Dynamic State Estimation of Power Systems with Quantization Effects: A Recursive Filter Approach

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Abstract— **In this paper, a recursive filter algorithm is developed to deal with the state estimation problem for power systems with quantized nonlinear measurements. The measurements from both the remote terminal units (RTUs) and the phasor measurement unit (PMUs) are subject to quantizations described by a logarithmic quantizer. Attention is focused on the design of a recursive filter such that, in the simultaneous presence of nonlinear measurements and quantization effects, an upper bound for the estimation error covariance is guaranteed and subsequently minimized. Instead of using the traditional approximation methods in nonlinear estimation that simply ignore the linearization errors, we treat both the linearization and quantization errors as norm-bounded uncertainties in the algorithm development so as to improve the performance of the estimator. For the power system with such kind of introduced uncertainties, a filter is designed in the framework of robust recursive estimation, and the developed filter algorithm is tested on the IEEE benchmark power system to demonstrate its effectiveness.**

Keywords— **Power systems; state estimation; quantized estimation; nonlinear systems; recursive filter**

I. Introduction

The power system state estimation (PSSE) serves as a key stage in monitoring and controlling power systems. In fact, the optimal state estimate is a prerequisite for energy management systems (EMS) to perform other important control and planning tasks including bad data detection and identification as well as power flow optimization. Traditionally, the PSSE is accomplished by static weighted least square estimators where a single-scan measurement is used to estimate the state of the system [1, 28]. The static method has been widely applied in control centers around the world due primarily to its merits of fast convergence and easy implementation. Nevertheless, the static method is incapable of predicting the future state of the power system as the system dynamics is ignored.

In response to the recent development of advanced measurement technologies such as phasor measurement units (PMUs), the dynamic state estimators (DSEs) have been attracting a rapidly growing research interest whose main idea is to provide a recursive update of the state estimate that can also track the changes occurring during normal system operation [21]. More specifically, given the measurements, DSEs yield not only the state estimate of the current time instant but also the state prediction of the next time instant [7]. Note that, sometimes, the measurement data of power grids may be imperfect or even unavailable due to various reasons, e.g., temporary malfunction of the communication system, meter outage/maintenance, bad data and cyber attacks. In these cases, a database with state prediction values is expected to provide a set of pseudo-states of the power system, which makes the EMS more robust and resilient against the external disturbances.

Advanced information and communication technologies such as wireless communication network and PMUs have been recently applied in power systems and become an integrated part of the so-called smart grids. These advanced technologies make the online monitoring popular and therefore lead to renewed research interests on the design of DSEs. Various kinds of DSEs have been proposed based on the estimation theory, see e.g. [6, 7, 18, 32, 38] and the references therein. Generally speaking, the Kalman filter-type DSEs can be applied when only PMU measurements are utilized [5]. The nonlinear filter-type DSEs, which are typically based on the extended Kalman filter (EKF) [7] and uncented Kalman filter [32], are suitable to the cases where the remote terminal unit (RTU) measurements are available. The adoption of different kinds of filters is largely dependent on the characteristics of the measurement models, for example, the *linearity* of the PMU measurement model and the *nonlinearity* of the RTU one. In [12, 20], the DSE research has been initialized for power systems in the time scale of low frequency electromechanical dynamics and the proposed algorithms have recently been applied on real-world data [15].

Quantization phenomenon is ubiquitous in power systems. Considering the measurements in power systems, the readings provided by digital meters (e.g, PMU and RTU) are practically the quantized values converted by the analog-to-digital converter (ADC) from the continuous original measurement signals. Quantization by the ADC adds errors to the measurement values. Due to its effects on power system monitoring, the quantization error has attracted a great deal of research attention. In [26], the effects of ADC-induced quantization error on the recovery of harmonic amplitudes and phases have been examined by theoretical investigation and simulation validation in order to determine the error limits of the instrumentation system design for power system monitoring and harmonic power-flow measurement. In [10], it has been revealed that, despite its impressive dynamic range, the 12-bit state-

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of-the-art digital recorder is insufficient to achieve the required precision of the measured loss in high voltage thyristor valves.

To mitigate the effects on quantization errors, a seemly natural way is to evaluate and improve the measurement reliability by re-calibrating the measurements. In [33], the reliability issue of PMU has been investigated by taking into account the data uncertainty representing the quantization error. In order to enhance the overall accuracy of measurements in a power transmission system, several mathematical techniques have been utilized in [36] within an integrated calibration process. Furthermore, on the software side, new algorithms for power system monitoring and control have been developed in consideration of the effects of quantization errors. In [14], the authors have proposed a fault location algorithm and tested its performance against the quantization error introduced by ADCs. In [24], the inherent limitation of quantization errors incurred by low-precision sensors on the accuracy of estimated fault locations has been further investigated. In [2, 11], the quantization errors have been assumed to be a range uncertainty with uniform distribution and new least-square state estimation algorithms have been developed. However, such an assumption is quite coarse as no prior knowledge on quantization process is utilized.

On the other hand, todays' power system together with the tightly integrated hi-tech devices constitutes a complex networked cyber-physical system, for which some practical issues are emerging that have rarely been considered before for the traditional power systems. One of these issues is to do with the transmission of massive measurement data over the communication network with limited capacity. For example, PMUs update the measurements with high frequency, and this puts enormous strain on the communication and data processing infrastructure of the grid. It has been recently reported in [3] that, a single PMU can take up almost 10% of the bandwidth of the substation. Due to the limited bandwidth of the communication networks, the measurement signals in the networked environment are typically quantized before being transmitted to the substation, and such a network-induced quantization phenomenon (in addition to the aforementioned device-induced one) should be properly taken in account when designing DSEs.

It is worth pointing out that, in the communities of system control and signal processing, a series of theoretical results have been obtained on quantized control and estimation [4,8,13,16,17,19,23,30,34]. In [13], it has been proved that the logarithmic quantizer performs better than the linear quantizer in the quantized control problem. The sector bound approach, which was first introduced in [17], has been extensively employed to solve the quantized control problem. Parallel to the quantized control problem, the quantized estimation problem has been widely investigated as well. In [16], the state estimation problem has been investigated for linear discrete-time systems with quantized measurements. In [19], a recursive filter has been designed for the systems with nonlinear dynamics subject to multiplicative noise, missing measurements and quantization effects such that the estimation error covariance has an upper bound. However, almost all published results on quantized estimation have been concerned with systems with the *linear* measurement model only. In reality, lots of practical systems have *nonlinear* measurements. Taking the power system as an example, the RTU measurements are strongly nonlinear with respect to the state variables, see Section II of this paper for more details. As such, there is a gap between the theoretical results and the practical application of DSE design problems in power systems due to the quantization issue, and our aim of this paper is to shorten this gap by initiating a study on this challenging issue.

Motivated by the above discussion, we aim to design a recursive filter algorithm for power systems with quantized nonlinear measurement. First, the quantized nonlinear measurement model of power systems is presented where the quantization is assumed to be of logarithmic type. In the filter design, the composite errors caused by linearization of nonlinear measurements and quantization effects are taken into consideration and represented by several norm-bounded uncertainty matrices. Subsequently, a recursive filter algorithm is designed for the system with the introduced uncertainties such that an upper bound of the estimation error is guaranteed and then minimized by appropriately designing the filter gains. *The main contribution of this paper is threefold: (1) An explicit model for power system with quantized nonlinear measurement is proposed that is closer to the engineering practice; (2) a recursive estimation algorithm is developed for the system with consideration of both the nonlinear measurements and quantization effects; (3) and the developed recursive algorithm is computational efficient and suitable for online application in power systems.*

The reminder of this paper is organized as follows. In Section II, the dynamic model of power systems with quantized measurements is briefly introduced, and the structure of the proposed filter is presented. In Section III, the gain matrices of the recursive filter are derived, which minimize the upper bound of the covariance matrix of the estimation errors, and the upper bound at each time instant is given explicitly. In Section IV, the results of case studies performed on the 14-bus IEEE benchmark system are presented and analyzed. Finally, the paper is concluded in Section V.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. I denotes the identity matrix of compatible dimension, and $I_{m,n}$ denotes the $m\times n$-dimensional matrix with all elements equal to 1. I_m (0_m) denotes the $m \times m$-dimensional identity (zeros) matrix. A^T represents the transpose of A, and $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. $\text{diag}\{\cdots\}$ stands for a block-diagonal matrix and the notation $\text{diag}_n\{*\}$ is employed to stand for $\text{diag}\{\overbrace{*,\cdots,*}^{n}\}$. SVD$(\cdot)$ stands for the singular values (of a matrix).

