An Adaptive Penalty Scheme for Multiobjective Evolutionary Algorithm Based on Decomposition

Shouyong Jiang and Shengxiang Yang

Abstract—The multiobjective evolutionary algorithm based on decomposition (MOEA/D) decomposes a multiobjective optimization problem into a number of single-objective subproblems and solves them collaboratively. Since its introduction, MOEA/D has gained increasing research interest and has become a benchmark for validating new designed algorithms. Despite that, some recent studies have revealed that MOEA/D faces some difficulties to solve problems with complicated characteristics. In this paper, we study the influence the penalty-based boundary intersection (PBI) approach, one of the most popular decomposition approaches used in MOEA/D, on individuals' convergence and diversity, showing that the fixed same penalty value for all the subproblems is not very sensible. Based on this observation, we propose to use adaptive penalty values to enhance the balance between population convergence and diversity. Experimental studies show that the proposed adaptive penalty scheme can generally improve the performance of the original PBI when solving the problems considered in this paper.

I. INTRODUCTION

IN the field of multiobjective optimization, multiobjective evolutionary algorithms (MOEAs) are one of the most important classes of methods as they can solve problems without any knowledge of connectivity or differentiability of the problems in question. MOEAs generally employ a population of candidate individuals and evolve them collaboratively so as to obtain a set of solutions instead of only one solution in a single run. The optimal solution set of a multiobjective optimization problem (MOP) is called the Pareto-optimal set and its image in the objective space is called the Pareto-optimal front (POF).

There are two main goals in multiobjective optimization: (1) minimizing the gap between the obtained POF and the true POF, and (2) maximizing the diversity of the obtained POF. Most often than not, these two goals conflict with each other and any effort to improve one goal inevitably impairs the other. Thus, the design of good MOEAs is to seek proper balance between them.

The MOEA based on decomposition (MOEA/D) is one of the most representative MOEAs that can reach a good balance between convergence and diversity. Unlike Pareto-dominance based MOEAs, e.g., the nondominated sorting genetic algorithm II (NSGA-II) and strength Pareto evolutionary algorithm 2 (SPEA2), MOEA/D makes use of the

concept of decomposition and converts an MOP into a number of single-objective subproblems that are easier to solve than MOPs. By properly structuring subproblems and their interaction, MOEA/D can efficiently solve the decomposed subproblems in a collaborative manner while providing a good distribution of the obtained POF points. Since its introduction, MOEA/D has received increasing research interest, and various variants have been proposed in the literature [1], [9], [13], [18], and the idea of decomposition has been exploited in a number of studies [4], [11], [12], [17].

In MOEA/D, the resulting approximation performance, such as distribution and convergence, depends largely on the chosen decomposition approach. The weighted sum, the weighted Tchebycheff, and the penalty boundary intersection (PBI) are three mainly and widely used decomposition approaches. The weighted sum is unable to approximate problems with the entire concave POF [18], but it has shown better performance than the weighted Tchebycheff for many-objective problems (with at least four objectives) in a recent study [8]. The weighted Tchebycheff approach can approximate both convex and concave POFs and perform well when solving standard benchmark problems like ZDT [19] and DTLZ [5]. However, some recent investigations have revealed that this approach has difficulties in uniformly distributing solutions on the boundary regions of the POF when solving complex problems [9], [14].

On the other hand, the PBI approach gains a firm foothold in MOEA/D as it can provide a more uniform distribution of POF than the weighted Tchebycheff for three- and higher-dimensional problems [4], [7], [11], [18]. The performance of PBI is largely determined by its penalty factor, which controls the balance between convergence and diversity. A small penalty favours convergence whereas a large one stresses diversity. The penalty value is commonly set to 5.0 in most studies, e.g., [4], [11], [18]. However, in [8], PBI with a penalty value of 0.1 was reported to outperform that with a penalty value of 5.0 on multiobjective knapsack problems. This means, the optimal value for the penalty factor may vary dramatically from problem to problem. In [15], the author argued that, if the obtained ideal point is far from the true ideal point (probably due to the loss of boundary solutions), PBI will face difficulties to approximate the entire POF.

Another issue is the same penalty setting for all subproblems. The same penalty value may be suitable for solving problems whose POF is not extremely shaped. That is, if all POF points have similar distances to the ideal point, it makes sense to use the same penalty value for all the subproblems to penalise possible poor diversity. However, the use of the same penalty value is unfair for all subproblems if their POF points have significantly different distances to the ideal

The authors are with the Centre for Computational Intelligence (CCI), School of Computer Science and Informatics, De Montfort University, The Gateway, Leicester, LE1 9BH, U.K. (email: shouyong.jiang@email.dmu.ac.uk, syang@dmu.ac.uk).

