_{(m-1)m}\}$.

Definition 8 (Similarity-trust Network) An undirected weighted similarity-trust network is a 3-tuple, $G_{ST} = \langle E, L, ST \rangle$, that represents a set of vertexes, E , connected by a set of edges, L , with a set of similarity-trust weights, ST , attached to L .

By constructing the similarity-trust network, the visualization of the expert interrelation, including their closeness of preferences and trustworthiness, is performed.

Example 5 (Continuation of Example 4) Based on the similarity-trust matrix $ST^{0.5}$, the similarity-trust network of the set of ministers applying the Multi-dimensional Scaling (MDS) technique is presented in Figure 1.

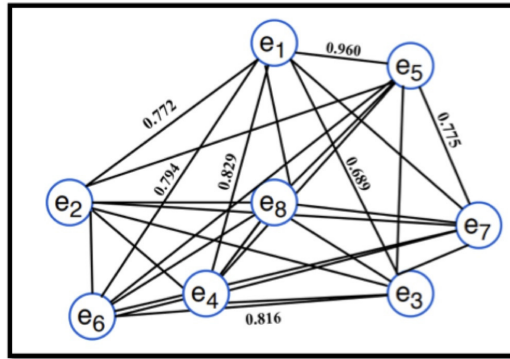


FIGURE 1 The undirected weighted similarity-trust network for $\beta = 0.5$.

3.2 | Centrality Measure

In SNA, the centrality concept has been widely utilized by researchers to determine the most important individual in a network. Through the visualization of similarity-trust network in the previous section (Section 3.1), the most important expert can be observed. The most important expert is the one having the highest centrality index from the network, which reflects that he/she has the most similar with and is most trusted by other experts.

Definition 9 (Similarity-trust Centrality) *The similarity-trust centrality index of vertex i , $A(e_i)$, of the undirected weighted similarity-trust network G_{ST} is*

$$A(e_i) = \frac{1}{m-1} \sum_{j \neq i} ST_{ij}. \quad (7)$$

Example 6 (Continuation of Example 5) *The ministers' similarity-trust centrality indices at $\beta = 0.5$ are*

$$A(e^1) = 0.797; A(e^2) = 0.856; A(e^3) = 0.808; A(e^4) = 0.872; A(e^5) = 0.827; A(e^6) = 0.857; A(e^7) = 0.835; A(e^8) = 0.876.$$

The ranking of importance of ministers is $e^8 > e^4 > e^6 > e^2 > e^7 > e^5 > e^3 > e^1$. From this ranking order, the minister e^8 is the most influential and trusted person in making the decision making process.

4 | INTEGRATION OF SIMILARITY-TRUST IN A CLUSTERING BASED CGDM FRAMEWORK

Determining the consensus level among a group of experts is one of the main procedures in a CGDM. The consensus level expresses the agreement status between individuals involved in the decision making process. In order to systematically evaluate the consensus level when a large group of experts is involved, its division into a few subgroups or clusters makes the process more efficient [13]. In this context, several feasible clustering methods have been developed, including k-mean, spectral, agglomerative hierarchical clustering, divisive hierarchical and grid-based algorithms, to mention a few. The agglomerative hierarchical clustering procedure is quite straightforward and produce explicit illustration [31]. Its consecutive steps are based on similarity-trust are:

The Consecutive Steps of the Agglomerative Hierarchical Clustering Procedure Based on Similarity-Trust

Step 1: Start the clustering with partition $K^m = \{C_1, C_2, \dots, C_m\}$, where each cluster C_i has exactly one element e_i :

$$K^m = \{\{e_1\}, \{e_2\}, \dots, \{e_m\}\} = \{\{C_1\}, \{C_2\}, \dots, \{C_m\}\}$$

Step 2: Identify clusters C_i and C_j in K^m with maximum similarity-trust value:

$$(C_i, C_j) = \arg \max_{C_i, C_j \in K^m} ST(C_i, C_j).$$

This

Step 3: Merge clusters C_i and C_j to get cluster C_r : $C_r = C_i \cup C_j$. Build a new partition K^{m-1} by adding C_r and removing C_i and C_j .