II. Problem Formulation and Preliminaries

A. Dynamic System Model

In this paper, the following dynamic equation is used to model the power system containing N buses:

$$x(k+1) - u = A(x(k) - u) + \omega(k) \tag{1}$$

where the state $x(k) \in \mathbb{R}^n$ is the vector of phasor voltages of all buses, and $u \in \mathbb{R}^n$ is the nominal centre of the normal state. $\omega(k)$ is a Gaussian sequence with zero mean and covariance matrix $W(k)$. A is a known matrix with appropriate dimensions which represents how fast the transitions between states are. The initial value of state $x(0)$ is a white Gaussian noise with mean value $\bar{x}(0)$ and covariance matrix $\Sigma(0|0)$. For a power system with N buses, the state $x(k)$ can be chosen as $x(k) = \begin{bmatrix} x_{r,1}(k) & x_{r,2}(k) & \cdots & x_{r,N}(k) & x_{i,1}(k) & x_{i,2}(k) & \cdots & x_{i,N}(k) \end{bmatrix}^T$ where $x_{r,l}(k)$ and $x_{i,l}(k)$ represent the real and imaginary voltage of the lth bus, respectively.

For the purpose of simplicity, (1) can be rewritten in the following compact form:

$$x(k+1) = Ax(k) + Bu + \omega(k) \tag{2}$$

where $B \triangleq I - A$ is associated with the trend behavior of the state trajectory.

The measurement $z^{(r)}(k) \in \mathbb{R}^{m_1}$ collected by RTUs is given as follows

$$z^{(r)}(k) = [V^T(k) \;\; P^T(k) \;\; Q^T(k) \;\; P_f^T(k) \;\; Q_f^T(k)]^T$$

where $V(k) = [V_1(k) \; V_2(k) \; \cdots \; V_{n_v}(k)]^T$ denotes the bus voltage magnitude measurements, $P(k) = [P_1(k) \; P_2(k) \; \cdots \; P_{n_p}(k)]^T$ and $Q(k) = [Q_1(k) \; Q_2(k) \; \cdots \; Q_{n_p}(k)]^T$ stand for the real and reactive bus power injections measurements, and $P_f(k) = [P_{f1}(k) \; P_{f2}(k) \; \cdots \; P_{fn_l}(k)]^T$ and $Q_f(k) = [Q_{f1}(k) \; Q_{f2}(k) \; \cdots \; Q_{fn_l}(k)]^T$ are the real and reactive transmission line power flows, respectively. n_v, n_p and n_l are equal to the number of voltage meters, power meters installed at the buses and power flow meters installed at the lines, respectively. Assuming the general two-port π-model for the network branches, the explicit element for each aforementioned measurement is given as follows (the symbol of time instant, k, is omitted for brevity):

$$\begin{aligned}
V_s &= \sqrt{x_{r,s}^2 + x_{i,s}^2} \\
P_s &= x_{r,s} \sum_{j \in \mathbb{N}_s} (G_{sj} x_{r,j} - B_{sj} x_{i,j}) \\
&\quad + x_{i,s} \sum_{j \in \mathbb{N}_s} (G_{sj} x_{i,j} + B_{sj} x_{r,j}) \\
Q_s &= x_{i,s} \sum_{j \in \mathbb{N}_s} (G_{sj} x_{r,j} - B_{sj} x_{i,j}) \\
&\quad - x_{r,s} \sum_{j \in \mathbb{N}_s} (G_{sj} x_{i,j} + B_{sj} x_{r,j}) \\
P_{fs} &:= P_{ftj} = (x_{r,t}^2 + x_{i,t}^2)(g_{tj}^0 + g_{tj}) - x_{r,t} x_{r,j} g_{tj} \\
&\quad - x_{i,t} x_{i,j} g_{tj} - x_{i,t} x_{r,j} b_{tj} + x_{r,t} x_{i,j} b_{tj} \\
Q_{fs} &:= Q_{ftj} = -(x_{r,t}^2 + x_{i,t}^2)(b_{tj}^0 + b_{tj}) - x_{i,t} x_{r,j} g_{tj} \\
&\quad + x_{r,t} x_{i,j} g_{tj} + x_{r,t} x_{r,j} b_{tj} + x_{i,t} x_{i,j} b_{tj}
\end{aligned} \tag{3}$$

where $G_{sj} + jB_{sj}$ is the sjth element of the complex bus admittance matrix, $g_{tj} + jb_{tj}$ is the admittance of the series branch connecting bus t and j, $g_{tj}^0 + jb_{tj}^0$ is the half admittance of the shunt branch of the line collecting bus t and j in the π-model circuit, and $\mathbb{N}_s$ is the set of bus numbers which are directly connected to bus s. Generally, there are m measurements and n state variables with $m > n$.

Taking the measurement noise into consideration, RTU measurements can be rewritten in the following compact form:

$$z^{(r)}(k) = h(x(k)) + v_1(k) \tag{4}$$

where the nonlinear function $h(x(k))$ is determined by (3), $v_1(k)$ is the RTU measurement noise which is also a Gaussian noise with zero mean and covariance matrix $R_1(k)$. Assume that $\omega(k)$ and $v_1(k)$ are uncorrelated with $x(0)$ and with each other.

Recently, PMUs have been increasingly deployed in power systems as PMUs are able to provide more accurate and timely measurements than RTUs. A PMU measures not only the voltage phasor of the bus where it is installed but also the current flows of the lines connecting to the bus. In theory, the PMU measurements are inherently in the rectangular form [29], and it is therefore suggested that PMUs should provide the data in both angular and rectangular form in the IEEE standard c37.118-2005 [27]. In this paper, both the state variables and measured variables are in the rectangular form, which makes a linear PMU measurement model. Assume that the buses installed PMUs are labeled by $s_1, s_2, \ldots, s_M$. The measurement $z_{s_l}^{(p)} \in \mathbb{R}^{2(1+N_l)}$ obtained from the lth PMU deployed at the bus s_l can be described as follows

$$z_{s_l}^{(p)} = \begin{bmatrix} z_{r,s_l}^{(p)} & z_{i,s_l}^{(p)} & z_{r,t_1^{s_l}}^{(p)} & z_{i,t_1^{s_l}}^{(p)} & \cdots & z_{r,t_{N_l}^{s_l}}^{(p)} & z_{i,t_{N_l}^{s_l}}^{(p)} \end{bmatrix}^T.$$

To be more specific, the voltage measurement in the above vector is given as follows

$$z_{r,s_l}^{(p)} = x_{r,s_l}, \quad z_{i,s_l}^{(p)} = x_{i,s_l} \tag{5}$$

where $z_{r,s_l}^{(p)}$ and $z_{i,s_l}^{(p)}$ are the real and imaginary voltage measurements of bus s_l, respectively.