This work was supported by the Engineering and Physical Sciences Research Council (EPSRC) of UK under Grant EP/K001310/1.

point, because those close to the ideal point are overpenalised whereas those far from the ideal point are underpenalised.

To overcome these drawbacks, in this paper, we propose an adaptive penalty scheme (APS) to specify the penalty value. APS takes into consideration the landscape of each subproblem and adaptively adjust each subproblem's penalty value during the search. This way, the balance between convergence and diversity in each subproblem can be reached. Experimental studies validate the effectiveness of the proposed scheme.

The rest of this paper is organized as follows. Section II presents related work and illustrates the influence of the penalty factor used in PBI. The proposed APS is given in Section III. Experimental studies are presented in Section IV. Finally, Section V concludes this paper and discusses some relevant future work.

II. RELATED WORK

A. MOEA/D Algorithm

MOEA/D employs a set of predefined weight vectors that uniformly partition the entire objective space to specify a number of search directions, and defines a single-objective problem or a multiobjective subproblem by decomposition approaches for each search direction. For each search direction, MOEA/D also specifies T closest neighbours beforehand, which helps to efficiently solve the associated single-objective problem in a collaborative manner. During the course of search, mating selection and replacement are considered among solutions associated with the T neighbouring search directions. MOEA/D is a steady-state algorithm and updates solutions one by one, so it approximates the true POF quickly. Algorithm 1 presents a brief description of the original MOEA/D with the PBI [18]. For more details, the interested readers are referred to [18].

B. The PBI Approach

Decomposition approaches play a key role in converting an MOP into a number of scalar optimization subproblems in decomposition-based MOEAs. There are three popular decomposition approaches, i.e., the weighted sum, weighted Tchebycheff and PBI. The weighted sum approach favours convex problems and fails to approximate problems with a concave POF, whereas the weighted Tchebycheff approach can handle both convex and concave problems. The PBI approach has its advantages in obtaining a good distribution of solutions in the objective space [18] and handling many-objective problems [4], [8], but its performance is very sensitive to the setting of the penalty factor [8]. The scalar optimization problem of PBI is defined as follows:

$$\begin{aligned} \min \quad & g^{pbi}(\mathbf{x}|\mathbf{w}, \mathbf{z}^*) = d_1 + \theta d_2 \\ \text{s.t.} \quad & \mathbf{x} \in \Omega_x \end{aligned} \quad (1)$$

where

$$d_1 = \frac{\|(\mathbf{F}(\mathbf{x}) - \mathbf{z}^*)^T \mathbf{w}\|}{\|\mathbf{w}\|} \quad (2)$$

$$d_2 = \|\mathbf{F}(\mathbf{x}) - (\mathbf{z}^* + d_1 \frac{\mathbf{w}}{\|\mathbf{w}\|})\| \quad (3)$$

Algorithm 1 MOEA/D

1: **Input:**

- *MaxIteration*: the stopping criterion
- N : the number of subproblems considered in MOEA/D
- T : the neighbourhood size

2: **Output:** approximated Pareto-optimal set

- 3: **Initialization:** Generate a uniform spread of N weight vectors: $w^1, \dots, w^N$ and then compute the T closest weight vectors to each weight vector by the Euclidean distance. For each $i = 1, \dots, N$, set $B(i) = \{i_1, \dots, i_T\}$ where $w^{i_1}, \dots, w^{i_T}$ are the T closest weight vectors to w^i
 - 4: Generate an initial population $P = \{x^1, \dots, x^N\}$ by uniformly randomly sampling from the decision space
 - 5: **while** $gen := 1$ to $MaxIteration$ **do**
 - 6: **for** $i := 1$ to N **do**
 - 7: Randomly select two indexes r_1 and r_2 from $B(i)$
 - 8: Apply genetic operators on individuals r_1, r_2 to produce a new solution y
 - 9: If y is better than any individual x^j in $B(i)$ (measured by the fitness of the i -th subproblem), then x^j is replaced by y
 - 10: **end for**
 - 11: **end while**
 - 12: Output P
-

Fig. 1. Illustration of PBI.

Ω_x is the search space and $\mathbf{z}^* = (z_1^*, \dots, z_M^*)$ is the ideal point in the objective space for which the i th component can be computed by $z_i^* = \min_{\mathbf{x} \in \Omega_x} f_i(\mathbf{x})$, and θ is a user-defined penalty factor. d_1 and d_2 are the length of the projection of vector $(\mathbf{F}(\mathbf{x}) - \mathbf{z}^*)$ on the weight vector $\mathbf{w}$ and the perpendicular distance from $\mathbf{F}(\mathbf{x})$ to $\mathbf{w}$, respectively. Fig. 1 offers a brief illustration of the PBI approach. It is not difficult to see that θ takes the responsibility for balancing convergence (measured by d_1) and diversity (measured by d_2), and the PBI approach drives the search toward the obtained ideal point $\mathbf{z}^*$ by minimizing g^{pbi} .