Step 4: Update the similarity-trust values between C_r and C_k :

$$ST(C_r, C_k) = \begin{cases} \min_{C_k \in K^{l-1}} \{ST(C_i, C_k), ST(C_j, C_k)\} & \text{if } k \neq r; \\ 1 & \text{otherwise.} \end{cases}$$

Step 5: Repeat Steps 2-4 until K^1 is obtained.

clustering algorithm results in m different possible partitions of experts. In this work, we exclude the clustering solution K^1 and K^m : in partition K^1 all experts are placed in the same cluster, while in partition K^m each of the expert belongs to its own cluster. The best partition will be the one that maximised the consensus level, which is elaborated next.

Partition K^2 *dots*, K^{m-1} can be represented as a set of α -levels, $L = \{\alpha_l ; l = 2, \dots, m-1\}$. The set of clusters at the α_l -level can be expressed as $C_l = \{C_{lk} ; k = 1, \dots, l\}$. Let $\#C_{lk}$ denotes the cardinality of cluster C_{lk} . In order to determine the optimal consensus level of the group, five (5) types of indices are needed [32]:

(i) Cluster internal cohesion index:

$$\delta_{int}(C_{lk}) = \frac{\sum_{i \in C_{lk}} \sum_{j \in C_{lk}} ST_{ij}}{(\#C_{lk})^2} \quad (8)$$

(ii) Cluster external cohesion index:

$$\delta_{ext}(C_{lk}) = \frac{\sum_{i \in C_{lk}} \sum_{j \notin C_{lk}} ST_{ij}}{\#C_{lk} (m - \#C_{lk})} \quad (9)$$

(iii) Cluster-consensus index:

$$\delta_{CC}(C_{Ik}) = \frac{\#C_{Ik} \cdot \delta_{int}(C_{Ik})}{m} + \frac{(m - \#C_{Ik}) \cdot \delta_{ext}(C_{Ik})}{m} \quad (10)$$

(iv) Group consensus index:

$$\delta_{LC}(I) = \frac{\#C_{Ik} (\delta_{int}(C_{Ik}) - \delta_{ext}(C_{Ik}))}{m} + \delta_{ext}(C_{Ik}) \quad (11)$$

(v) Optimal group consensus index:

$$\delta_G(I) = \max_{\delta_G \in LC} \delta_{LC}(I). \quad (12)$$

Example 7 (Continuation of Example 6) At each α -level and cluster level, C , the internal cohesion index, δ_{int} , the external cohesion index, δ_{ext} , the cluster consensus index, δ_{CC} , the group consensus index, δ_{CL} and the optimal group consensus index, δ_G are computed and presented in Table 1. This table considers the similarity-trust input at $\beta = 0.5$. The optimal ministers group consensus, shown in boldface in Table 1, is achieved at the α -level 5.

Notice that all clusters at all levels, δ_{int} is always greater than δ_{ext} , which is explained by the agglomerative clustering technique classifying experts in the same cluster using maximum of similarity-trust values. Assuming that the optimal group consensus is above the group consensus threshold, then the selection of the best alternative is activated, which is described in the next section. Otherwise, a feedback process would be activated, which is not considered in the present study.

5 | RESOLUTION PHASE

When the experts' similarity preferences and trust relations have been analyzed and the optimal group consensus level has been determined, the alternatives are ranked from best to worst from which the final solution of consensus is obtained. The final ranking of alternatives is derived from the collective preferences obtained aggregating the experts' individual preferences.

5.1 | An Extended IOWA-based Aggregation Operator

As mentioned before, the IOWA is an operator useful in GDM since it allows to implement expert's importance in the aggregation process:

Definition 10 (IOWA operator) An IOWA operator of dimension n is a function $\Phi_W: (\mathbb{R} \times \mathbb{R})^n \rightarrow \mathbb{R}$, with associated weighting vector $W = (\omega_1, \dots, \omega_n)$ such that $\omega_i \in [0, 1]$ and $\sum_i \omega_i = 1$, a list of n 2-tuples $\{\langle u_1, p_1 \rangle, \dots, \langle u_n, p_n \rangle\}$ to the