The current measurement of the line collecting the bus s_l and $t_i^{s_l}$ is as follows

$$\begin{cases}
z_{r,t_i^{s_l}}^{(p)} = x_{r,s_l} g_{s_l,t_i^{s_l}}^0 - x_{i,s_l} b_{s_l,t_i^{s_l}}^0 \\
\qquad + (x_{r,s_l} - x_{r,t_i^{s_l}}) g_{s_l,t_i^{s_l}} - (x_{i,s_l} - x_{i,t_i^{s_l}}) b_{s_l,t_i^{s_l}} \\
z_{i,t_i^{s_l}}^{(p)} = x_{i,s_l} g_{s_l,t_i^{s_l}}^0 + x_{r,s_l} b_{s_l,t_i^{s_l}}^0 \\
\qquad + (x_{i,s_l} - x_{i,t_i^{s_l}}) g_{s_l,t_i^{s_l}} + (x_{r,s_l} - x_{r,t_i^{s_l}}) b_{s_l,t_i^{s_l}}
\end{cases} \tag{6}$$

where $z_{r,t_i^{s_l}}^{(p)}$ and $z_{i,t_i^{s_l}}^{(p)}$ are the real and imaginary current measurements, respectively.

Taking the measurement noise into consideration, PMU measurements can be represented in the following compact vector form:

$$z^{(p)}(k) = H^{(p)} x^p(k) + v_2(k) \tag{7}$$

where $z^{(p)} = [z^{(p)}_{s_1}, \ldots, z^{(p)}_{s_M}] \in \mathbb{R}^{m_2}$ with $m_2 = 2(M + N_1 + N_2 + \ldots + N_l)$. $x^{(p)}(k) \in \mathbb{R}^{n_1}$ is the state vector related to the PMU measurements. $x^{(p)}(k)$ is a subset of the state of the whole power systems, and $x^{(p)}(k) = Tx(k)$, where $T \in \mathbb{R}^{n_1 \times n}$, $n_1 \leq n$. matrix T is determined by both the configuration of PMUs and the power network's topologies. $v_2(k)$ is the PMU measurement noise, which is also a Gaussian noise with zero mean and covariance matrix $R_2(k)$. $H^{(p)}$ can be obtained directly from (5) and (6). Different from the RTU measurement, PMU measurement $z^{(p)}(k)$ is linearly related to the state $x^{(p)}(k)$. For clarity, (7) can be rewritten as below:

$$z^{(p)}(k) = Hx(k) + v_2(k) \tag{8}$$

where $H = H^{(p)}T$.

Defining the overall measurement vector $z(k) = [z^{(r)}(k)^T \; z^{(p)}(k)^T]^T$, where $z(k) \in \mathbb{R}^m, m = m_1 + m_2$, we can obtain

$$z(k) = g(x(k)) + v(k) \tag{9}$$

where $g(x(k)) \triangleq \left[h^T(x(k)) \;\; (Hx(k))^T\right]^T$, and $v(k) \triangleq \left[v_1^T(k) \;\; v_2^T(k)\right]^T$. Correspondingly, the covariance matrix $R(k)$ of $v(k)$ are $R(k) \triangleq \mathrm{diag}\{R_1(k), R_2(k)\}$.

Remark 1: In RTU measurements, one bus is usually chosen as the reference bus for all the other buses to obtain the relative phase angles, while in PMU measurements, all PMU measurements provide the direct phase angles with respect to the time reference provided by the GPS system. In this paper, we use both RTU and PMU measurements, and therefore all the bus phase angles are relative to the reference from the GPS [25]. As a result, no reference buses are needed. Traditionally, the phasor angles and magnitudes are treated separately as state variables, whereas an alternative representation (i.e. the real and imaginary voltages of the buses) (see [5]) is adopted as state variables in this paper.

B. Quantized Measurement

In this paper, the quantization effect on measurements is considered, and the map of the quantization process is given by

$$\tilde{z}(k) = q(z(k)) = [q_1(z^{(1)}(k)) \; q_2(z^{(2)}(k)) \; \ldots \; q_m(z^{(m)}(k))].$$

The quantizer is assumed to be of the logarithmic type, that is, for each $q_j(\cdot)(j = 1, 2, \ldots, m)$, the set of the quantization levels is described by

$$\mathcal{U}_j = \left\{\pm u_i^{(j)}, u_i^{(j)} = {\chi_j}^i u_0^{(j)}, i = 0, \; \pm 1, \; \pm 2, \; \cdots \right\} \cup \{0\},$$
$$0 < \chi_j < 1, \quad u_0^{(j)} > 0,$$

where $\chi_j (j = 1, 2, \ldots, m)$ is called the quantization density. Each of the quantization level corresponds to a segment such that the quantizer maps the whole segment to this quantization level. The logarithmic quantizer $q_j(\cdot)$ is defined as

$$q_j(z^{(j)}(k)) = \begin{cases} u_i^{(j)}, \; \frac{1}{1+\delta_j} u_i^{(j)} < z^{(j)}(k) \leq \frac{1}{1-\delta_j} u_i^{(j)} \\ 0, \; z^{(j)}(k) = 0 \\ -q_j(-z^{(j)}(k)), \; z^{(j)}(k) < 0 \end{cases}$$

with $\delta_j = (1 - \chi_j)/(1 + \chi_j)$.

It can be easily seen from the above definition that $q_j(z^{(j)}(k)) = (1 + \Delta_k^{(j)}) z^{(j)}(k)$ for certain $\Delta_k^{(j)}$ satisfying $|\Delta_k^{(j)}| \leq \delta_j$. According to the above transformation, the quantization effects have been transformed into sector-bounded uncertainties [17]. Defining $\Delta_k = \mathrm{diag}\{\Delta_k^{(1)}, \cdots, \Delta_k^{(m)}\}$, the measurement after quantization can be expressed as

$$\tilde{z}(k) = (I + \Delta_k) z(k). \tag{10}$$

By defining $\Lambda = \mathrm{diag}\{\delta_1, \ldots, \delta_m\}$ and setting $F(k) = \Delta_k \Lambda^{-1}$, we can know that $F(k)$ is a real-value time-varying matrix satisfying $F(k)F^T(k) \leq I$.

Remark 2: As for state estimation with quantized measurements in power systems, the conventional way is to treat the quantization error as a range uncertainty with uniform distribution without in-depth characterization of the error [2, 11]. This assumption in quantization errors is quite coarse, hence making the estimation conservative. In addition, to the best of the author's knowledge, all the results on quantized PSSE have been done in the frame of static state estimation while none has been done in the frame of DSE, and this paper initializes the first attempt on DSE with quantized measurements in power systems.

III. Main results

A. Filter Structure

In this paper, we aim to design a filter with the following two properties: 1) the filter has a recursive structure and hence is suitable for online DSE in power systems; 2) despite the nonlinear measurement and quantization effects, the estimated state should be precise with a confidence interval, that is, the estimation error covariance should fall in a bounded interval. Meanwhile, we want to minimize such a bound by appropriately designing the filter gain at every time instant.