C. Influence of the Penalty Factor of PBI

There is a key factor, i.e., θ , which is of great importance for balancing population convergence and diversity, in PBI. A

Fig. 2. Illustration of insufficient penalty for weight vectors where $w_1 = (0.1, 0.9)^T$, $w_2 = (0.2, 0.8)^T$ and $\theta = 1.0$ is used in PBI.

small value of θ favours convergence whereas a large one is beneficial for diversity. Fig. 2 shows the influence of different θ values on convergence and diversity, where the bold curve is the true POF and points A to G in the alphabetical order are scattered POF points.

In Fig. 2, θ is set to 1.0, and its influence is illustrated by two adjacent boundary weight vectors, i.e., $w_1 = (0.1, 0.9)^T$ and $w_2 = (0.2, 0.8)^T$. Intuitively, B and C are ideal optimal points for subproblems associated with w_1 and w_2 , respectively. However, the solution B of the subproblem associated with w_1 will be replaced with C since $g^{pbi}(C|w_1, z^*) = 0.60$ is smaller than $g^{pbi}(B|w_1, z^*) = 0.75$. This means, due to insufficient penalty, POF points far away from the obtained ideal point z^* are replaced by those close to z^* , thus the diversity of solutions declines. Even worse, for extremely-convex problems that have a sharp peak and long tail [9], the above impact is more vital, and boundary points on the POF cannot be approximated so as to decrease the spread of the POF.

On the other hand, one may argue that the value of θ is not enough for PBI to solve this kind of problem. However, if we simply increase the θ value, population diversity will be surely improved, but population convergence will deteriorate. This is because there is very little room left for generating a good solution to update parents when subproblems' diversity is overemphasised. This inevitably slows down the search process. So, neither small or large values are good for θ .

The above illustration shows that the improper setting of θ can cause imbalance between population diversity and convergence. Intuitively, θ is used to balance between the values d_1 and d_2 , as illustrated in Fig. 1. If all subproblems' d_1 values are similar, then it is sensible to use the same θ value for all subproblems to penalise solutions' poor diversity. However, it is unfair to use the same θ value when subproblems' d_1 values are dissimilar, because the same θ value is likely to overpenalise solutions with small d_1 values and underpenalise solutions with large d_1 values. Thus, when the POF points have significantly different distances to the

ideal point, their corresponding penalty values should be distinct for fairness. In other words, different subproblems have different landscapes, thereby requiring different amount of penalty to balance diversity and convergence.

III. ADAPTIVE PENALTY SCHEME (APS)

The proposed APS takes into account both population diversity and convergence. Suppose that $d_{i,1}$ and $d_{i,2}$ are the d_1 and d_2 values, as shown in Fig. 1, of the i -th individual, respectively. If all individuals' $d_{i,2}$ values are very small, it implies the population diversity is very good. If all $d_{i,1}$ values are very similar as well, it means that all subproblems' landscapes are probably similar. In this situation, it is desirable to give all subproblems an identical penalty value. Otherwise, each subproblem should have its own penalty value that can strike the balance between diversity and convergence.

Bearing this in mind, we propose the following APS to adjust θ_i , the penalty value of the i -th subproblem, at each generation:

$$\theta_i^{t+1} = \begin{cases} \min_{1 \leq j \leq N} \{\theta_j^t\}, & \text{if similar landscapes,} \\ \theta_i^t + \Delta\theta_i, & \text{otherwise.} \end{cases} \quad (4)$$

where θ_i^t is the θ_i value at generation t , and $\Delta\theta_i$ is calculated by:

$$\Delta\theta_i = \text{sgn}(\alpha - 1) \cdot \alpha \cdot \theta_i^{\min} \quad (5)$$

where $\alpha = \frac{d_{i,2}}{\bar{d}_2}$ in which $\bar{d}_2$ is the average of all $d_{i,2}$ values. $\text{sgn}()$ is the sign function, and $\theta_i^{\min}$ is the minimum allowable value of θ_i . Correspondingly, the maximum allowable value of θ_i is denoted as $\theta_i^{\max}$. After that, θ_i^{t+1} obtained from 4 should be repaired to keep its value in the range $[\theta_i^{\min}, \theta_i^{\max}]$, which can be achieved by:

$$\theta_i^{t+1} = \begin{cases} \theta_i^{\min}, & \text{if } \theta_i^{t+1} < \theta_i^{\min}, \\ \theta_i^{\max}, & \text{if } \theta_i^{t+1} > \theta_i^{\max}, \\ \theta_i^{t+1}, & \text{otherwise.} \end{cases} \quad (6)$$

where the initial penalty value is set to $\theta_i^1 = \theta_i^{\min}$ for all the subproblems in our experimental studies.