TABLE 1 The cluster internal and external cohesions, cluster consensus and group consensus indexes for $\beta = 0.5$

| α -level | Cluster level | Experts | Internal Cohesion | External Cohesion | Cluster Consensus | Group Consensus |
|-----------------|---------------|---------------------------|-------------------|-------------------|-------------------|-----------------|
| 2 | 1 | e_2, e_4, e_6, e_8, e_3 | 0.8965 | 0.7875 | 0.8693 | 0.8524 |
| | 2 | e_1, e_5 | 0.9798 | 0.7875 | 0.8356 | |
| 3 | 1 | e_2, e_4, e_6, e_8 | 0.9381 | 0.8259 | 0.8820 | 0.8538 |
| | 2 | e_1, e_5 | 0.9798 | 0.7875 | 0.8356 | |
| | 3 | e_3, e_7 | 0.9599 | 0.8052 | 0.8439 | |
| 4 | 1 | e_4, e_6, e_8 | 0.9488 | 0.8462 | 0.8847 | 0.8595 |
| | 2 | e_1, e_5 | 0.9798 | 0.7875 | 0.8356 | |
| | 3 | e_3, e_7 | 0.9599 | 0.8052 | 0.8439 | |
| | 4 | e_2 | 1 | 0.8560 | 0.8740 | |
| 5 | 1 | e_1, e_5 | 0.9798 | 0.7875 | 0.8356 | 0.8649 |
| | 2 | e_6, e_8 | 0.9663 | 0.8554 | 0.8831 | |
| | 3 | e_3, e_7 | 0.9599 | 0.8052 | 0.8439 | |
| | 4 | e_2 | 1 | 0.8560 | 0.8740 | |
| | 5 | e_4 | 1 | 0.8717 | 0.8878 | |
| 6 | 1 | e_1, e_5 | 0.9798 | 0.7875 | 0.8356 | 0.8614 |
| | 2 | e_6, e_8 | 0.9663 | 0.8554 | 0.8831 | |
| | 3 | e_2 | 1 | 0.8560 | 0.8740 | |
| | 4 | e_3 | 1 | 0.8078 | 0.8318 | |
| | 5 | e_4 | 1 | 0.8717 | 0.8878 | |
| | 6 | e_7 | 1 | 0.8353 | 0.8559 | |
| 7 | 1 | e_1, e_5 | 0.9798 | 0.7875 | 0.8356 | 0.8645 |
| | 2 | e_2 | 1 | 0.8560 | 0.8740 | |
| | 3 | e_3 | 1 | 0.8078 | 0.8318 | |
| | 4 | e_4 | 1 | 0.8717 | 0.8878 | |
| | 5 | e_6 | 1 | 0.8568 | 0.8747 | |
| | 6 | e_7 | 1 | 0.8353 | 0.8559 | |
| | 7 | e_8 | 1 | 0.8761 | 0.8916 | |

value

$$\Phi_W \{ \langle u_1, p_1 \rangle, \dots, \langle u_n, p_n \rangle \} = \sum_{i=1}^n \omega_i \cdot p_{\sigma(i)} \tag{13}$$

being $\sigma: \{1, \dots, n\} \rightarrow \{1, \dots, n\}$ the permutation verifying $u_{\sigma(i)} \geq u_{\sigma(i+1)}, \forall i = 1, \dots, n - 1$.

Thus, an IOWA reorder the values to aggregate, $\{p_1, \dots, p_n\}$, by using the corresponding ordering of an auxiliary set of associated numbers, $\{u_1, \dots, u_n\}$, known as order inducing values. Using the network G_{ST} similarity-trust centrality index values, $\{A(e^1), A(e^2), \dots, A(e^m)\}$, as the set of order inducing values of the experts' preference relations,

the similarity-trust centrality IOWA (STC-IOWA) operator, Φ_w^{STC} , and similarity-trust centrality collective preference relation are obtained.

Definition 11 (STC-IOWA operator) The STC-IOWA operator of dimension m , Φ_w^{STC} , is the IOWA operator with the set of similarity-trust centrality indexes of the edges of a network $\{A(e^1), A(e^2), \dots, A(e^m)\}$ as its set of order inducing values.

Definition 12 (Similarity-trust centrality collective Preference Relation) The similarity-trust centrality collective experts' preference relation is the aggregation result of the application of the STC-IOWA operator of dimension m , Φ_w^{STC} , to the individual preferences of the experts:

$$p_{ij}^c = \Phi_w^{ST} \left(\langle A(e^1), p_{ij}^1 \rangle, \langle A(e^2), p_{ij}^2 \rangle, \dots, \langle A(e^m), p_{ij}^m \rangle \right) \quad (14)$$

The question of which weighting vector to use in the application of an OWA/IOWA operator was resolved by Yager [33] with the implementation of the soft majority concepts represented by a regular increasing monotone quantifier, in particular the use of quantifier $Q = r^{1/2}$ to represent the fuzzy linguistic quantifier 'most of' is implemented in the following examples, which means that the collective of preferences represent 'the preference of an alternative over another for most of the important expert in the similarity-trust network'. When the linguistic quantifier Q is used to derive the weighting vector of an OWA operator, this is denoted Φ_Q .