For the system (2) with measurement model (9), the recursive filter is designed as follows:

$$\begin{aligned} \hat{x}(k+1|k) &= A\hat{x}(k|k) + Bu & (11) \\ \hat{x}(k+1|k+1) &= \hat{x}(k+1|k) + K(k+1) \\ &\quad \times (\tilde{z}(k+1) - g(\hat{x}(k+1|k))) & (12) \end{aligned}$$

where $\hat{x}(k+1|k+1)$ is the estimate of $x(k+1)$ with $\hat{x}(0|0) = \bar{x}(0)$, $\hat{x}(k+1|k)$ is the one-step state prediction at time instant k, $K(k+1)$ is the filter gain to be determined. The one-step prediction and filtering error and the

corresponding covariance matrices are defined as

$$\begin{aligned}&\tilde{x}(k+1|k)=x(k+1)-\hat{x}(k+1|k),\\&\Sigma(k+1|k)=\mathbb{E}\{\tilde{x}(k+1|k)\tilde{x}^T(k+1|k)\}\\&\tilde{x}(k+1|k+1)=x(k+1)-\hat{x}(k+1|k+1),\\&\Sigma(k+1|k+1)=\mathbb{E}\{\tilde{x}(k+1|k+1)\tilde{x}^T(k+1|k+1)\}\end{aligned}\tag{13}$$

Remark 3: The filter presented above inherits the basic recursive structure of the Kalman filter, and hence it is suitable for online computation. However, due to the nonlinear RTU measurement function and the nonlinearity induced by quantization effects, to design an appropriate filter gain K is quite challenging, which is accomplished in the subsequent subsection.

B. Filter Design

To introduce our main results, we need the following two lemmas.

Lemma 1: [35] Given matrices A, H, F, and M with compatible dimensions such that $FF^T \leq I$. Let U be a symmetric positive-definite matrix and a be an arbitrary positive constant such that $a^{-1}I - MUM^T > 0$, then the following matrix inequality holds:

$$\begin{aligned}&(A+HFM)U(A+HFM)^T\\&\leq A(U^{-1}-aM^TM)^{-1}A^T+a^{-1}HH^T.\end{aligned}\tag{14}$$

Lemma 2: For $0 \leq k \leq N$, suppose that $X = X^T > 0$, $Y = Y^T > 0$, $S_k(X) = S_k^T(X) \in \mathbb{R}^{n\times n}$. If

$$S_k(Y) \geq S_k(X), \quad \forall X \leq Y,\tag{15}$$

then the solutions M_k and N_k to the following difference equations

$$M_{k+1} \leq S_k(M_k),\ N_{k+1} = S_k(N_k),\ M_0 = N_0 > 0\tag{16}$$

satisfy

$$M_k \leq N_k.$$

This lemma can be easily derived from Lemma 3.2 in [31], and hence the derivation is omitted here.

In this section, the filter is designed for the power system with quantized nonlinear measurements. First, the one-step prediction and filtering error covariances are calculated, wherein the specific difficulties caused by the composite of the measurement nonlinearity and the quantization are pointed out. Second, a special effort is made to cope with these difficulties in terms of some robust filtering techniques. At last, an upper bound of the filtering error covariance is obtained and a filter gain is designed to guarantee that such an upper bound is minimized.

To begin with, substituting (12) into (13), we have

$$\tilde{x}(k+1|k) = A\tilde{x}(k|k) + \omega(k),\tag{17}$$

and the corresponding covariance matrix is easily obtained,

$$\Sigma(k+1|k) = A\Sigma(k|k)A^T + W(k).\tag{18}$$

Similarly, the filtering error can be written as

$$\tilde{x}(k+1|k+1) = \tilde{x}(k+1|k) - K(k+1)(\tilde{z}(k+1) - g(\hat{x}(k+1|k)))$$

where

$$\begin{aligned}&\tilde{z}(k+1)-g(\hat{x}(k+1|k))\\=&q(g(x(k+1))+v(k+1))-g(\hat{x}(k+1|k))\\=&(I+\Delta_{k+1})(g(x(k+1))+v(k+1))-g(\hat{x}(k+1|k)).\end{aligned}\tag{19}$$

Expanding $g(x(k+1))$ in a Taylor series around $\hat{x}(k+1|k)$, we can have

$$\begin{aligned}g(x(k+1))=&g(\hat{x}(k+1|k))+G(k+1)\tilde{x}(k+1|k)\\&+o(|\tilde{x}(k+1|k)|)\end{aligned}\tag{20}$$

where $G(k+1) \triangleq \frac{\partial g(x)}{\partial x}|_{x=\hat{x}(k+1|k)}$ and $o(|\tilde{x}(k+1|k)|)$ represents the high-order terms of the Taylor series expansion. From the results of [9, 22], the high-order terms are transformed into the following easy-to-handle formulation:

$$o(|\tilde{x}(k+1|k)|) = C(k+1)\aleph(k+1)L(k+1)\tilde{x}(k+1|k)\tag{21}$$

where $C(k+1) \in \mathbb{R}^{m\times n}, L(k+1) \in \mathbb{R}^{n\times n}$ are problem-dependent scaling matrices, and $\aleph(k+1) \in \mathbb{R}^{n\times n}$ is an unknown time-varying matrix representing the linearization errors of the measurement model that satisfies

$$\aleph(k+1)\aleph^T(k+1) \leq I.\tag{22}$$

Combining the equations (19), (20), (21) and(22), we can obtain the filtering errors in the following form:

$$\begin{aligned}&\tilde{x}(k+1|k+1)\\=&\Phi(k+1)\tilde{x}(k+1|k)\\&-K(k+1)(I+\Delta_{k+1})v(k+1)\\&-K(k+1)F(k+1)\Lambda g(\hat{x}(k+1|k))\end{aligned}\tag{23}$$

where

$$\begin{aligned}\Phi(k+1)\triangleq\;& I-K(k+1)\big(C(k+1)\aleph(k+1)L(k+1)\\&G(k+1)+F(k+1)\Lambda G(k+1)\\&+M(k+1)L(k+1)\big)\\M(k+1)\triangleq\;& F(k+1)\Lambda C(k+1)\aleph(k+1)\end{aligned}$$

It can be easily found that $M(k+1)$ satisfies

$$M^T(k+1)M(k+1) \leq \gamma I\tag{24}$$

for certain scalar γ. To ensure the condition (24) is fulfilled, γ is chosen as $\gamma = 10\max\big(\mathrm{svd}(\Lambda C)\big)$, where $\max\big(\mathrm{svd}(\cdot)\big)$ indicates the maximum singular value (of a matrix). The covariance of the filtering error can be written as follows:

$$\begin{aligned}&\Sigma(k+1|k+1)\\=&\Phi(k+1)\Sigma(k+1|k)\Phi^T(k+1)\\&-\Phi(k+1)\tilde{x}(k+1|k)g^T(\hat{x}(k+1|k))\Lambda F^T(k+1)K^T(k+1)\\&-K(k+1)F(k+1)\Lambda g(\hat{x}(k+1|k))\tilde{x}^T(k+1|k)\Phi^T(k+1)\\&+K(k+1)F(k+1)\Lambda g(\hat{x}(k+1|k))\\&\times g^T(\hat{x}(k+1|k))\Lambda F^T(k+1)K^T(k+1)\\&+K(k+1)(I+F(k+1)\Lambda)R(k+1)\\&\times(I+F(k+1)\Lambda)^TK^T(k+1)\end{aligned}\tag{25}$$

$$\begin{aligned}
K(k+1) &= (1+\varepsilon_1)\big(P^{-1}(k+1|k)-\lambda_{1,k+1}\tilde{L}^T(k+1)\tilde{L}(k+1)\big)^{-1}G^T(k+1)\\
&\quad\times\Big[(1+\varepsilon_1)G(k+1)\big(P^{-1}(k+1|k)-\lambda_{1,k+1}\tilde{L}^T(k+1)\tilde{L}(k+1)\big)^{-1}G^T(k+1)\\
&\quad+(1+\varepsilon_1)\lambda_{1,k+1}^{-1}\tilde{C}(k+1)\tilde{C}^T(k+1)+(1+\varepsilon_1^{-1})\mathrm{tr}\big(\Psi(k+1|k)I\big)\\
&\quad+(R^{-1}(k+1)-\lambda_{2,k+1}\Lambda\Lambda)^{-1}+\lambda_{2,k+1}^{-1}I\Big]^{-1}, \qquad (26)\\
P(k+1|k) &= AP(k|k)A^T+W(k), \qquad (27)\\
P(k+1|k+1) &= (1+\varepsilon_1)(I-K(k+1)G(k+1))\big(P^{-1}(k+1|k)-\lambda_{1,k+1}\tilde{L}^T(k+1)\tilde{L}(k+1)\big)^{-1}\\
&\quad\times(I-K(k+1)G(k+1))^T\\
&\quad+(1+\varepsilon_1)\lambda_{1,k+1}^{-1}K(k+1)\tilde{C}(k+1)\tilde{C}^T(k+1)K^T(k+1)\\
&\quad+(1+\varepsilon_1^{-1})K(k+1)\mathrm{tr}\big(\Psi(k+1|k)\big)K^T(k+1)\\
&\quad+K(k+1)\Big[(R^{-1}(k+1)-\lambda_{2,k+1}\Lambda\Lambda)^{-1}+\lambda_{2,k+1}^{-1}I\Big]K^T(k+1). \qquad (28)
\end{aligned}$$

The main result of this section is summarized in the following theorem.