It is clear to see from Eq. (4) that, when subproblems have dissimilar landscapes, their penalty values depend on the diversity of individuals. If an individual's diversity is less (the $d_{i,2}$ value is larger) than the average, its subproblem can increase penalty to stress diversity, and vice versa. When all the subproblems have very similar landscapes, they can have the same penalty value, which is actually the minimum of all θ_i^t values. It is possible to judge whether or not all the subproblems have similar landscapes if the true POF of the considered problem is known in advance. However, we often have little knowledge of a problem's landscape beforehand. In this situation, a landscape estimator is required to judge subproblems' similarity. The following gives the definition of subproblems' landscape similarity:

Definition 1. Subproblems are said to be landscape-similar if their optimal solutions have similar $d_{i,1}$ values.

According to Definition 1, a perfect landscape-similar problem is the one having a spherical POF as all POF points have similar distances to the ideal point. In the course of evolution, subproblems' landscape similarity can be judged only

Fig. 3. An illustration of the condition of landscape similarity in the 2-objective case.

when all solutions have achieved good diversity, indicating a good coverage of the POF. When all $d_{i,2}$ values (or $\bar{d}_2$) are very small, we then consider the $d_{i,1}$ values. If all the $d_{i,1}$ values are similar, the subproblems can be considered to be landscape-similar. In this paper, the condition of landscape similarity is stated as:

$$\bar{d}_2 < \epsilon \cap d_{max,1} < \sqrt{M} \cdot d_{min,1} \quad (7)$$

where ϵ is a very small non-negative threshold value. For simplicity, ϵ is set to 10^{-3} , indicating all $d_{i,2}$ values are very small and good population diversity has been achieved. $d_{min,1}$ and $d_{max,1}$ are the minimum and maximum value of $d_{i,1}$, respectively, and M is the number of objectives. Similar landscapes often occur in concave problems, where the POF is very similar to a sphere. In the worst case, boundary POF points have $d_{min,1}$ whereas intermediate POF points have $d_{max,1}$. Correspondingly, the largest possible value of $d_{max,1}$ is less than $\sqrt{\sum_{j=1}^M d_{min,1}^2} = \sqrt{M} \cdot d_{min,1}$. Fig. 3 presents a 2-objective case of the judgement condition of landscape similarity.

IV. EXPERIMENTAL STUDY

A. Test Problems

To examine the effectiveness of our proposed scheme, we use six complex problems with irregular Pareto fronts instead of some well-known test suites such as ZDT [19] or DTLZ [5] for empirical comparison. Some of them have been tested in [9] and [4], showing that the original MOEA/D faces difficulties in solving these kinds of problems due to their complex characteristics. The detailed description of these problems is presented in Table I.

B. Performance Metrics

In the experiments, we use three performance measures, which are described below.

1) *Maximum spread (MS)*: The MS, first introduced by Zitzler *et al.* [19], measures to what extent the extreme members in an approximated Pareto front POF^* has been reached. Goh and Tan [6] proposed a modified version of

MS by taking into account the proximity of POF^* towards the true Pareto front POF , as follows:

$$MS' = \sqrt{\frac{1}{M} \sum_{k=1}^M \left[\frac{\min[\overline{POF}_k, \overline{POF}_k^*] - \max[\underline{POF}_k, \underline{POF}_k^*]}{\overline{POF}_k^* - \underline{POF}_k^*} \right]^2} \quad (8)$$

where $\overline{POF}_k$ and $\underline{POF}_k$ are the maximum and minimum of the k th objective in POF , respectively; Similarly, $\overline{POF}_k^*$ and $\underline{POF}_k^*$ are the maximum and minimum of the k th objective in POF^* , respectively. A large value of MS' indicates a good spread of POF^* , and MS' will have a value of one when POF^* covers the whole POF .

2) *Inverted Generational Distance (IGD)*: The IGD [22] is an effective performance indicator since it can provide reliable information on both the diversity and convergence of obtained solutions. Let PF be a set of solutions uniformly sampled from the true POF, and PF^* be the approximated solutions in the objective space, the IGD metric measures the gap between PF^* and PF , which is calculated as follows:

$$IGD(PF^*, PF) = \frac{\sum_{p \in PF} d(p, PF^*)}{|PF|} \quad (9)$$

where $d(p, PF^*)$ is the distance between the member p of PF and the nearest member of PF^* . In this paper, 500 points uniformly sampled from the true POF are used as the reference set for IGD, which can be done by generating a large volume of points and then pruning them to the desirable size by the k th nearest neighbour method proposed in SPEA2 [20].