Example 8 (Continuation of Example 7) The similarity-trust centrality indices for all ministers in descending order are:

$$A(e^8) = 0.876, A(e^4) = 0.872, A(e^6) = 0.857, A(e^2) = 0.856, A(e^7) = 0.835, A(e^5) = 0.827, A(e^3) = 0.808, A(e^1) = 0.797.$$

The corresponding weighting vector is:

$$W = (0.361, 0.149, 0.112, 0.095, 0.082, 0.074, 0.066, 0.061).$$

Thus, the similarity-trust centrality collective preference relation for all ministers over the 6 sectors is:

$$P^C = \begin{bmatrix} 1 & 0.417 & 0.321 & 0.456 & 0.530 & 0.621 \\ 0.583 & 1 & 0.294 & 0.687 & 0.353 & 0.470 \\ 0.679 & 0.706 & 1 & 0.590 & 0.243 & 0.304 \\ 0.544 & 0.313 & 0.410 & 1 & 0.703 & 0.618 \\ 0.470 & 0.647 & 0.757 & 0.297 & 1 & 0.513 \\ 0.379 & 0.530 & 0.696 & 0.382 & 0.487 & 1 \end{bmatrix}.$$

5.2 | Ranking of Alternatives

The final ranking of alternatives is obtained from the similarity-trust centrality collective preference relation by implementing an appropriate score function. We utilize the Quantifier Guided Dominance Degree (QGDD) based on the OWA operator guided by a linguistic quantifier Q representing the fuzzy majority concept desired to implement at this stage:

Definition 13 (Quantifier Guided Dominance Degree [34]) Consider a collective preference relation $P^c = (p_{ij}^c)$ on a set of alternatives $X = \{x_1, x_2, \dots, x_n\}$ and Q a linguistic quantifier representing a fuzzy majority concept. The quantifier

guided dominance degree of alternative x_i , $QGDD(x_i)$, quantifies the degree of dominance that such alternative over the fuzzy majority of the rest of alternatives by the OWA operator Φ_Q :

$$QGDD(x_i) = \Phi_Q(p_{ij}^c, j = 1, \dots, n, j \neq i) \tag{15}$$

The solution to the similarity-trust network clustering based CGDM model is the set of alternatives with maximum dominance over the fuzzy majority of alternatives:

$$X^{QGDD} = \left\{ x \mid x \in X, QGDD(x) = \sup_{x \in X} QGDD(x) \right\}, \tag{16}$$

Notice that a different linguistic quantifier (and therefore fuzzy major concept) to the one used to obtain the collective preferences can be applied to compute the quantifier guided dominance degree.

Example 9 (Continuation of Example 8) *The sector of consensus with maximum dominance over 'most of' sectors for 'most of' of the ministers based on their similarity-trust network is obtained as follows:*

- The OWA weights are computed: $W_Q = (0.4472, 0.1852, 0.1421, 0.1198, 0.1056)$.
- The QGDD are obtained: $QGDD = (0.5247, 0.5554, 0.5876, 0.5885, 0.6190, 0.5645)$.
- The final ranking of sectors is: $x_5 > x_4 > x_3 > x_6 > x_2 > x_1$.
- The final solution of consensus is: $x_5 = \text{health care}$.

6 | ANALYSIS OF RESULTS

To analyse the impact of the trust integrated in the CGDM framework, the ranking of ministers (based on their STC values), the optimal cluster, the optimal group consensus index and ranking of sectors for 5 different values of the β -parameter are provided in Table 2.