Theorem 1: Consider the one-step prediction error and the filtering error covariances in (18) and (25), respectively. Assume that (22) holds. Let γ, $\lambda_{1,k}$, $\lambda_{2,k}$ and ε_1 be positive scalars, and $K(k)$ be calculated recursively shown in (26) at the top of the next page. If there exist positive-definite solutions $P(k+1|k)$, $P(k+1|k+1)$ with initial condition $P(0|0)=\Sigma(0|0)$ to the Riccati difference equations shown in (27)-(28) at the top of the next page, subject to

$$\begin{cases} \lambda_{1,k+1}^{-1}I-\tilde{L}(k+1)P(k+1|k)\tilde{L}^T(k+1)>0\\ \lambda_{2,k+1}^{-1}I-\Lambda R(k+1)\Lambda>0\end{cases} \qquad (29)$$

where

$$\tilde{L}(k+1)\triangleq\big[L^T(k+1)\ \ (\Lambda G(k+1))^T\ \ L^T(k+1)\ \big]^T \qquad (30)$$

$$\tilde{C}(k+1)\triangleq[C(k+1)\quad I\quad \gamma I] \qquad (31)$$

$$\Psi(k+1|k)\triangleq\Lambda g(\hat{x}(k+1|k))g^T(\hat{x}(k+1|k))\Lambda \qquad (32)$$

then the matrix $P(k|k)$ is an upper bound for $\Sigma(k|k)$, that is,

$$\Sigma(k|k)\le P(k|k).$$

Moreover, the filter with gain $K(k+1)$ given by (26) minimizes the upper bound $P(k|k)$.

Proof: To begin with, from (27) and (28), we can view the covariance matrices $P(k+1|k+1)$ as a function of $P(k|k)$, that is

$$P(k+1|k+1)=\varphi_k\big\{P(k|k)\big\} \qquad (33)$$

where $\varphi_k\{\,\cdot\,\}$ denotes the specific functional relationship between $P(k+1|k+1)$ and $P(k|k)$. Then, it is not difficult to verify that

$$\varphi_k(Y)\ge\varphi_k(X), \qquad (34)$$

for all $X\le Y$, $X=X^T>0$, and $Y=Y^T>0$.

Now, let's consider the right side of (25) term by term. Representing $\Phi(k+1)$ in the following form:

$$\begin{aligned}
&\Phi(k+1)\\
=&I-K(k+1)G(k+1)-K(k+1)\tilde{C}(k+1)\\
&\times\begin{bmatrix}\aleph(k+1) & 0 & 0\\ 0 & F_{k+1} & 0\\ 0 & 0 & 1/\gamma M(k+1)\end{bmatrix}\tilde{L}(k+1)
\end{aligned}$$

from Lemma 1, we can obtain

$$\begin{aligned}
&\Phi(k+1)\Sigma(k+1|k)\Phi^T(k+1)\\
\le&\big(I-K(k+1)G(k+1)\big)\big(\Sigma^{-1}(k+1|k)-\lambda_{1,k+1}\\
&\times\tilde{L}^T(k+1)\tilde{L}(k+1)\big)^{-1}\big(I-K(k+1)G(k+1)\big)^T\\
&+\lambda_{1,k+1}^{-1}K(k+1)\tilde{C}(k+1)\tilde{C}^T(k+1)K^T(k+1)
\end{aligned} \qquad (35)$$

if

$$\lambda_{1,k+1}^{-1}I-\tilde{L}(k+1)P(k+1|k)\tilde{L}^T(k+1)>0$$

for arbitrary positive scalars $\lambda_{1,k+1}$.

Recall the following fundamental inequality

$$ab^T+ba^T\le\varepsilon_1aa^T+\varepsilon_1^{-1}bb^T \qquad (36)$$

where $\varepsilon_1>0$ is a scalar, a and b are two vectors with arbitrary dimension. Taking (36) into consideration, and noticing $F(k+1)F^T(k+1)\le I$, the second and third terms on the right side of (25) can be rearranged as follows:

$$\begin{aligned}
&-\Phi(k+1)\tilde{x}(k+1|k)g^T(\hat{x}(k+1|k))\Lambda F^T(k+1)K^T(k+1)\\
&-K(k+1)F(k+1)\Lambda g(\hat{x}(k+1|k))\tilde{x}_{k+1|k}^T\Phi^T(k+1)\\
&\le\varepsilon_1\Phi(k+1)\Sigma(k+1|k)\Phi^T(k+1)\\
&+\varepsilon_1^{-1}K(k+1)F(k+1)\Psi(k+1|k)F^T(k+1)K^T(k+1)\\
&\le\varepsilon_1\Phi(k+1)\Sigma(k+1|k)\Phi^T(k+1)\\
&+\varepsilon_1^{-1}K(k+1)\mathrm{tr}(\Psi(k+1|k))K^T(k+1)
\end{aligned} \qquad (37)$$

Similarly, the fourth term on the right side of (25) can be tackled as follows:

$$\begin{aligned}&K(k+1)F(k+1)\Lambda g(\hat{x}(k+1|k))\\ &\quad\times g^T(\hat{x}(k+1|k))\Lambda F^T(k+1)K^T(k+1)\\ \leq\ &K(k+1)\mathrm{tr}\big(\Psi(k+1|k)\big)K^T(k+1).\end{aligned} \tag{38}$$

As to the last term of the right side of (25), the following inequality can be easily derived from Lemma 1,

$$\begin{aligned}&K(k)(I+F(k)\Lambda)R(k)(I+F(k)\Lambda)^TK^T(k)\\ &\leq K(k)\big[(R^{-1}(k)-\lambda_{2,k}\Lambda\Lambda)^{-1}+\lambda_{2,k}^{-1}I\big]K^T(k)\end{aligned} \tag{39}$$

if

$$\lambda_{2,k}^{-1}I-\Lambda R(k)\Lambda>0$$

for arbitrary positive scalars $\lambda_{2,k+1}$.