3) *Hypervolume (HV)*: HV [21] measures the size of the objective space dominated by the approximated solutions S and bounded by a reference point $R = (R_1, \dots, R_M)^T$ that is dominated by all points on the POF, and is computed by:

$$HV(S) = Leb\left(\bigcup_{x \in S} [f_1(x), R_1] \times \dots \times [f_M(x), R_M]\right) \quad (10)$$

where $Leb(A)$ is the Lebesgue measure of a set A . In our experiments, R is set to $(1.2, 1.2)^T$ and $(1.2, 1.2, 1.2)^T$ for bi- and three-objective test instances, respectively.

C. Parameter Settings

As we aim to improve the performance of PBI, we would like to test and compare the proposed approach, i.e., PBI with the proposed APS, denoted APBI, with the original PBI. A recently-developed MOEA/D variant [10], i.e., MOEA/D with a stable matching (STM) model (denoted MOEA/D-STM), is also used to validate APBI. In both the original MOEA/D and MOEA/D-STM, the penalty value was set to 5.0, as recommended by [18]. In APBI, the lower and upper bounds of the penalty value for all subproblems were set to 0.1 and 100, that is, $\theta_i^{min} = 0.1$ and $\theta_i^{max} = 100$. This kind of setting makes sure that the penalty value of 5.0 is contained in the range. Note that, θ_i^{min} should be positive so as to evoke diversity maintenance. In this paper, the simulated binary crossover (SBX) [2] and polynomial mutation [3] are chosen as our recombination and mutation operators, respectively. The crossover probability is $p_c = 1.0$ and its distribution index is $\eta_c = 20$. The mutation probability is $p_m = 1/n$ and its distribution $\eta_m = 20$, where n is the number of decision variables.

TABLE I. TEST INSTANCES

Instance	Description	Domain	Number of Variables	Notes
F1	$f_1(x) = (1 + g(x))x_1$ $f_2(x) = (1 + g(x))(1 - \sqrt{x_1})^3$ $g(x) = 2 \sin(0.5\pi x_1)(n - 1 + \sum_{i=2}^n (y_i^2 - \cos(2\pi y_i)))$ where $y_{i=2:n} = x_i - \sin(0.5\pi x_i)$ POF: $f_2 = (1 - \sqrt{f_1})^3$ POS: $x_i = \sin(0.5\pi x_i), i = 2, \dots, n$	$[0, 1]^n$	20	Uni-modal Convex
F2	$f_1(x) = (1 + g(x))x_1$ $f_2(x) = (1 + g(x))\sqrt{1 - x_1^5}$ $g(x) = 2 \sin(0.5\pi x_1)(n - 1 + \sum_{i=2}^n (y_i^2 - \cos(2\pi y_i)))$ where $y_{i=2:n} = x_i - \sin(0.5\pi x_i)$ POF: $f_2 = \sqrt{1 - f_1^5}$ POS: $x_i = \sin(0.5\pi x_i), i = 2, \dots, n$	$[0, 1]^n$	20	Uni-modal Convex
F3	$f_1(x) = (1 + g(x))x_1$ $f_2(x) = \frac{1}{2}(1 + g(x))(1 - x_1^{0.1} + (1 - \sqrt{x_1})^2 \cos^2(3\pi x_1))$ $g(x) = 2 \sin(0.5\pi x_1)(n - 1 + \sum_{i=2}^n (y_i^2 - \cos(2\pi y_i)))$ where $y_{i=2:n} = x_i - \sin(0.5\pi x_i)$ POF: $f_2 = \frac{1}{2}(1 - f_1^{0.1} + (1 - \sqrt{f_1})^2 \cos^2(3\pi f_1))$ POS: $x_i = \sin(0.5\pi x_i), \forall x_i \in \mathbf{x}_{\Pi}$	$[0, 1]^n$	20	Multi-modal Disconnected
F4	$f_1(x) = (1 + g(x))(x_1 + 0.05 \sin(6\pi x_1))^2$ $f_2(x) = (1 + g(x))(1 - x_1 + 0.05 \sin(6\pi x_1))^2$ $g(x) = 2 \sin(0.5\pi x_1)(n - 1 + \sum_{i=2}^n (y_i^2 - \cos(2\pi y_i)))$ where $y_{i=2:n} = x_i - \sin(0.5\pi x_i)$ POF: $f_1^{0.5} + f_2^{0.5} = 1 + 0.1 \sin(3\pi(f_1^{0.5} - f_2^{0.5} + 1))$ POS: $x_i = \sin(0.5\pi x_i), \forall x_i \in \mathbf{x}_{\Pi}$	$[0, 1]^n$	20	Multi-modal mixed
F5	$f_1(x) = (1 + g(x))(x_1 + 0.05 \sin(6\pi x_1))^{0.2}$ $f_2(x) = (1 + g(x))(1 - x_1 + 0.05 \sin(6\pi x_1))^{10}$ $g(x) = 2 \sin(0.5\pi x_1)(n - 1 + \sum_{i=2}^n (y_i^2 - \cos(2\pi y_i)))$ where $y_{i=2:n} = x_i - \sin(0.5\pi x_i)$ POF: $f_1^5 + f_2^{0.1} = 1 + 0.1 \sin(3\pi(f_1^5 - f_2^{0.1} + 1))$ POS: $x_i = \sin(0.5\pi x_i), \forall x_i \in \mathbf{x}_{\Pi}$	$[0, 1]^n$	20	Multi-modal mixed
F6	$f_1(x) = ((1 + g(x)) \cos(0.5\pi x_1) \cos(0.5\pi x_2))^4$ $f_2(x) = ((1 + g(x)) \cos(0.5\pi x_1) \sin(0.5\pi x_2))^4$ $f_3(x) = ((1 + g(x)) \sin(0.5\pi x_1))^2$ $g(x) = \sum_{i=3}^n (x_i - 0.5)^2$ POF: $\sqrt{f_1} + \sqrt{f_2} + f_3 = 1$ POS: $x_i = 0.5, i = 3, \dots, n$	$[0, 1]^n$	20	Uni-modal Convex