TABLE 2 Ministers' rankings, the optimal cluster, the optimal consensus index and ranking of sectors for 5 different levels of β .

| β parameter | Ranking of Ministers | Optimal Cluster | Optimal Consensus | Ranking of Sectors |
|-------------------|---|-----------------|-------------------|-------------------------------------|
| 0 | $e_8 > e_4 > e_6 > e_2 > e_7 > e_5 > e_3 > e_1$ | 7 | 0.8182 | $x_5 > x_4 > x_2 > x_6 > x_3 > x_1$ |
| 0.25 | $e_8 > e_4 > e_6 > e_2 > e_7 > e_5 > e_3 > e_1$ | 5 | 0.8415 | $x_5 > x_4 > x_3 > x_6 > x_2 > x_1$ |
| 0.5 | $e_8 > e_4 > e_6 > e_2 > e_7 > e_5 > e_3 > e_1$ | 5 | 0.8649 | $x_5 > x_4 > x_3 > x_6 > x_2 > x_1$ |
| 0.75 | $e_4 > e_8 > e_2 > e_6 > e_7 > e_5 > e_3 > e_1$ | 7 | 0.8876 | $x_5 > x_3 > x_4 > x_2 > x_6 > x_1$ |
| 1 | $e_4 > e_8 > e_2 > e_6 > e_7 > e_5 > e_3 > e_1$ | 4 | 0.9068 | $x_5 > x_3 > x_4 > x_2 > x_6 > x_1$ |

To obtain variations in the ranking of ministers using different values of parameter β is not a surprising result. Ministers e^8 and e^4 occupy the first-second places always, so they are the most trust-similarity influential ministers. It is noticed that as the β -parameter value increases, the most important minister shifted from minister e^8 ($\beta=0,0.25$ and 0.50) to minister e^4 ($\beta=0.75$ and 1). Thus, minister e^8 is most influential in terms of trust while minister e^4 is most influential in terms of preferences/opinions. Different rankings of ministers resulted in different weighting vectors in the computation of the collective preferences, which eventually impacted in both the final ranking of sectors but not

on the final solution to the problem (x_5) in this particular example, which cannot be guaranteed to be always the case.

Different values of parameter β affects directly to the weights associated with the link between ministers in the constructed similarity-trust network. Consequently, there is a direct influence in the identified optimal cluster: for $\beta=0, 0.25, 0.50, 0.75$ and 1 the optimal cluster were Cluster 7, Cluster 5, Cluster 5, Cluster 7 and Cluster 4, respectively. Regarding the optimal group consensus index, it is noticed that this increases when the parameter β increases its value. Figure 2 suggests a positive linear correlation in this case between the optimal group consensus index and the parameter β , which can be interpreted as the group consensus index being affected negatively by the lack of trust between experts.

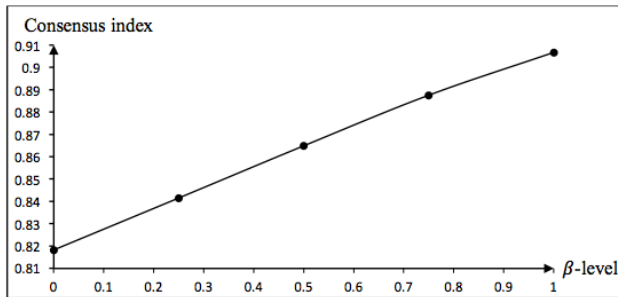


FIGURE 2 The optimal group consensus index with respect to β -levels.

7 | CONCLUSIONS

In recent years, the element of trust between experts has become an important factor to be considered in GDM context. In this paper, a new decision model that integrates the similarity of experts' preferences and trust relations between experts is introduced. The undirected similarity-trust network is constructed and the measure of similarity-trust centrality is proposed. We identify the expert with the highest centrality degree, who is the most important expert and trusted by the others.

The agglomerative hierarchical clustering algorithm is adapted using the information of experts' preference similarities and trustworthiness. From the clustering solutions, the optimal group consensus index, which represents the level of agreement of a group is obtained. The similarity-trust centrality is fused with the IOWA operator to introduce a particular IOWA, STC-IOWA operator, to aggregate all individual experts' preferences into a collective one. After the aggregation process is carried out, the final ranking of the alternatives is determined and represents the agreed solution for all experts. An analysis of results with different integration approaches illustrates the effect of implementing the element of trust in the experts' centrality, consensus level and ranking of the alternatives. These findings indicate that the inclusion of the trust relation in the preference similarity network is significant on the consensus level and the final ranking of alternatives.

In the future, the feedback mechanism for the proposed similarity-trust network CGDM framework is to be investigated to address those cases where the optimal group consensus index is unacceptable.

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