It then follows from (35), (37), (38) and (39) that

$$\begin{aligned}&\Sigma(k+1|k+1)\\ \leq\ &(1+\varepsilon_1)\big(I-K(k+1)G(k+1)\big)\\ &\times\big(\Sigma^{-1}(k+1|k)-\lambda_{1,k+1}\tilde{L}^T(k+1)\tilde{L}(k+1)\big)^{-1}\\ &\times\big(I-K(k+1)G(k+1)\big)^T\\ &+(1+\varepsilon_1)\lambda_{1,k+1}^{-1}K(k+1)\tilde{C}(k+1)\tilde{C}^T(k+1)K^T(k+1)\\ &+(1+\varepsilon_1^{-1})K(k+1)\mathrm{tr}\big(\Psi(k+1|k)\big)K^T(k+1)+K(k+1)\\ &\times\big[\big(R^{-1}(k+1)-\lambda_{2,k+1}\Lambda\Lambda\big)^{-1}+\lambda_{2,k+1}^{-1}I\big]K^T(k+1)\end{aligned} \tag{40}$$

In other words, we have obtained that $\Sigma(k+1|k+1)\leq\varphi_k\big\{\Sigma(k|k)\big\}$. Recall the condition (33) and (34). Based on Lemma 2, we can therefore conclude that

$$\Sigma(k|k)\leq P(k|k).$$

Having determined the upper bound $P(k|k)$, we are now ready to show the filter gain given by (26) is optimal as it minimizes the upper bound $P(k|k)$. Taking partial derivatives of $P(k+1|k+1)$ with respect to $K(k+1)$ as follows:

$$\begin{aligned}&\frac{\partial tr\big(P(k+1|k+1)\big)}{\partial K(k+1)}\\ =&-2(1+\varepsilon_1)\big(I-K(k+1)G(k+1)\big)\\ &\big(P^{-1}(k+1|k)-\lambda_{1,k+1}\tilde{L}^T(k+1)\tilde{L}(k+1)\big)^{-1}G^T(k+1)\\ &+2(1+\varepsilon_1)\lambda_{1,k+1}^{-1}K(k+1)\tilde{C}(k+1)\tilde{C}^T(k+1)\\ &+2(1+\varepsilon_1^{-1})K(k+1)\mathrm{tr}\big(\Psi(k+1|k)\big)\\ &+2K(k+1)\big[(R^{-1}(k+1)-\lambda_{2,k+1}\Lambda\Lambda)^{-1}+\lambda_{2,k+1}^{-1}I\big]\end{aligned} \tag{41}$$

and setting $\frac{\partial tr(P(k+1|k+1))}{\partial K(k+1)}=0$, through some straightforward algebraic manipulation, we obtain the optimal filter gain, as shown in (26). This completes the proof of Theorem 1. ■

Remark 4: It can be seen that the linearization has been enforced to facilitate the recursive filtering algorithm developments. From (18) and (25), the filtering error covariance can be obtained in consideration of the quantization effect. Unfortunately, due to the simultaneous presences of the measurement nonlinearity and the quantization, the uncertainty matrices $\aleph(k)$, $M(k)$, and $F(k)$ are involved in the error covariance in (25). As such, it is impossible to calculate the accurate covariance matrix $\Sigma(k|k)$, and an alternative approach is proposed to find an upper bound of the covariance matrix at every time instant through designing an appropriate filtering gain $K(k|k)$ for the filter.

IV. Simulation results

In this section, the proposed algorithm is tested in the case study of the IEEE 14-bus test system. The simulation is implemented in Matlab with the Matpower package [37]. First, the IEEE 14-bus test system can be model as (2) with parameters $A=\mathrm{diag}_{28}\{0.98\}$, $B=\mathrm{diag}_{28}\{0.02\}$ and $W(k)=\mathrm{diag}_{28}\{0.1^2\}$. The nominal centre u of the normal state is the base-case voltages given in Table I. Furthermore, assume that the initial voltages of all buses are at flat start, that is, $x_{r,l}(0)=1$ p.u, $x_{i,l}(0)=0$ for all $l=1,2,\ldots,14$, and $\Sigma(0|0)=10^{-4}I_{28}$.

The measurement configuration is shown in Fig. 1, which has been adopted in [25]. The measurement system includes both conventional RTUs and PMUs, in which RTU measurements consist of three categories: the voltage magnitude at bus 1, power injections at the bus 3, 5, 13 and 14, and power flows at branches 1-2, 1-5, 2-5, 3-4, 4-7, 4-9, 6-11, 6-12, 6-13, 7-8, 7-9, 9-10, 9-14, 10-11, 12-13 and 13-14, and PMUs are deployed at buses 2, 7 and 9. Furthermore, the covariance matrices of RTU and PMU measurement noise are $R_1(k)=\mathrm{diag}_{43}\{0.1^2\}$ and $R_2(k)=\mathrm{diag}_{28}\{0.01^2\}$, respectively.

In the simulation, The parameters are chosen as $C=\begin{bmatrix}0.01I_{28\times43} & 0_{28}\end{bmatrix}^T, L=0.001I_{28}, \varepsilon_1=0.6, \lambda_{1,k}=0.01, \lambda_{2,k}=100$. The parameters of the logarithmic quantizers are $u_0^i=1$ and $\chi_i=0.8$, for $i=1,\ldots,71$.

Of all the buses, we choose bus 7 and 11 as the representative buses, as both PMU and RTU are installed at bus 7 while only RTU at bus 11. In this test system, two experiments regarding the estimation accuracy are carried out as follows:

Case 1) The proposed filter is implemented for the system with quantized measurements;
Case 2) The state estimations based on the proposed quantized filter and the traditional EKF without considering quantization effects are compared.

In order to have more general and significant experimental results, 100 Monte-Carol simulations are run. The notion mean square error (MSE) is adopted to evaluate the estimation accuracy, where MSE_i denotes MSE for the estimate of the ith state, i.e. $\mathrm{MSE}_i(k)=\frac{1}{100}\sum_{j=1}^{100}(x_i(k)-\hat{x}_i(k))^2$. To evaluate the average estimation performance of all states, average mean square error (AMSE) is defined as $\mathrm{AMSE}(k):=\frac{1}{n}\sum_{j=1}^{n}\mathrm{MSE}_j(k)$, where n is the number of the state variables. In all the figures, "R.V" and "I.V" denote the real and imaginary part of voltage, respectively.

TABLE I
THE NOMINAL VOLTAGE AT NORMAL STATES

Bus	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Real voltages	1.0600	1.0368	0.9609	0.9858	0.9958	1.0016	1.0022	1.0270	0.9827	0.9769	0.9850	0.9806	0.9755	0.9552
Imaginary voltages	0	0.0943	0.2173	0.1821	0.1563	0.2694	0.2512	0.2643	0.2743	0.2744	0.2724	0.2759	0.2748	0.2812

Fig. 1. IEEE 14 bus system and measurement configuration

A. Estimation Performance of The Proposed Filter

In this case, both the RTU and PMU measurements are assumed to be quantized according to the same quantizer level. In Fig. 2.a, $P_{7,8}$ is the RTU measurement of active power flow from bus 7 to bus 8, while in Fig. 2.b, $I_{7,8}$ is the PMU measurement of the real part of the current from bus 7 to bus 8. From the comparison, we can see that even with the same quantization level, the quantized RTU measurement is less accurate than the PMU counterpart. This is due to the nonlinearity of RTU measurement model which aggravates the quantization errors of RTU measurements, as the PMU measurements are linear functions of state variables while the RTU measurements are nonlinear functions of state variables.

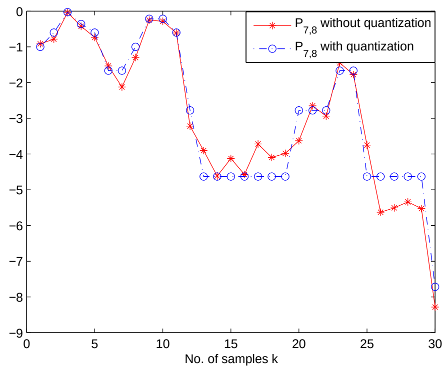
Fig. 3 shows the log(MSE) for the state at bus 7 and 11 as well as the upper bound, which confirms that the MSEs stay below their upper bounds. That means the estimated voltages of the systems are always close to the real values with a known upper bound on the estimate error. Moreover, The trajectories of the actual state $x_j(k), j = 7, 11$ and their estimation are plotted in Fig. 4, which illustrate the good performance of our proposed algorithm in estimating the system states. This is due to the specific efforts we have made to compensate the linearization errors of the nonlinear measurements as well as the quantization errors.