D. Comparison Results

Table II gives the results of MOEA/D with PBI, STM, and APBI, where best, mean, and worst values of three metrics are reported and the best values are marked in boldface. The Wilcoxon rank-sum test [16] is carried out to indicate significance between different results at the 0.05 significance level. The superscripts ‘†’ and ‘‡’ above the mean value in the table are used to indicate that APBI performs equivalently to and significantly outperforms the corresponding algorithm, respectively.

It can be clearly seen from the table that, APBI generally performs better than the original PBI and STM. Specifically, APBI helps enhance the coverage of population, as indicated by large MS metric values. Compared with PBI, STM can also improve the coverage of population for all the cases except F2 and F3, but its improvement is not very significant. On F2, the MS value of zero shows that all population members in STM are driven toward a single point/solution so as to provide very poor population distribution. In contrast, APBI can drastically boost the spread of population and achieve promising MS values.

Despite a slightly better coverage performance than PBI, STM obtains worse IGD values than PBI in some cases. This means the fine-grained stable matching in STM may cause

the deterioration of population convergence and distribution. APBI achieves better performance than PBI and STM in terms of the IGD metric, indicating that it can converge toward the POF well.

The HV metric shows that, compared with PBI and STM, APBI performs comparatively on two concave problems, i.e., F2 and F5, and significantly better on the rest of the test problems. In fact, F2 and F5 are the only two problems that are judged by APBI to have similar landscapes. This indicates our landscape similarity measure is able to detect similar landscapes during the search. The statistical testing suggests there is no much difference between PBI and APBI. Thus, APBI can also effectively solve subproblems having similar landscapes.

To have a better understanding of the exact performance of the three compared algorithms, we graphically report the approximated POF for each problem, as presented in Fig. 4. The figure clearly illustrates that, APBI is able to approximate more boundary solutions than PBI and STM. STM misses some regions of the POF and it has difficulty in converging toward the POF perfectly.

The poor performance of STM on the tested problems could be explained by two possible reasons. One is that, the test problems may be too complicated in terms of their