B. Traditional EKF VS The Proposed Filter

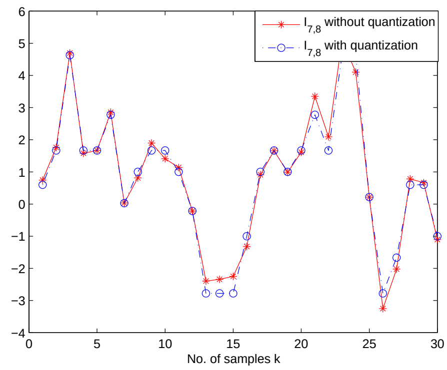
In Fig. 5, the estimation performances of the standard EKF and our proposed quantized filter algorithm are compared. One realization of the EKF and one realization of the proposed algorithm are simulated simultaneously, and the estimate errors of the real part of the voltage at bus 7 at both cases are illustrated in Fig. 5.a. We can find that, during the most of the time, the estimate error of EKF-based state estimation is bigger than the one of our proposed algorithm. Especially, when the accumulated error of EKF-based state estimation becomes bigger after several integrations, our proposed algorithm still yields accurate estimated states without accumulating the errors. The AMSEs of EKF and of the proposed algorithm are plotted in Fig. 5.b, from which we can find that our proposed algorithm performs much better than the EKF one. This is because the traditional EKF is sensitive to the quantization and linearization errors in measurements. However, due to specific considerations of these errors of measurement model and the designed robust filter gain, our proposed algorithm performs better.

V. CONCLUSION

In this paper, we have developed a recursive filter algorithm for power system dynamic state estimation. The system model with quantized RTU and PMU measurements is

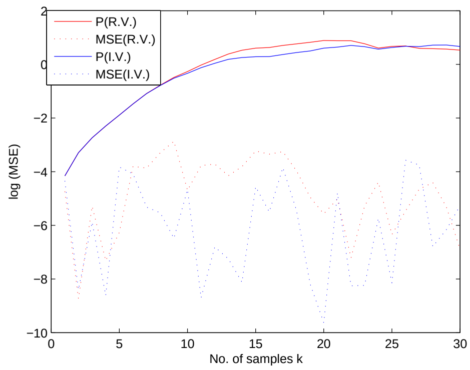


(a) RTU measurement $P_{7,8}$.

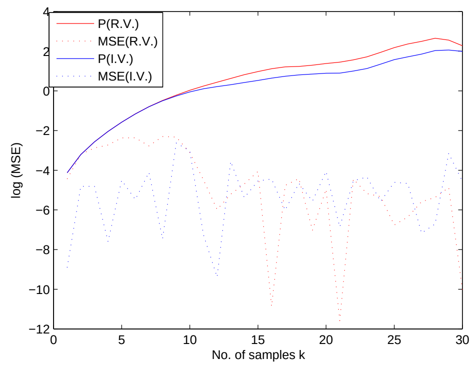


(b) PMU measurement $I_{7,8}$.

Fig. 2. The measurements with/without quantization

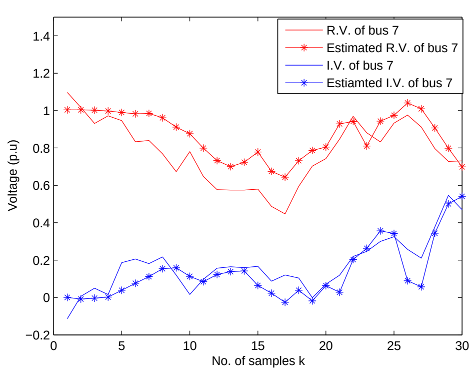


(a) Bus 7.

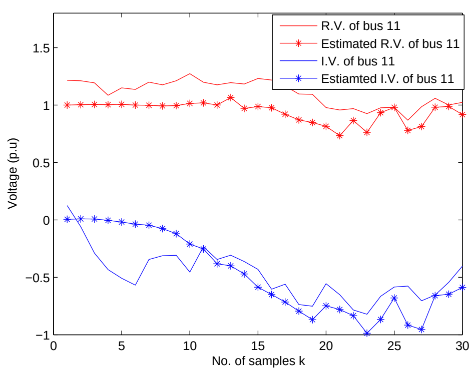


(b) Bus 11.

Fig. 3. Log(MSE) and its upper bound.

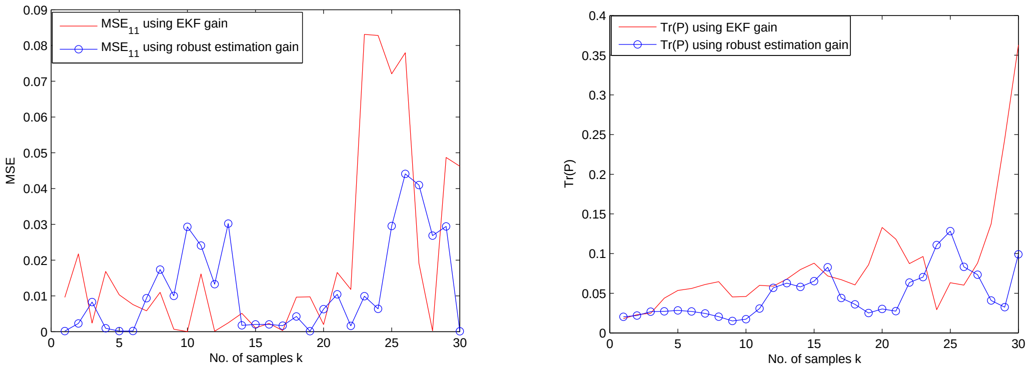


(a) Bus 7.



(b) Bus 11.

Fig. 4. The actual state and its estimation



(a) R.V of Bus 7. (b) AMSE of all states.

Fig. 5. The estimate error comparison

first proposed. In consideration of the quantization effect of nonlinear measurement, both the linearization and quantization errors are represented in terms of norm-bounded uncertainty matrices. Then, in the frame of robust estimation, a recursive filter is designed to guarantee that, despite the uncertainties existing in the derived model, the estimation error covariances are always less than a finite upper bound. Furthermore, the filter gain is designed such that the upper bound is minimized. Simulations have illustrated the performance of our proposed algorithm. Higher estimation accuracy can been achieved with our algorithm than that from the traditional EKF algorithm, which has confirmed the effectiveness of the propose filter algorithm.

References

[1] A. Abur and A. Exposito, *Power system state estimation: theory and implementation.* New York: Mercel Dekker, 2004.

[2] A. Al-Othman and M. Irving, “Uncertainty modelling in power system state estimation,” *IET Generation, Transmission & Distribution*, vol. 152, no. 2, pp. 233–239, 2005.

[3] P. Ashton, G. Taylor, M. Irving, I. Pisica, A. Carter, and M. Bradley, “Novel Application of Detrended Fluctuation Analysis for State Estimation Using Synchrophasor Measurements,” *IEEE Transactions on Power Systems*, vol. 28, no. 2, pp. 1930–1938, 2013.

[4] M. Basin, S. Elvira-Ceja, and E. Sanchez, Mean-square $\mathcal{H}_\infty$ filtering for stochastic systems: application to a 2DOF helicopter, *Signal Processing*, vol. 92, no. 3, pp. 801-806, Mar. 2012.

[5] X. Bian, X. Li, H. Chen, D. Gan, and J. Qiu, “Joint estimation of state and parameter with synchrophasorspart I: State tracking,” *IEEE Transactions on Power Systems*, vol. 26, no. 3, pp. 1196–1208, 2011.