TABLE II. BEST, MEAN, AND WORST PERFORMANCE MEASURE VALUES OBTAINED BY THREE METHODS ON SIX PROBLEMS

Prob.	MS			IGD			HV		
	PBI	STM	APBI	PBI	STM	APBI	PBI	STM	APBI
F1	0.5323	0.5375	0.8070	0.1045	0.1025	0.0274	1.3137	1.3139	1.3349
	(0.5252) [‡]	(0.5273) [‡]	0.7576	(0.1077) [‡]	(0.1068) [‡]	0.0378	(1.3123) [‡]	(1.3123) [‡]	1.3333
	0.5135	0.5130	0.7127	0.1125	0.1124	0.0534	1.3101	1.3099	1.3302
F2	0.9998	0.0000	0.9997	0.0074	0.7838	0.0106	0.5404	0.2400	0.5397
	(0.9970) [†]	(0.0000) [‡]	0.9984	(0.0116) [†]	(0.7838) [‡]	0.0177	(0.5388) [†]	(0.2400) [‡]	0.5368
	0.9845	0.0000	0.9532	0.7838	0.7838	0.0294	0.5334	0.2400	0.5138
F3	0.5988	0.4796	0.8441	0.1941	0.1978	0.0258	1.3334	1.3319	1.3552
	(0.4842) [‡]	(0.3311) [‡]	0.6304	(0.4040) [‡]	(0.4036) [‡]	0.1680	(1.2974) [‡]	(1.2976) [‡]	1.3364
	0.3290	0.3191	0.3870	0.4046	0.4051	0.3953	1.2962	1.2956	1.3145
F4	0.6192	0.6579	0.9178	0.0817	0.0805	0.0149	1.2549	1.2538	1.2652
	(0.6041) [‡]	(0.6066) [‡]	0.8744	(0.0838) [‡]	(0.0846) [‡]	0.0217	(1.2537) [‡]	(1.2525) [‡]	1.2634
	0.5988	0.5998	0.6862	0.0848	0.0861	0.0527	1.2527	1.2519	1.2584
F5	0.8659	0.8784	0.9295	0.0218	0.0253	0.0125	0.7530	0.7546	0.7579
	(0.8561) [‡]	(0.8719) [‡]	0.8842	(0.0230) [‡]	(0.0313) [†]	0.0213	(0.7479) [‡]	(0.7533) [†]	0.7493
	0.6367	0.8678	0.8738	0.1391	0.0406	0.0739	0.6168	0.7517	0.6840
F6	0.7108	0.7373	0.9367	0.0425	0.0389	0.0387	1.6705	1.6693	1.6717
	(0.6889) [‡]	(0.7094) [‡]	0.8849	(0.0469) [‡]	(0.0421) [†]	0.0418	(1.6644) [‡]	(1.6664) [‡]	1.6676
	0.6676	0.6847	0.8011	0.0495	0.0444	0.0493	1.6572	1.6625	1.6593

[†] indicates APBI performs equivalently to the corresponding algorithm.

[‡] indicates APBI performs significantly better than the corresponding algorithm.

POF shapes so that STM struggles to solve them. The other reason may be the inappropriate setting of the penalty value in PBI. As subproblems in MOEA/D are responsible for different landscapes of the objective space, they should be given different penalty values for different scenarios to balance convergence and diversity in PBI. Although STM uses a stable matching model to promote the balance between convergence and diversity, its convergence still depends on scalarizing functions. Thus, when PBI is chosen as the scalarizing function, the penalty value should be carefully specified so that the convergence (measured by the d_1 value in PBI) is not overemphasised or underemphasised.

V. CONCLUSIONS

The original PBI used in MOEA/D gains great success in solving MOPs, but it has been recently reported to face difficulties to solve problems with extremely-shaped POF. In this paper, we have investigated the influence of the penalty factor of PBI on population diversity and convergence, finding that each subproblem's penalty value should depend on the subproblem's landscape. Based on this finding, we have proposed an adaptive penalty scheme, i.e., APBI, to adjust each subproblem's penalty value during the search. Therefore, diversity and convergence that are determined by the penalty value can be easily balanced.

Experimental studies have been conducted on several irregular problems that have different POF characteristics. Empirical results have shown that the proposed APBI can approximate boundary regions which the original PBI cannot, and achieve generally better performance than the original PBI. Besides, APBI has also been compared with a recent MOEA/D variant, i.e., MOEA/D-STM, showing that improper setting the penalty value in PBI can significantly deteriorate the performance of STM, and our APBI is more suitable than STM for the problems considered in this paper.

This paper have presented a generic idea to adaptively specify penalty values for different subproblems. However, APBI requires a good knowledge of the whole landscape of the POF, which means the performance of APBI depends largely on the judgement of subproblems' landscape. Our

future work will focus on the improvement of the judgement condition of landscape similarity. Besides, there is also a need to enhance the adaptive update of the penalty value so as to make a more accurate update. It will be interesting to extend our experiments on high-dimensional and other kinds of problems.