[6] E. Blood, B. Krogh, and M. Ilic, “Electric power system static state estimation through Kalman filtering and load forecasting,” in *Proc. Power and Energy Society General Meeting*, Pittsburgh, USA, 2008, pp. 1–6.

[7] M. Brown Do Coutto Filho and J. de Souza, “Forecasting-aided state estimation Part I: Panorama,” *IEEE Transactions on Power Systems*, vol. 24, no. 4, pp. 1667–1677, 2009.

[8] R. Caballero-Aguila, A. Hermoso-Carazo, J. D. Jimenez-Lopez, J. Linares-Perez and S. Nakamori, Signal estimation with multiple delayed sensors using covariance information, *Digital Signal Processing*, vol. 20, no. 2, pp. 528-540, Mar. 2010.

[9] G. Calafiore, “Reliable localization using set-valued nonlinear filters,” *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*, vol. 35, no. 2, pp. 189–197, 2005.

[10] M. Cepek, J. Douville, G. Fecteau, and R. Malewski, “Loss measurement in high voltage thyristor valves,” *IEEE Transactions on Power Delivery*, vol. 9, no. 3, pp. 1222–1236, 1994.

[11] S. Chakrabarti and E. Kyriakides, “PMU measurement uncertainty considerations in wls state estimation,” *IEEE Transactions on Power Systems*, vol. 24, no. 2, pp. 1062–1071, 2009.

[12] P. Du, Z. Huang, Y. Sun, R. Diao, K. Kalsi, K. K. Anderson, Y. Li, and B. Lee, “Distributed dynamic state estimation with extended kalman filter,” in *Proc. North American Power Symposium*, 2011, pp. 1–6.

[13] N. Elia and S. K. Mitter, “Stabilization of linear systems with limited information,” *IEEE Transactions on Automatic Control*, vol. 46, no. 9, pp. 1384–1400, 2001.

[14] C. Y. Evrenosoglu and A. Abur, “Travelling wave based fault location for teed circuits,” *IEEE Transactions on Power Delivery*, vol. 20, no. 2, pp. 1115–1121, 2005.

[15] L. Fan, and Y. Wehbe, “Extended Kalman filtering based realtime dynamic state and parameter estimation using PMU data,” *Electric Power Systems Research*, vol. 103, pp. 168–177, Oct. 2013.

[16] M. Fu and C. E. de Souza, “State estimation for linear discrete-time systems using quantized measurements,” *Automatica*, vol. 45, no. 12, pp. 2937–2945, 2009.

[17] M. Fu and L. Xie, “The sector bound approach to quantized feedback control,” *IEEE Transactions on Automatic Control*, vol. 50, no. 11, pp. 1698–1711, 2005.

[18] E. Ghahremani and I. Kamwa, “Dynamic state estimation in power system by applying the extended Kalman filter with unknown inputs to phasor measurements,” *IEEE Transactions on Power Systems*, vol. 26, no. 4, pp. 2556–2566, 2011.

[19] J. Hu, Z. Wang, B. Shen, and H. Gao, “Quantised recursive filtering for a class of nonlinear systems with multiplicative noises and missing measurements,” *International Journal of Control*, vol. 86, no. 4, pp. 650–663, 2013.

[20] Z. Huang, K. Schneider, and J. Nieplocha, “Feasibility studies of applying kalman filter techniques to power system dynamic state estimation,” in *Proc. the 8th International Power Engineering Conference*, Singapore, 2007, pp. 376–382.

[21] Y.-F. Huang, S. Werner, J. Huang, N. Kashyap and V. Gupta, “State estimation in electric power grids: meeting new challenges presented by the requirements of the future grid,” *IEEE Signal Processing Magazine*, vol. 29, no. 5, pp. 33–43, Sept. 2012.

[22] X. Kai, C. Wei, and L. Liu, “Robust extended Kalman filtering for nonlinear systems with stochastic uncertainties,” *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*, vol. 40, no. 2, pp. 399–405, 2010.

[23] H. R. Karimi, Robust $\mathcal{H}_\infty$ filter design for uncertain linear systems over network with network-induced delays and output quantization, *Modeling, Identification and Control*, vol. 30, no. 1, pp. 27-37, 2009.

[24] M. Korkalı and A. Abur, “Robust fault location using least-absolute-value estimator,” *IEEE Transactions on Power Systems*, vol. 28, no. 4, 2013.

[25] G. Korres and N. Manousakis, “State estimation and bad data processing for systems including PMU and SCADA measurements,” *Electric Power Systems Research*, vol. 96, pp. 253-260, 2012.

[26] V. Kuhlmann, A. Sinton, M. Dewe, and C. Arnold, “Effects of sampling rate and adc width on the accuracy of amplitude

and phase measurements in power-quality monitoring," *IEEE Transactions on Power Delivery*, vol. 22, no. 2, pp. 758–764, 2007.

[27] K. Martin, D. Hamai, M. Adamiak, S. Anderson, M. Begovic, G. Benmouyal, G. Brunello, J. Burger, J. Cai, B. Dickerson *et al.*, "Exploring the IEEE standard c37.118–2005 synchrophasors for power systems," *IEEE Transactions on Power Delivery*, vol. 23, no. 4, pp. 1805–1811, 2008.

[28] A. Monticelli, *State estimation in electric power systems: a generalized approach*. Boston: Kluwer, 1999.

[29] A. G. Phadke and J. S. Thorp, *Synchronized phasor measurements and their applications*. Springer, 2008.

[30] B. Shen, Z. Wang, H. Shu, and G. Wei, "Robust H_∞ finite-horizon filtering with randomly occurred nonlinearities and quantization effects," *Automatica*, vol. 46, no. 11, pp. 1743–1751, 2010.

[31] Y. Theodor and U. Shaked, "Robust discrete-time minimum-variance filtering," *IEEE Transactions on Signal Processing*, vol. 44, no. 2, pp. 181–189, 1996.

[32] G. Valverde and V. Terzija, "Unscented Kalman filter for power system dynamic state estimation," *IET Generation, Transmission & Distribution*, vol. 5, no. 1, pp. 29–37, 2011.

[33] Y. Wang, W. Li, P. Zhang, B. Wang, and J. Lu, "Reliability analysis of phasor measurement unit considering data uncertainty," *IEEE Transactions on Power Systems*, vol. 27, no. 3, pp. 1503–1510, 2012.

[34] Z. Wang, H. Dong, B. Shen, and H. Gao, "Finite-horizon H_∞ filtering with missing measurements and quantization effects," *IEEE Transactions on Automatic Control*, vol. 58, no. 7, pp. 1707–1718, 2013.

[35] L. Xie, Y. C. Soh, and C. E. de Souza, "Robust Kalman filtering for uncertain discrete-time systems," *IEEE Transactions on Automatic Control*, vol. 39, no. 6, pp. 1310–1314, 1994.

[36] Q. Zhang, V. Vittal, G. T. Heydt, N. Logic, and S. Sturgill, "The integrated calibration of synchronized phasor measurement data in power transmission systems," *IEEE Transactions on Power Delivery*, vol. 26, no. 4, pp. 2573–2581, 2011.

[37] R. D. Zimmerman, C. E. Murillo-Sánchez, and R. J. Thomas, "Matpower: Steady-state operations, planning, and analysis tools for power systems research and education," *IEEE Transactions on Power Systems*, vol. 26, no. 1, pp. 12–19, 2011.

[38] S. Zonouz and W. Sanders, "A Kalman-based coordination for hierarchical state estimation: Agorithm and analysis," in *Proc. the 41st Annual Conference on System Sciences*, Hawaii, USA, 2008, pp. 187–187.