REFERENCES

- [1] M. Asafuddoula, T. Ray, and R. Sarker, "A decomposition based evolutionary algorithm for many objective optimization," *IEEE Transactions on Evolutionary Computation*, vol. 19, no. 3, pp. 445–460, 2015.
- [2] K. Deb, *Multi-Objective Optimization Using Evolutionary Algorithms*. Chichester: John Wiley and Sons, 2001.
- [3] K. Deb and M. Goyal, "A combined genetic adaptive search (GeneAS) for engineering design," *Comput. Sci. and Inform.*, vol. 26, pp. 30–45, 1996.
- [4] K. Deb and H. Jain, "An evolutionary many-Objective optimization algorithm using reference-point based non-dominated sorting approach, Part I: Solving problems with box constraints," *IEEE Transactions on Evolutionary Computation*, vol. 18, no. 4, pp. 577–601, 2014.
- [5] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, "Scable test problems for evolutionary multi-objective optimization," in *Evolutionary Multiobjective Optimization*. London: Springer, 2005, pp. 105–145.
- [6] C. Goh and K. C. Tan, "An investigation on noisy environments in evolutionary multiobjective optimization," *IEEE Transactions on Evolutionary Computation*, vol. 11, no. 3, pp. 354–381, 2007.
- [7] R. H. Gomez and C. A. Coello Coello, "Improved metaheuristic based on the r2 indicator for many-objective optimization," in *Proceedings of the 2015 Annual Conference on Genetic and Evolutionary Computation (GECCO 15)*, 2015, pp. 679–686.
- [8] H. Ishibuchi, N. Akedo, and Y. Nojima, "Behavior of multi-objective evolutionary algorithms on many-objective knapsack problems," *IEEE Transactions on Evolutionary Computation*, vol. 19, no. 2, pp. 264–283, Apr. 2015.
- [9] S. Jiang and S. Yang, "An improved multi-objective optimization evolutionary algorithm based on decomposition for complex Pareto fronts," *IEEE Transactions on Cybernetic*, 2015, in Press. DOI: 10.1109/TCYB.2015.2403131.
- [10] K. Li, Q. Zhang, S. Kwong, M. Li, and R. Wang, "Stable matching-based selection in evolutionary multiobjective optimization," *IEEE Transactions on Evolutionary Computation*, vol. 18, no. 6, 2014.
- [11] K. Li, K. Deb, Q. Zhang, and S. Kwong, "An evolutionary many-objective optimization algorithm based on dominance and decom-

Fig. 4. Approximated POFs with the lowest IGD values among 30 runs on F1-F6.

- position," *IEEE Transactions on Evolutionary Computation*, 2015, in Press. DOI: 10.1109/TEVC.2014.2373386.
- [12] H. Liu, F. Gu, and Q. Zhang, "Decomposition of a multiobjective optimization problem into a number of simple multiobjective sub-problems," *IEEE Transactions on Evolutionary Computation*, vol. 18, no. 3, pp. 450–455, Jun. 2014.
 - [13] H. Li, Q. Zhang, "Multiobjective optimization problems with complicated pareto sets, MOEA/D and NSGA-II," *IEEE Transactions on Evolutionary Computation*, vol. 13, no. 2, pp. 284–302, 2009.
 - [14] T. Qi, X. Ma, F. Liu, L. Jiao, J. Sun, and J. Wu, "MOEA/D with adaptive weight adjustment," *Evolutionary Computation*, vol. 22, no. 2, pp. 231–264, 2014.
 - [15] H. Sato, "Inverted PBI in MOEA/D and its impact on the search performance on multi and many-objective optimization," in *Proceedings of 2014 Annual Conference on Genetic and Evolutionary Computation (GECCO 14)*, 2014, pp. 645–652.
 - [16] F. Wilcoxon, "Individual comparisons by ranking methods," *Biometrics*, vol. 1, no. 6, pp. 80–83, 1945.
 - [17] Y. Yuan, H. Xu, B. Wang, and X. Yao, "A new dominance relation based evolutionary algorithm for many-objective optimization," *IEEE Transactions on Evolutionary Computation*, 2015, in press. DOI: 10.1109/TEVC.2015.2420112.
 - [18] Q. Zhang and H. Li, "MOEA/D: A multiobjective evolutionary algorithm based on decomposition," *IEEE Transactions on Evolutionary Computation*, vol. 11, no. 6, pp. 712–731, 2007.
 - [19] E. Zitzler, K. Deb, and L. Thiele, "Comparison of multiobjective evolutionary algorithms: empirical results," *Evolutionary Computation*, vol. 8, no. 2, pp. 173–195, 2000.
 - [20] E. Zitzler, M. Laumanns, and L. Thiele, "SPEA2: Improving the strength Pareto evolutionary algorithm for multiobjective optimization," in *Proceedings of Evolutionary Methods for Design, Optimisation and Control with Application to Industrial Problems (EUROGEN 2001)*, vol. 3242, no. 103, 2002, pp. 95–100.
 - [21] E. Zitzler and L. Thiele, "Multiobjective evolutionary algorithms: A comparative case study and the strength pareto approach," *IEEE Transactions on Evolutionary Computation*, vol. 3, no. 4, pp. 257–271, Nov. 1999.
 - [22] E. Zitzler, L. Thiele, M. Laumanns, C.M. Fonseca, and V. G. da Fonseca, "Performance assessment of multiobjective optimizers: An analysis and review," *IEEE Transactions on Evolutionary Computation*, vol. 7, no. 2, pp. 117–132, 2003.